Measuring β* Directly

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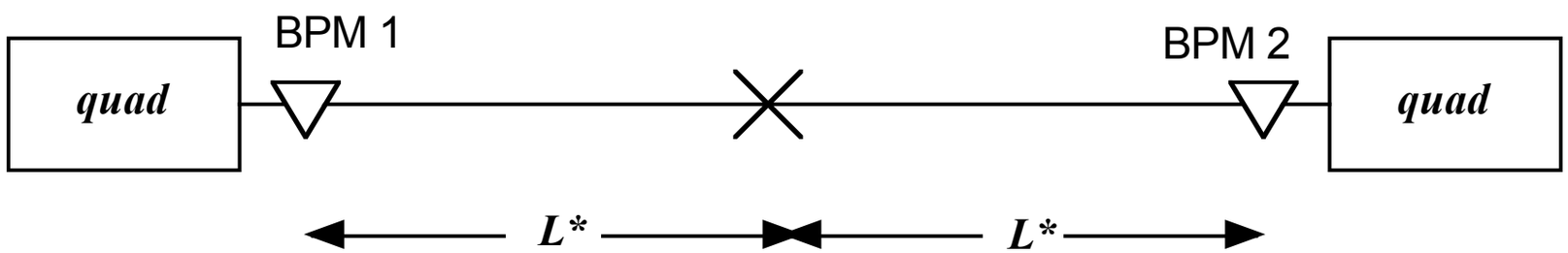
FNAL/AD

Lumi Mtg 7/20/05

Monitors at the IP's



Generate a coherent betatron oscillation, and measure $x_1^{(i)}$ and $x_2^{(i)}$, "turn-by-turn," for turn numbers $i = 0 \ldots N$.

on each turn,

$$x' = \frac{x_2 - x_1}{2L^*}$$

Then, from this, can compute β, α at each monitor...

Single Particle Dynamics

- Hill's Equation: $$x'' + K(s)x = 0$$

$$\boxed{K(s) = \frac{e}{p}\frac{\partial B_y}{\partial x}}$$

- Solution: $$x(s) = A\sqrt{\beta(s)}\,\cos[\psi(s) + \delta]$$

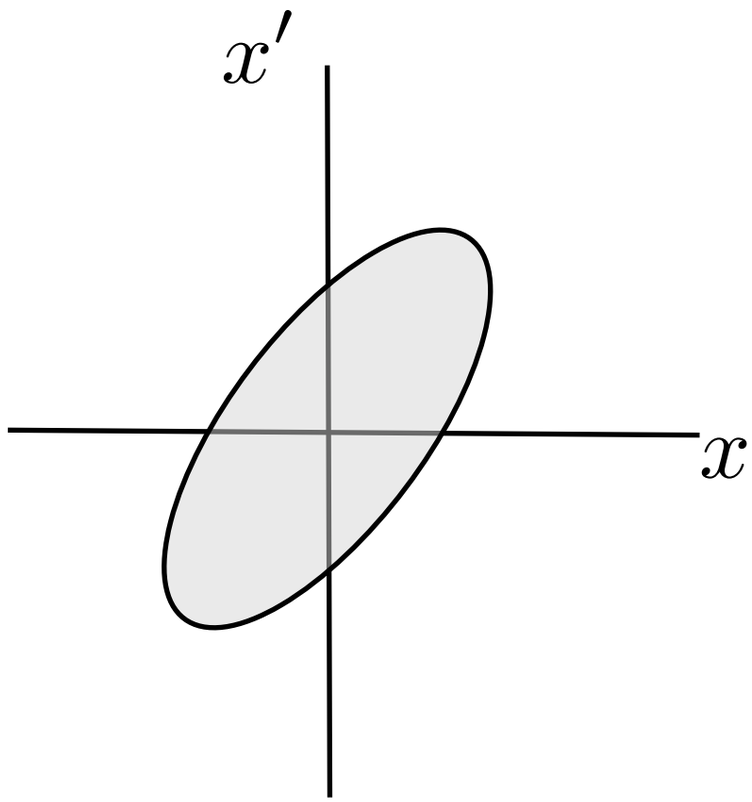
$$\boxed{{}' = \frac{d}{ds}}$$

$$\longrightarrow \psi' = 1/\beta, \quad K\beta = \gamma + \alpha'$$

$$\text{where} \quad \alpha = -\frac{1}{2}\beta', \quad \gamma = \frac{1+\alpha^2}{\beta}$$

α, β, γ are the Courant-Snyder parameters

Phase Space



$$\gamma x^2 + 2\alpha x x' + \beta x'^2 = A^2$$

Area of ellipse:
$$\epsilon = \pi A^2$$

Amplitude of motion:
$$a = A\sqrt{\beta}$$

Moments of Single Particle Motion

from betatron oscillation

- Position:

$$
\begin{aligned}
x &= A\sqrt{\beta}\cos\psi \\
x^2 &= A^2\beta\cos^2\psi \\
\langle x^2\rangle &= A^2\beta\langle\cos^2\psi\rangle \\
&= \frac{1}{2}A^2\beta
\end{aligned}
$$

Slope:

$$
\begin{aligned}
x &= A\sqrt{\beta}\cos\psi \\
x' &= -\frac{A}{\sqrt{\beta}}(\alpha\cos\psi+\sin\psi) \\
\langle x'^2\rangle &= \frac{1}{2}A^2\left(\frac{\alpha^2+1}{\beta}\right) \\
&= \frac{1}{2}A^2\gamma
\end{aligned}
$$

Averages are taken over time, or phase of oscillation

Correlation Moment

$$
\begin{aligned}
xx' &= [A\sqrt{\beta}\cos\psi]\left[-\frac{A}{\sqrt{\beta}}(\alpha\cos\psi+\sin\psi)\right] \\
&= -A^2(\alpha\cos^2\psi+\cos\psi\sin\psi)
\end{aligned}
$$

$$
\begin{aligned}
\langle xx'^2\rangle &= -A^2\alpha\langle\cos^2\psi\rangle \\
&= -\frac{1}{2}A^2\alpha
\end{aligned}
$$

- So, if could measure turn-by-turn position and slope of a single particle at a particular location, and compute the above moments from the data, could in principle determine Courant-Snyder parameters at that location

What about *A*? -- "Single Particle" Emittance

$$\langle x^2\rangle\langle x'^2\rangle - \langle xx'\rangle^2 = A^4(\beta\gamma - \alpha^2)/4 = \left(\frac{A^2}{2}\right)^2$$

or,

$$\frac{1}{2}A^2 = \sqrt{\langle x^2\rangle\langle x'^2\rangle - \langle xx'\rangle^2}$$

Thus, can substitute above expression for A^2 on previous pages to obtain ...

So, from turn-by-turn measurements, ...

$$\beta = \frac{\langle x^2 \rangle}{\sqrt{\langle x^2 \rangle \langle x'^2 \rangle - \langle x x' \rangle^2}}$$

$$\gamma = \frac{\langle x'^2 \rangle}{\sqrt{\langle x^2 \rangle \langle x'^2 \rangle - \langle x x' \rangle^2}}$$

$$\alpha = -\frac{\langle x x' \rangle}{\sqrt{\langle x^2 \rangle \langle x'^2 \rangle - \langle x x' \rangle^2}}$$

- In terms of the positions measured at the two BPM's,

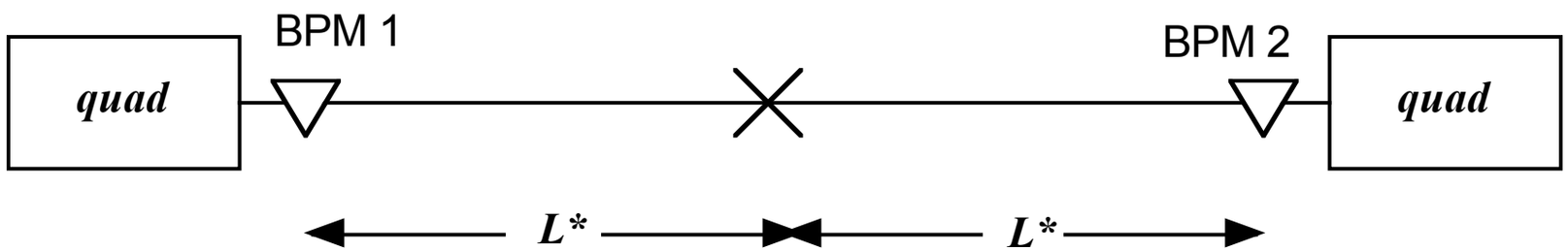
$$\beta_1 = \frac{2\langle x_1^2\rangle L^*}{\sqrt{\langle x_1^2\rangle\langle x_2^2\rangle - \langle x_1 x_2\rangle^2}} \qquad \beta_2 = \frac{2\langle x_2^2\rangle L^*}{\sqrt{\langle x_1^2\rangle\langle x_2^2\rangle - \langle x_1 x_2\rangle^2}}$$

$$\alpha_1 = \frac{\langle x_1^2\rangle - \langle x_1 x_2\rangle}{\sqrt{\langle x_1^2\rangle\langle x_2^2\rangle - \langle x_1 x_2\rangle^2}} \qquad \alpha_2 = -\frac{\langle x_2^2\rangle - \langle x_1 x_2\rangle}{\sqrt{\langle x_1^2\rangle\langle x_2^2\rangle - \langle x_1 x_2\rangle^2}}$$

Thus, take many turns-worth of data, and compute simple sums to determine values at the BPMs

The new Tevatron BPM electronics now allow us to do just that!

