Proton Driver Design Study Report

Chapter 3: Optics

**A.Drozhdin, A.Garren, N.Gelfand,
C.Johnstone, L.Michelotti, S.Ohnuma, G.Rees, D.Ritson**

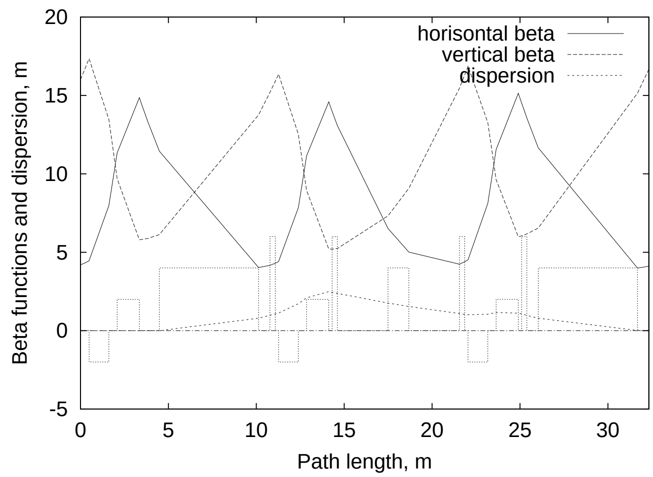
Constraints

Design of the PD2 lattice was constrained and influenced by a number of criteria, ranging from requirements to desiderata. These included:

- Length: 474.2 m
- Minimum spacing: quad-quad: 47 cm; dipole-quad: 85 cm
- RF: 7.05 m of space required for three cavities
- Superperiodicity: 2 or 3
- Horizontal phase advance / arc = $N \times 2\pi$
- Bused: all dipoles, same B; all quads, same $|B'|$
- Peak fields: dipoles: 1.5 T, quadrupoles: 10 T/m
- Avoid transition: with max $KE = 8$ GeV, $\gamma_t > 9.5$
- Physical aperture: $|x| < 3$ in and $|y| < 2$ in
- Dynamic aperture: $> 3 \times 40\pi$ mm-mr for $|\Delta p/p| \leq 0.01$
- Injection: $\beta_x \approx \beta_y \approx 10$ m at stripping foil
- Collimation: large β and D at primary collimators; $\Delta\psi > 180°$
- Time: Finish by April!

Arc Module

The arc module is the keystone of the PD2 design. Each arc comprises 5 modules of 3 cells. The phase advance across a module is $(\Delta\psi_x, \Delta\psi_y)|_{\text{module}} = (8\pi/5, 6\pi/5)$ which makes the average phase advance per cell $(\Delta\psi_x, \Delta\psi_y)|_{\text{cell}} = (8\pi/15, 2\pi/5) = (96^\circ, 72^\circ)$.



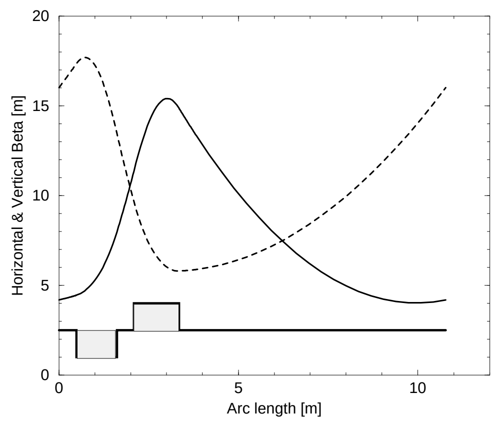
Total phase advance across an arc is $(\Delta\psi_x, \Delta\psi_y)|_{\text{arc}} = (8\pi, 6\pi)$; it is, to first order, optically transparent. Thus, the arcs will preserve lattice functions, including zero dispersion, across the straights.

The two outer cells of an arc module contain a large dipole (5.646 m, 16.2° bend) and the inner cell a small one (1.188 m, 3.4° bend). Their lengths were chosen so as to create a first order achromatic bend, thus zeroing the dispersion between modules.

Four chromaticity correcting sextupoles are placed in each arc module, replacing the four trim quadrupoles closest to the short dipole. It may be possible to build a correction package consisting of quadrupole and sextupole.