Computing β^*...



- Since we have a drift space between BPM 1 and BPM 2,
- $$K\beta = \gamma + \alpha' = 0 \qquad \longrightarrow \qquad \beta''' = 0$$
- and so the amplitude function is parabolic, the parameter γ is constant through the region, and we can write

$$\beta(s) = \beta_1 - 2\alpha_1 s + \gamma s^2$$

$$\alpha(s) = \alpha_1 - \gamma s$$

- The minimum value of the amplitude function and its distance from the center of the straight section are

$$\beta^* = \frac{2L^*}{\alpha_1 - \alpha_2} \qquad at \qquad \Delta s^* = \frac{\alpha_1 + \alpha_2}{\alpha_1 - \alpha_2} L^*$$

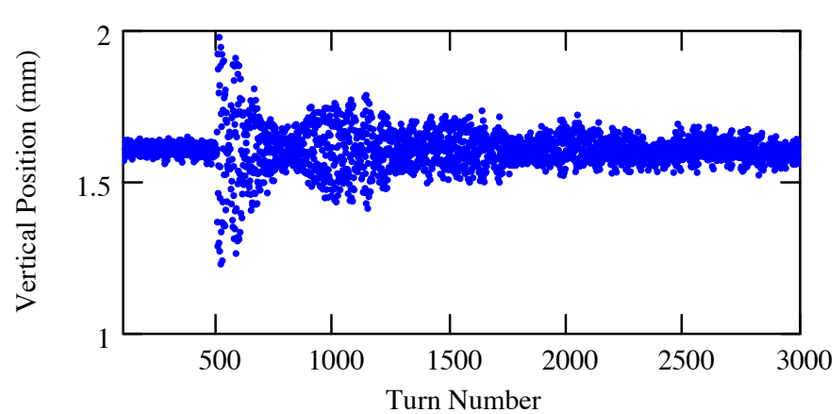
Above analysis assumes uncoupled motion between horiz/vert; in fact, this is not strictly the case, however OK if coupling is small;

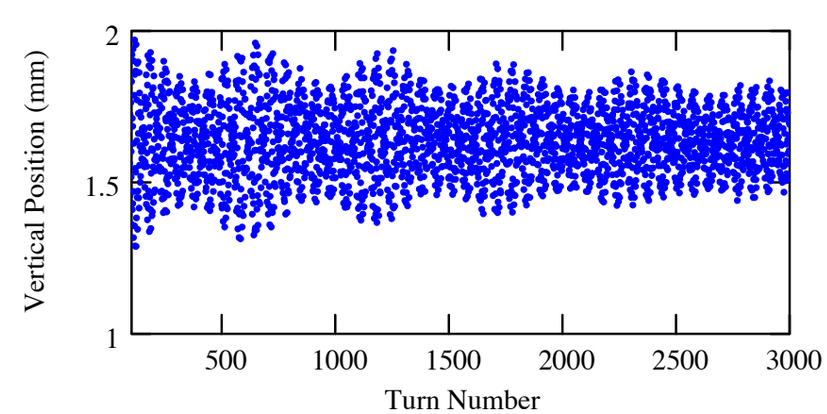
But since we have H/V measurements simultaneously, can untangle coupled motion; will pursue in future...

Kicked Beams and Decoherence

- To generate betatron oscillation of the beam, use fast 1-turn kicker magnet
- Particles in beam start out with coherent motion, but decohere due to non-linear fields, and due to chromaticity (osc. freq. *vs.* momentum)
- "signal" of a kicked, Gaussian beam:

large energy spread

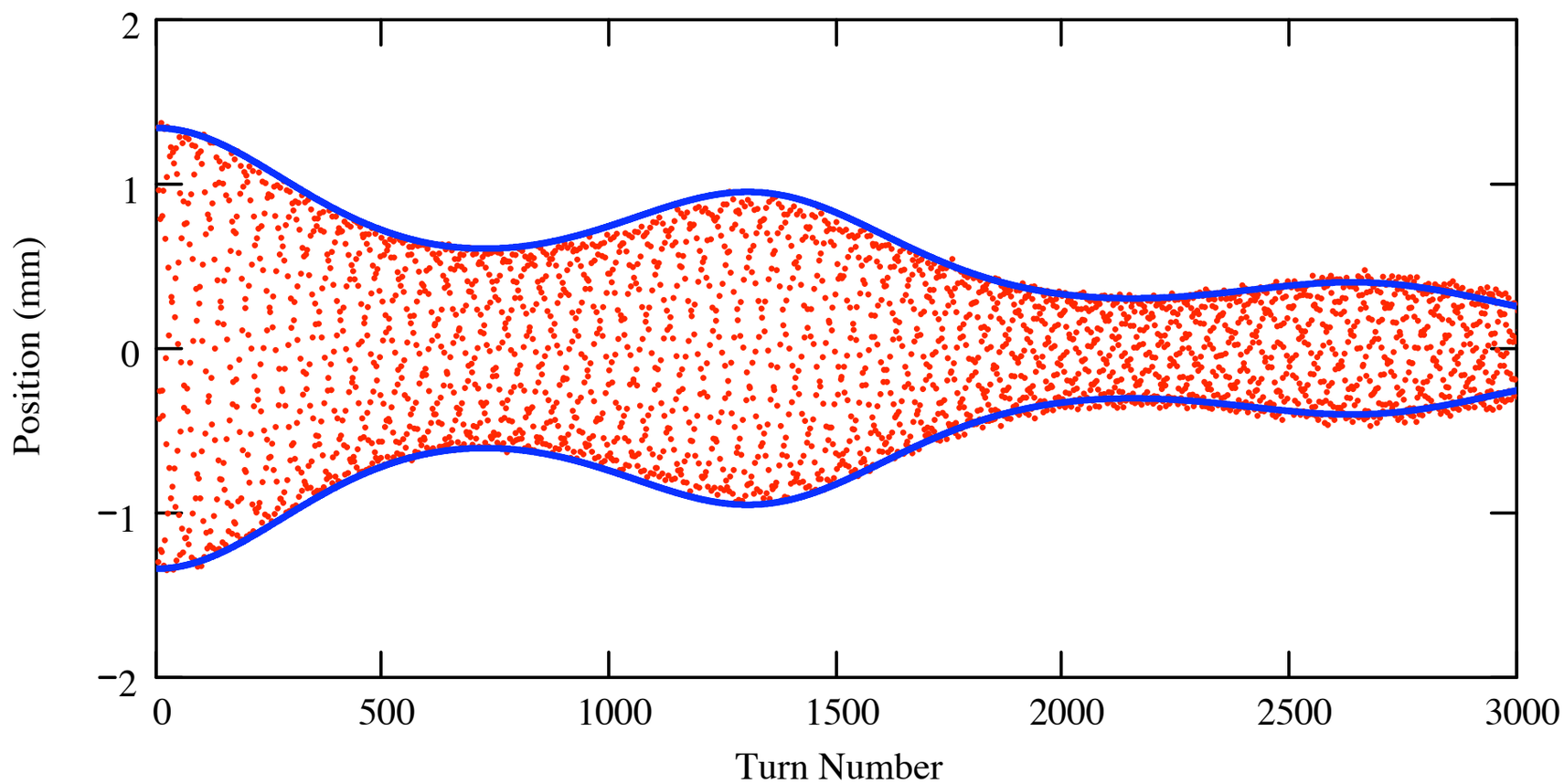




smaller energy spread

- Recorded position signal:

$$\overline{x}(n) = a \cdot e^{-[2\sigma_s \xi \nu_s^{-1} \sin(\pi \nu_s n)]^2/2} \cdot \left(\frac{1}{1+(\nu_p n)^2} \quad e^{-\frac{a^2}{2\sigma^2} \frac{(\nu_p n)^2}{1+(\nu_p n)^2}} \right) \cdot \cos[2\pi\nu_0 n + \Delta\overline{\phi}(n)]$$



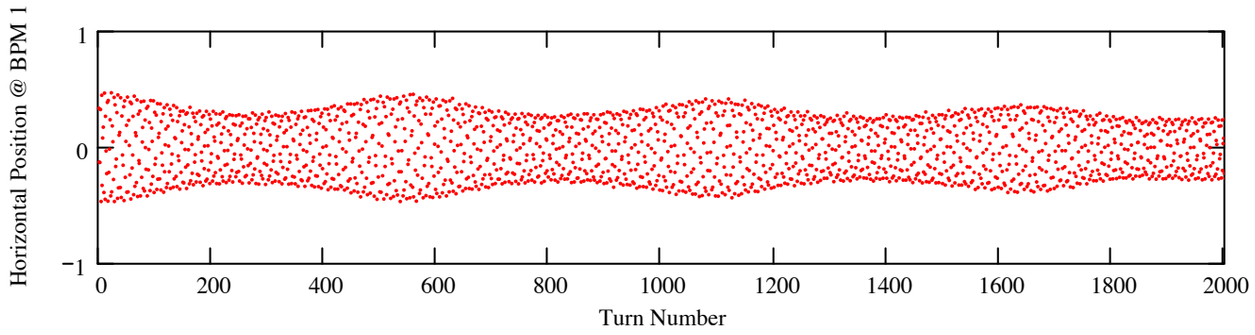
Parameters: $\xi \cdot \sigma_p/p = 0.00043$, $\nu_s = 1/1370$, $a = 1.34$ mm, $\nu_p = 0.00018$

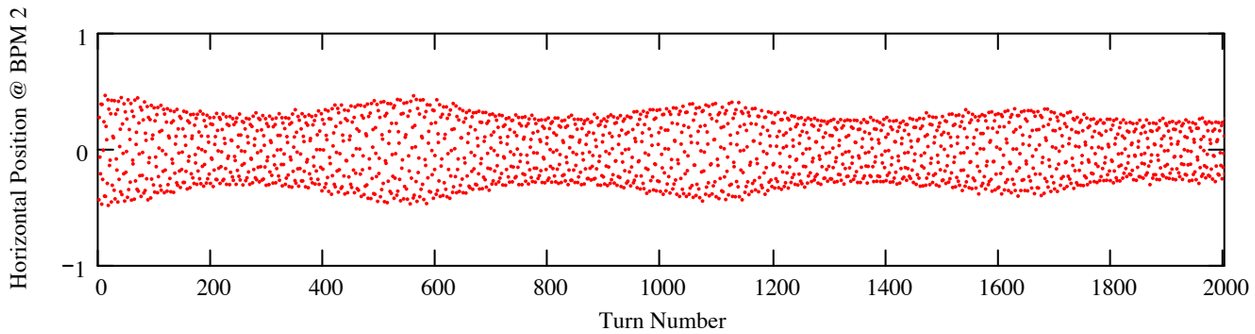
First Analyses

- Analyzed T-by-T measurements taken on three occasions (data obtained during studies periods for other purposes)...
  - 150 GeV, on May 2, 2005
    - uncoalesced beam, injection optics
  - 980 GeV, on May 31, 2005
    - coalesced beam, low-beta optics
  - 980 GeV, on June 23, 2005
    - uncoalesced beam, low-beta optics

Thanks!:
Yuri, Eliana, Vahid

150 GeV Measurement

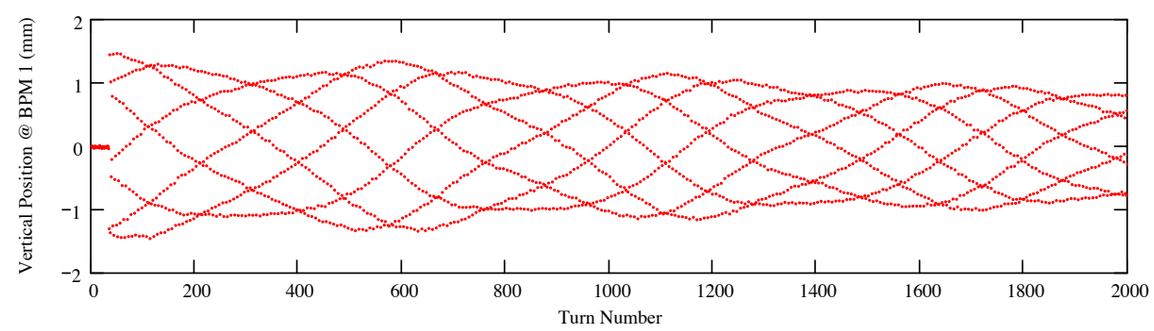


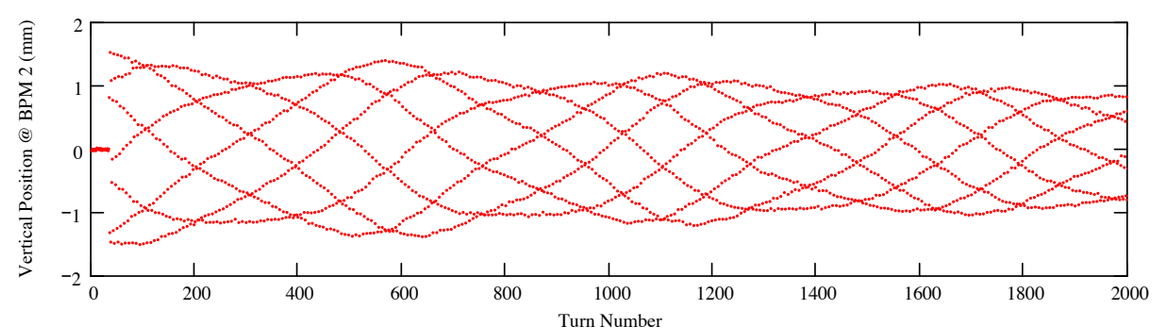


H

Here, vert. tune was very near 4/7

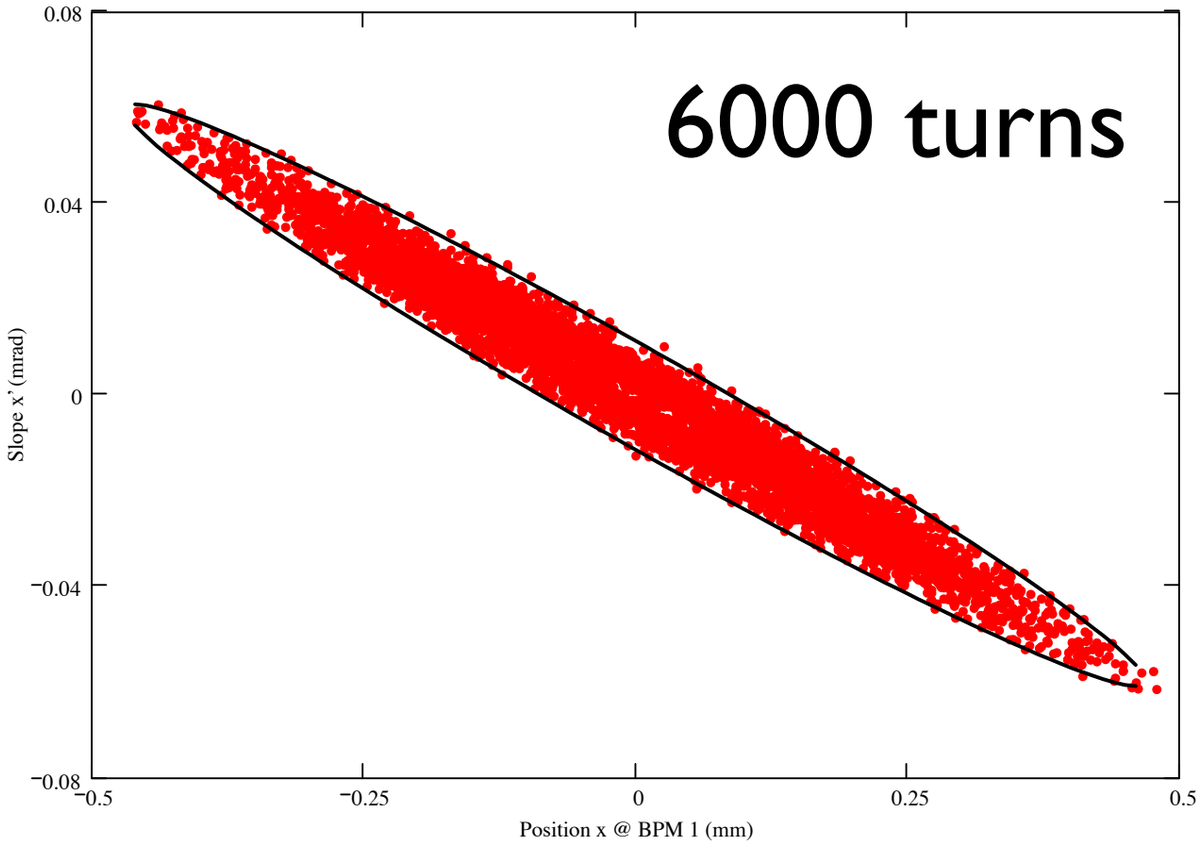
V



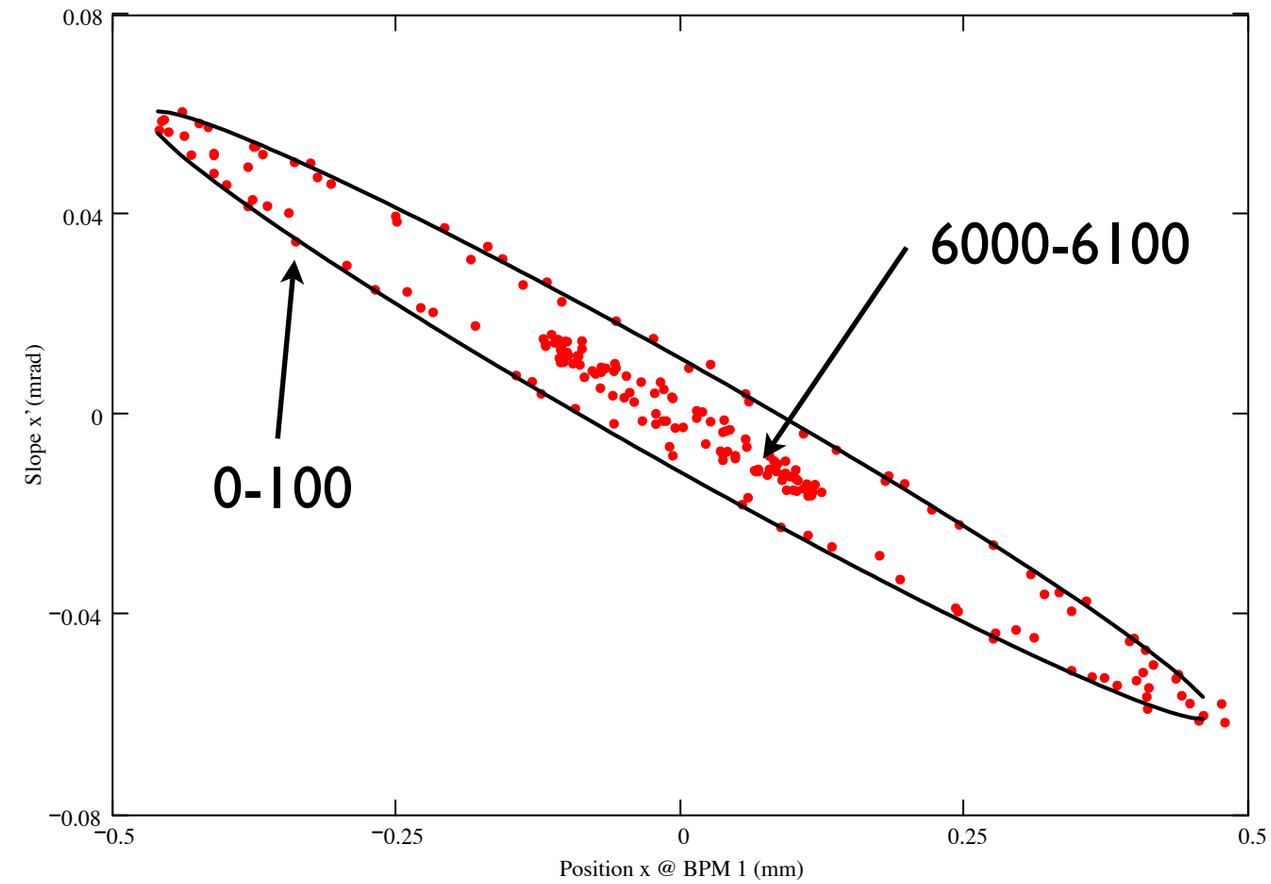


CDF (D0 similar)

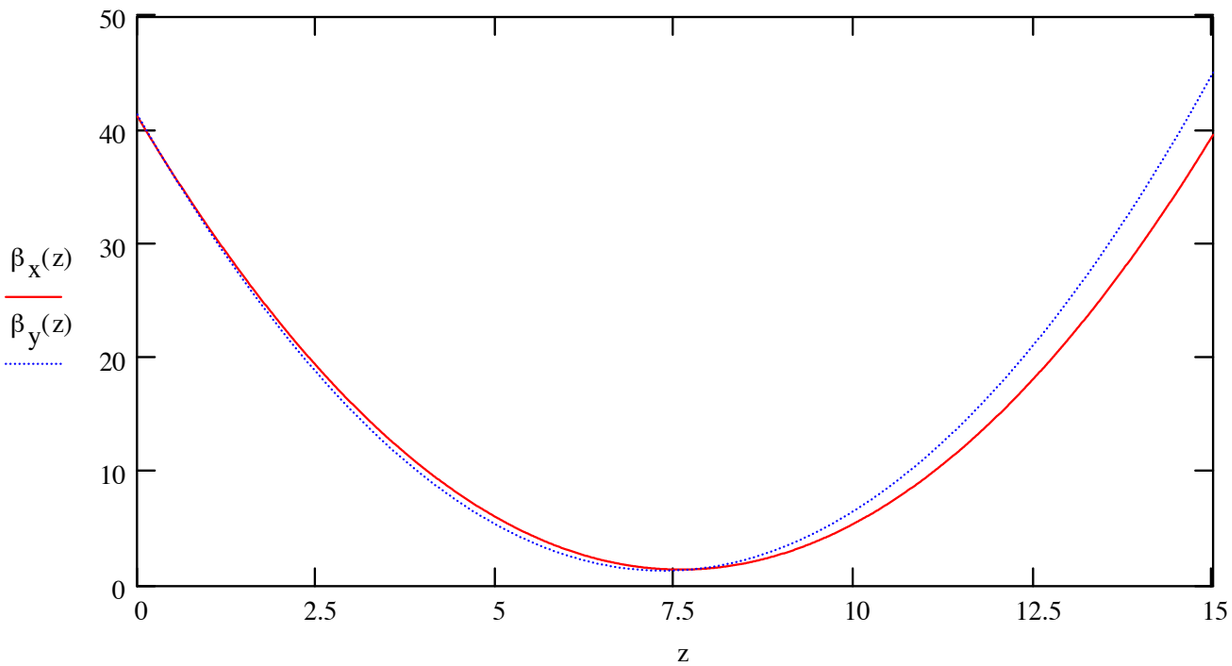
Phase Space



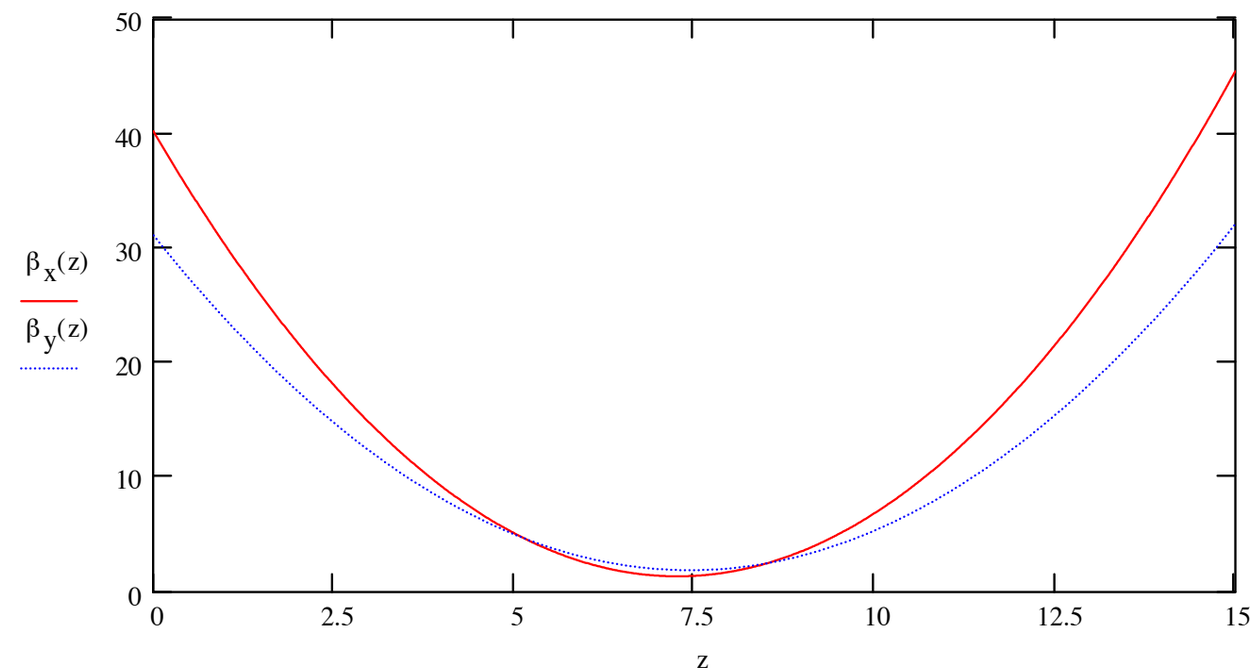
Elllipse drawn is derived from the analysis, using turns 0 - 3000



Amplitude Functions



CDF



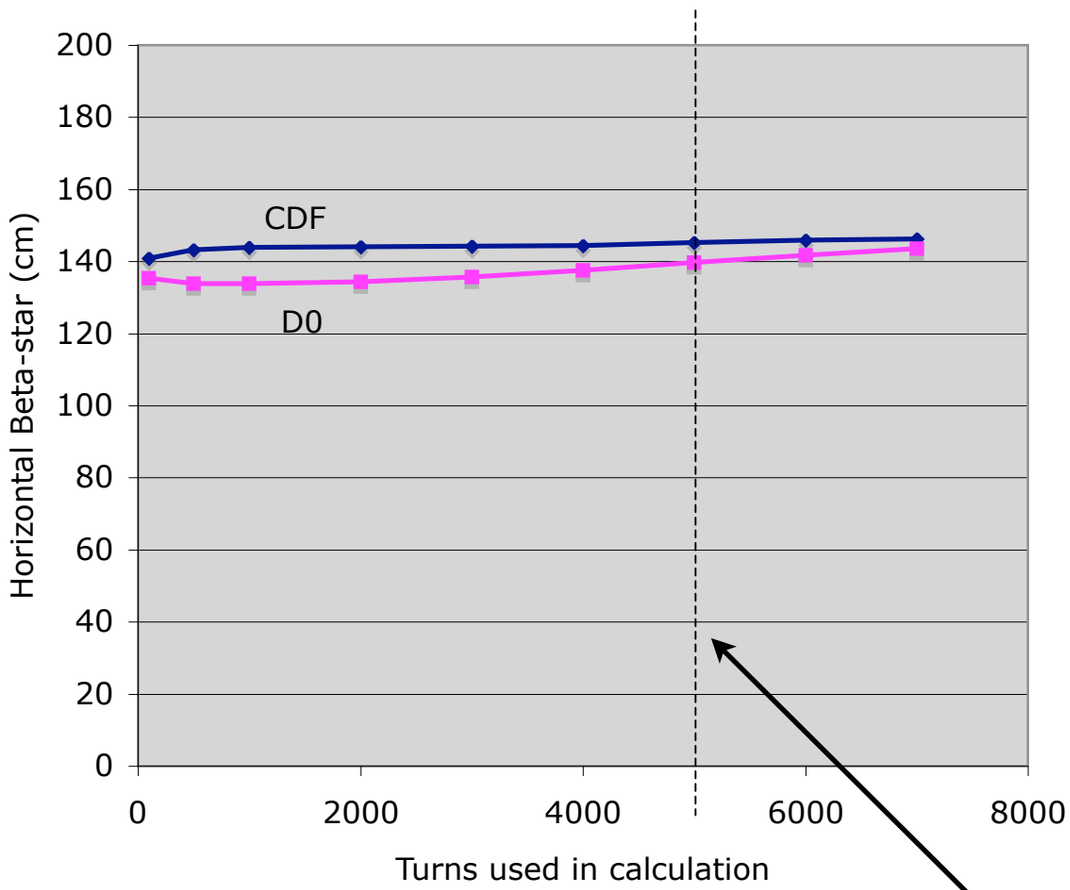
D0

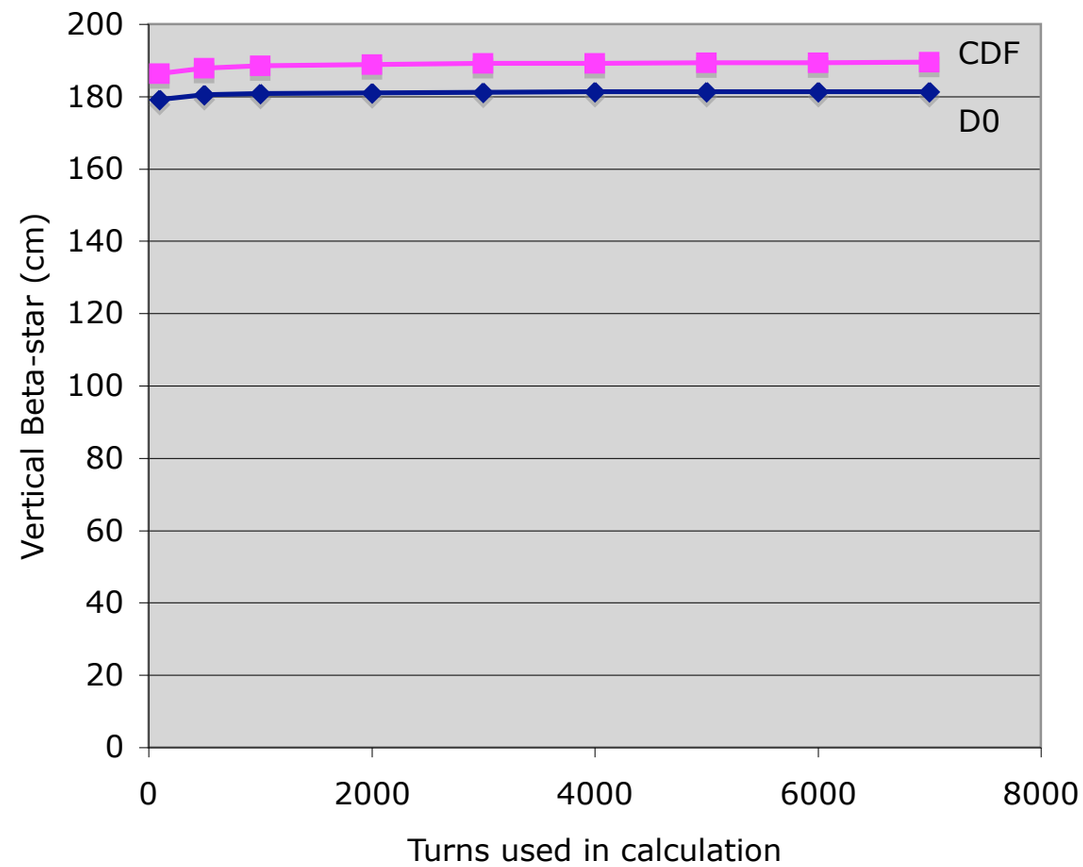
Table 1: Amplitude Functions at Injection

		β_1	β_2	β^*	Δs^*
CDF	Hor	41.2 m	40.1 m	144 cm	+5.49 cm
	Ver	31.0 m	33.2 m	181 cm	-13.4 cm
D0	Hor	40.2 m	46.0 m	136 cm	-26.1 cm
	Ver	31.2 m	30.6 m	189 cm	+3.56 cm

(150 GeV)

Result *vs.* Turn no.

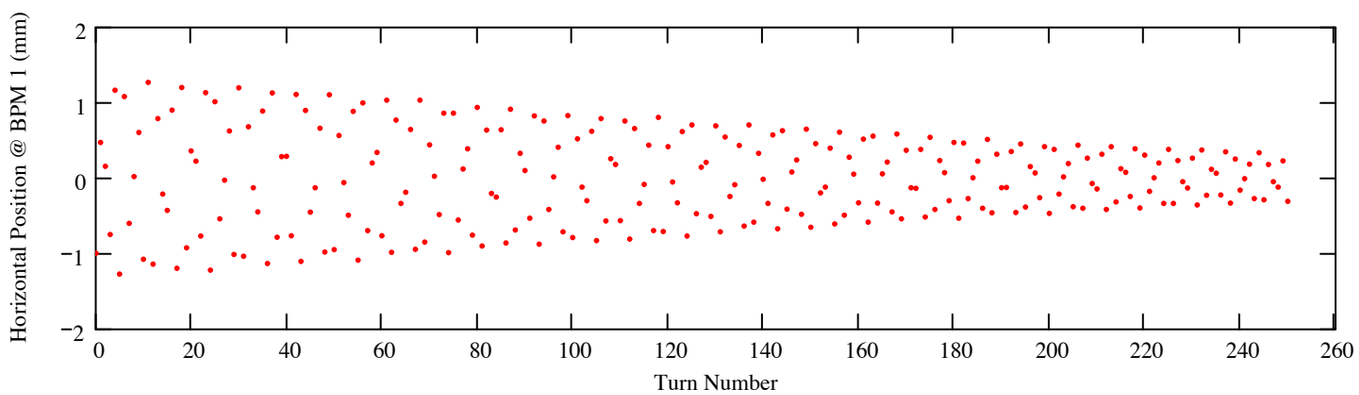




Decoherence time

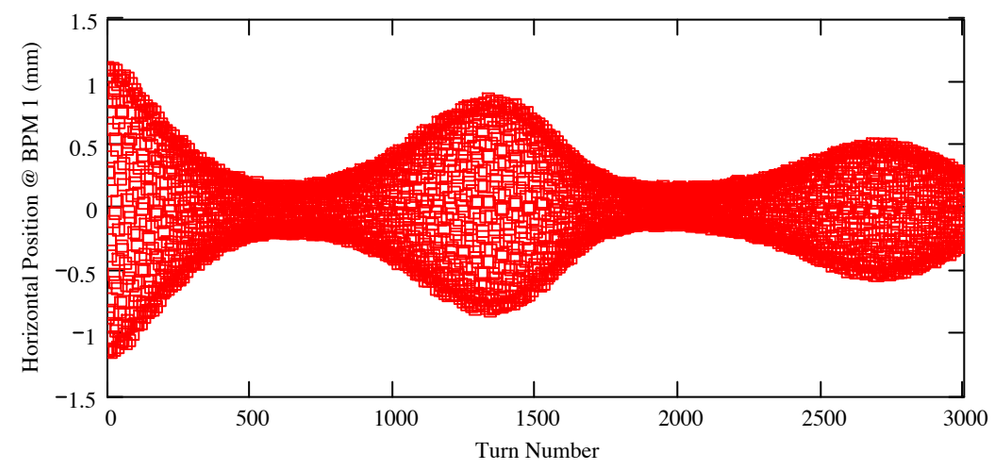
980 GeV Measurements

- Coalesced vs. Uncoalesced



Strong decoherence

(May 31)

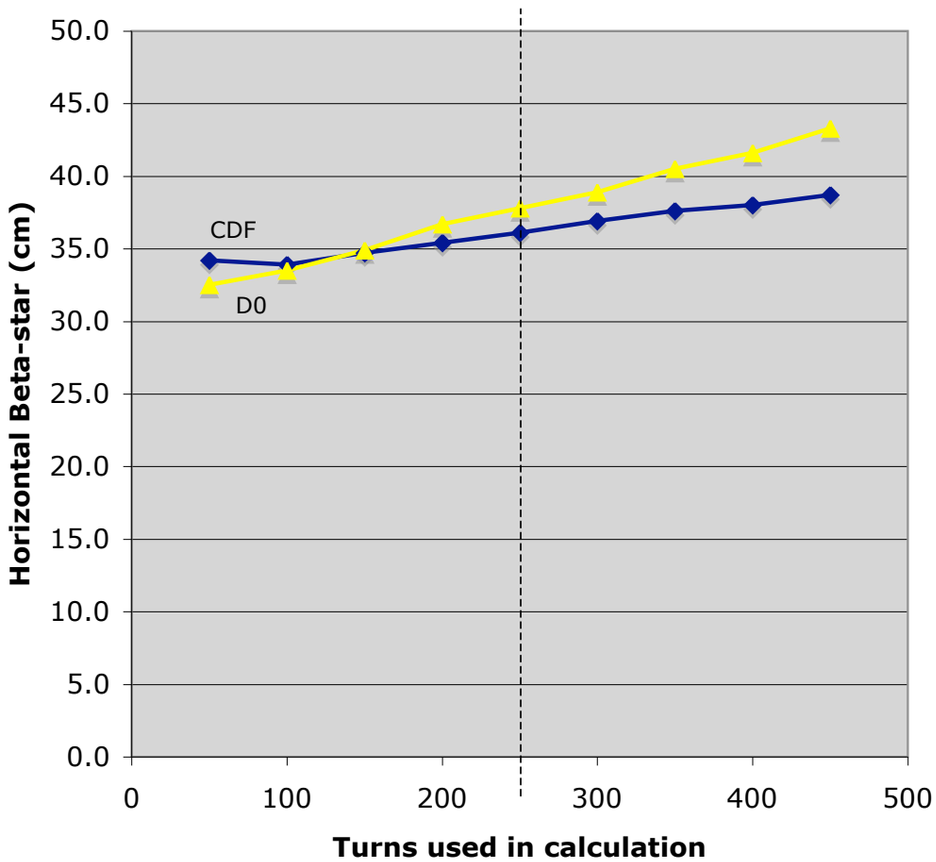


Much longer coherence;
here, 'large' chromaticity

(June 23)

Results for May 31, 2005

(coalesced)



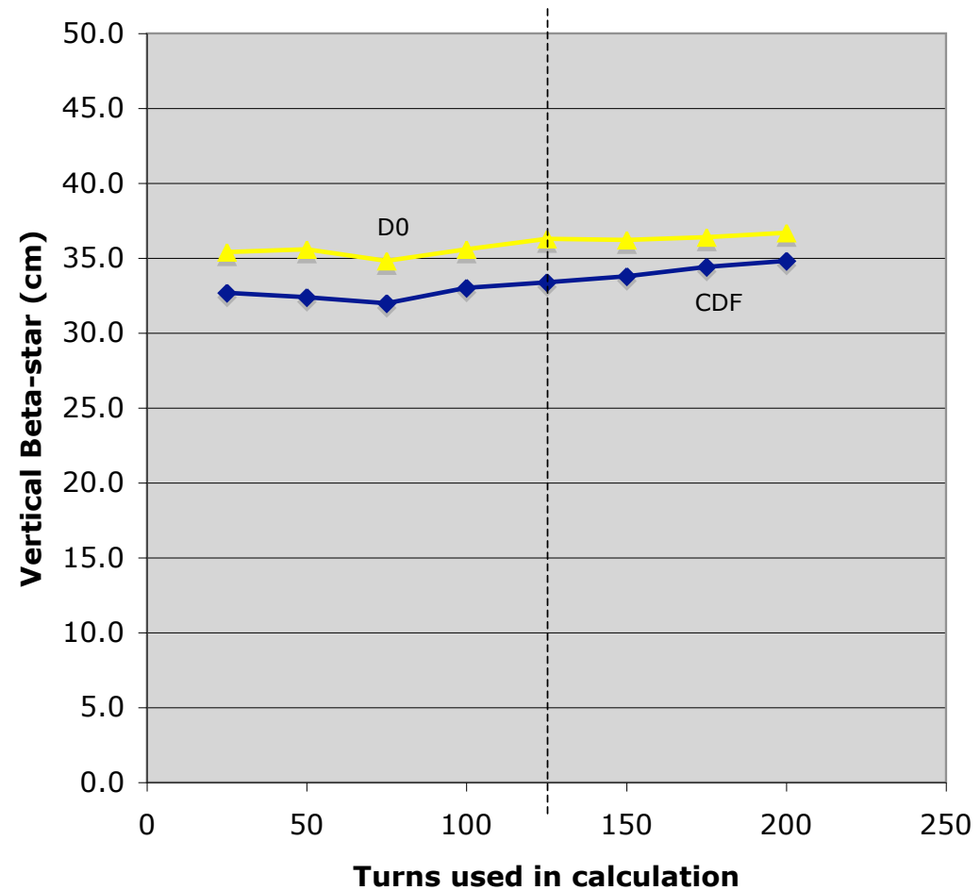


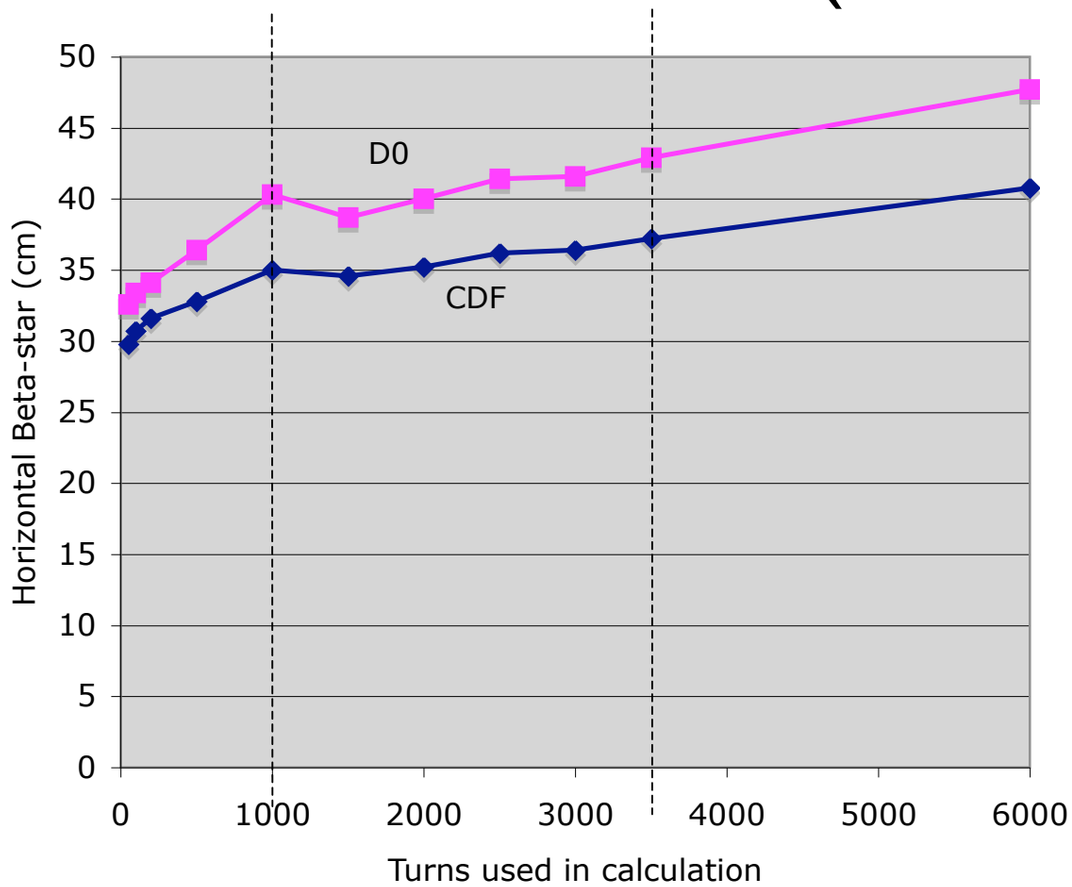
Table 2: 980 GeV, 250 turns for horizontal, 125 turns for vertical.

		β_1	β_2	β^*	$\delta\beta^*$	Δs^*	$\delta\Delta s^*$
CDF	Hor	173 m	141 m	35 cm	± 1 cm	$+40$ cm†	± 1 cm
	Ver	164 m	165 m	33 cm	± 1 cm	-2.4 cm	± 1 cm
D0	Hor	151 m	149 m	35 cm	± 3 cm	$+1.6$ cm	± 1 cm
	Ver	153 m	150 m	36 cm	± 1 cm	$+3.5$ cm	± 1 cm

† *more later*

Results for June 23, 2005

(uncoalesced)



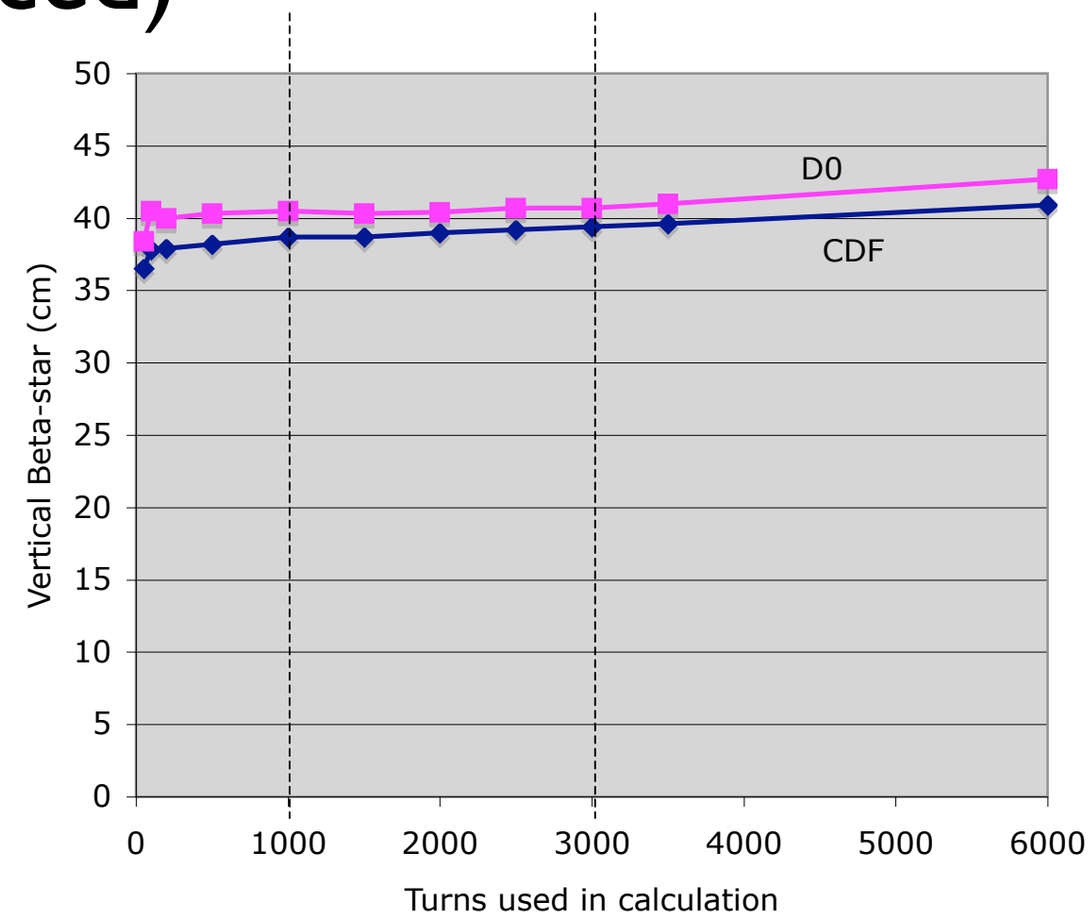
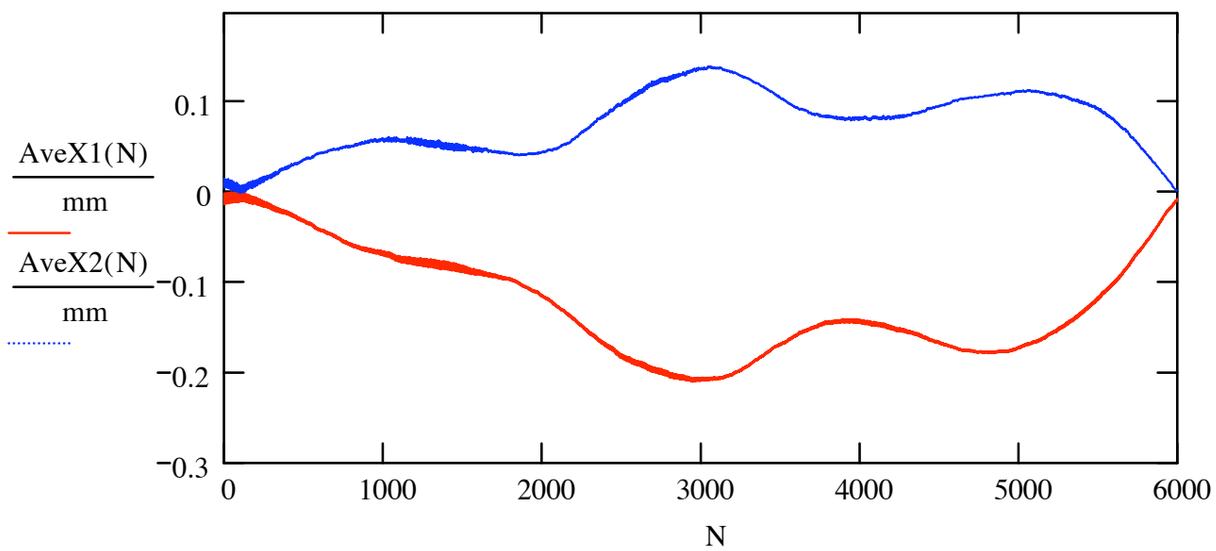


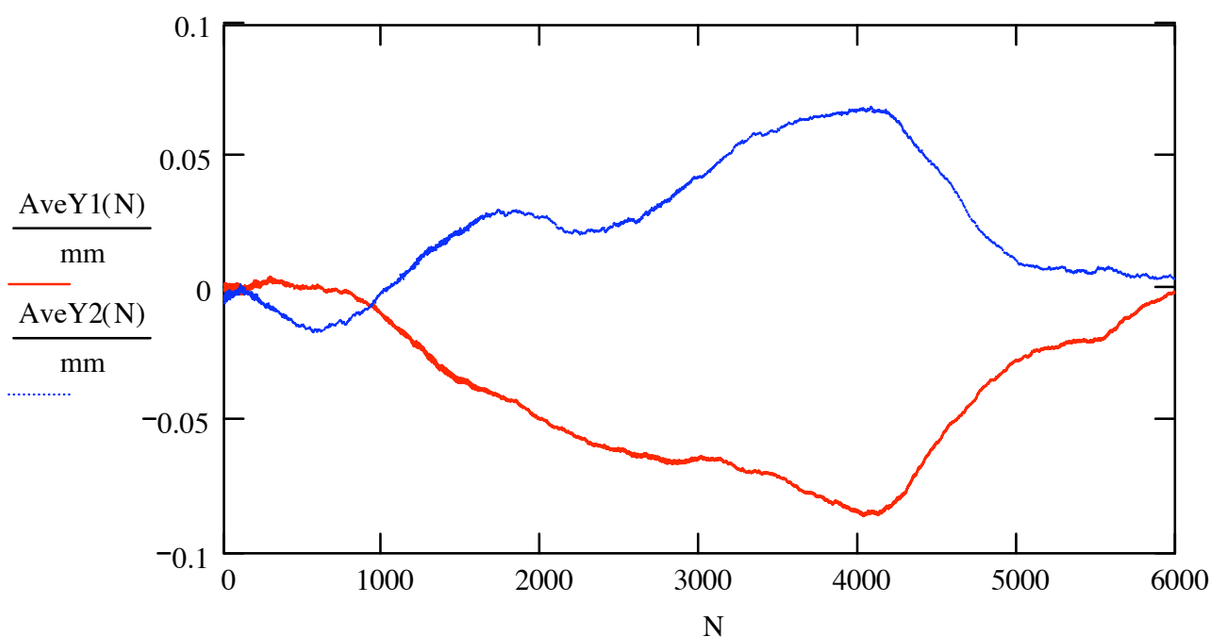
Table 3: Amplitude Functions at 980 GeV, 2005 June 23, using 1500 turns.

		β^*	$\delta\beta^*$	Δs^*	$\delta\Delta s^*$
CDF	Hor	34.6 cm	±0.6 cm	+34.2 cm†	±0.4 cm
	Ver	38.7 cm	±0.3 cm	-4.8 cm	±0.4 cm
D0	Hor	38.7 cm	±1.6 cm	+1.1 cm	±0.4 cm
	Ver	40.3 cm	±0.2 cm	+2.3 cm	±0.2 cm

† *more later*

Orbit Motion during Measurement





Here, each point represents a 100-turn running average, showing motion of the closed orbit during the measurement on the scale of 0.1-0.2 mm; more pronounced in Horizontal

D0 BPM's; similar at CDF; June 23 data

Effect of BPM Scaling

- BPM 'measurement' reports: $x = 26 \cdot \frac{A - B}{A + B} [\mathrm{mm}]$
- *A*, *B*, voltages on plates; response becomes non-linear for large offsets
- CDF results above show large horizontal displacement of waist
  - look at average offsets at detectors:
    - CDF: <x1> = 0.1 mm <x2> = 8 mm; <y1> = -1 <y2> = 1.6 mm
    - D0: <x1> = -0.15 mm <x2> = 2.5 mm; <y1> = -2 <y2> =0.6 mm

Sensitivity of Results on BPM scaling

- Assume BPM1 reads correctly, but BPM2 scale is off by scale factor $r = 1 - \delta$
- Then, would find ...

$$d\Delta s^* \approx \frac{L^*}{2} \cdot \delta$$

$$\frac{d\beta^*}{\beta^*} \approx -\frac{1}{4}\delta^2$$

So, if downstream BPM at CDF reading 10% too low, then would generate misinterpretation of waist location on order of (7.5 m/2)(0.1) = 38 cm, while β^* value would be off by only 0.25%

(consistent with 8 mm offset)

Solenoid Field

- Do the detector solenoid fields affect the measurement?

$$\begin{pmatrix} x_2 \\ x_2' \\ y_2 \\ y_2' \end{pmatrix} = \begin{pmatrix} 1 & \ell & \frac{1}{2}\theta & -\frac{1}{2}\theta\ell \\ 0 & 1 & 0 & \frac{1}{2}\theta \\ -\frac{1}{2}\theta & \frac{1}{2}\theta\ell & 1 & \ell \\ 0 & -\frac{1}{2}\theta & 0 & 1 \end{pmatrix} \begin{pmatrix} x_1 \\ x_1' \\ y_1 \\ y_1' \end{pmatrix}$$

CDF solenoid: $\ell = 4.8$ m, $B_s = 1.5$ T
D0 solenoid: $\ell = 2.7$ m, $B_s = 2.0$ T
Characteristic angle, $\theta = B_s\ell/B\rho$,
$\rightarrow \theta = 11\text{-}14$ mrad at 150 GeV,
$\theta = 1.7\text{-}2.2$ mrad at 980 GeV
for D0 and CDF respectively.

$$\frac{\Delta x'}{\hat{x'}} = \frac{1}{2}\theta\left(\frac{\hat{y'}}{\hat{x'}}\right) = \frac{1}{2}(0.0022)\left(\frac{1}{5}\right) = 0.02\%.$$

Beam-Beam Interaction

- During these measurements, no beam-beam
- During store, b-b interaction acts (to first order) as a focusing lens, with focal length: $\frac{1}{f} = 4\pi\xi/\beta^*$

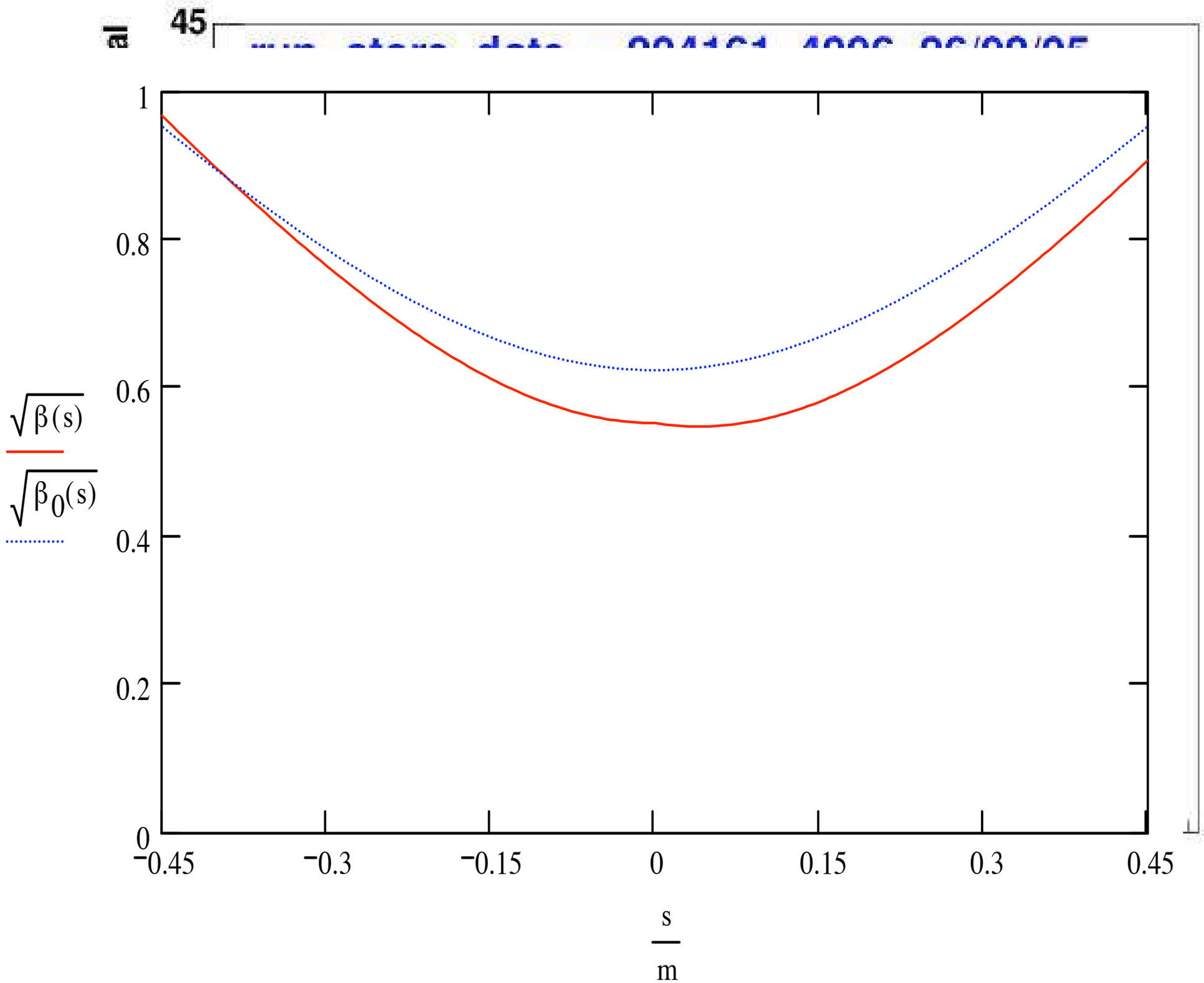
$$\xi = 3Nr_0/2\epsilon_N = \text{'beam-beam parameter'} \approx 0.01$$

Thus, distorts beta function:

$$\left|\frac{\Delta\beta}{\beta}\right| \approx \frac{\beta_0/f}{2|\sin 2\pi\nu|} = \frac{2\pi\xi}{|\sin 2\pi\nu|} \approx 10\%$$

This is 1st order estimate; slightly more complicated; however, points out that conditions are different during stores, as opposed to during beam studies, on this scale

Snap Shot



nominal $\beta_0(0 \cdot cm) = 39\,cm$ $\beta(0 \cdot cm) = 30.593\,cm$

Summary

- Method is promising as potential on-line measurement of beta-star during studies periods
- Results depend upon "clean" beam, *i.e.*, low chromaticity, low momentum spread, irrational tunes, ...
- Direct measurement -- does not rely upon modeling, fitting, etc.
- BPM scaling is issue when large offsets of closed orbit
- Does not measure beta-star during true "collision conditions"
  - Perhaps could be performed eventually in a special study?
- Full coupled motion analysis forthcoming
- See ***Beams-doc-1880*** for more detailed discussion of these measurements