Straight section cells

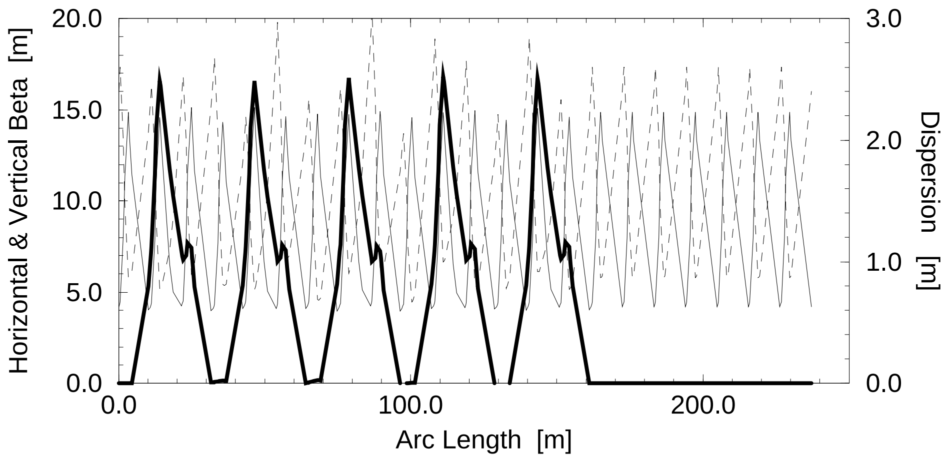
The seven cells in each straight section do not contain dipoles, and the absence of edge focusing distorts the lattice functions (esp., β_y) slightly.



Lattice functions for a single straight section cell, treated as a periodic unit, are shown above. Its phase advance is $(\,\Delta\psi_x,\,\Delta\psi_y\,)|_{\text{cell}} = (\,8\pi/15,\,0.96\cdot(2\pi/5)\,)$. In fact, these *are* its lattice functions in the base configuration, because of the arcs' optical transparency.

Complete Lattice Functions

The complete lattice functions for half of the racetrack, with one arc joined to one straight section, are shown below. The vertical beta wave is caused by edge focusing in the dipoles, or, alternatively, its absence in the straight section cells. The arcs' optical transparency



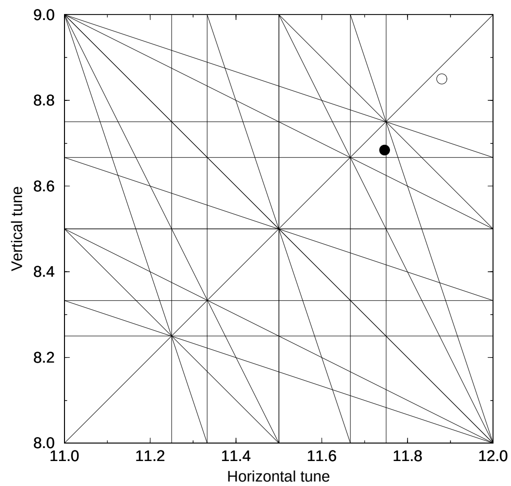
confines the wave; it does not propagate into the straight sections.

Attribute	Value
γ_t	13.8
max (β_x, β_y) [m]	(15.1, 20.3)
max D [m]	2.5
(ν_x, ν_y)	(11.747, 8.684)
$(\xi_x, \xi_y)_{\text{natural}}$	(-13.6, -11.9)

At injection, assuming $\epsilon_{\text{inv}} = 40\pi$ mm-mr, $|x| < 5$ cm, divided equally between dispersion and transverse emittance, and $|y| < 2.2$ cm.

Base and Tuned Configurations

The actual tunes associated with the base configuration are shifted from $(11.73, 8.80)$ to $(11.747, 8.684)$. That point is shown as a dark circle below. Families of trim quadrupoles in the straight section



were used to move the tunes to $(11.880, 8.850)$, without breaking superperiodicity.

Space charge will reduce the tunes of particles in the core of the beam by an amount that will depend on painting. Protons undergoing large amplitude oscillations will be less affected by space charge, but their tunes will increase (slightly) due to the presence of chromaticity correcting sextupoles (and octupole fields). The combined effects will spread the tunes away from the displayed points in opposite directions. As the beam's energy increases, the distribution will collapse into the working point. Space charge forces will decrease, as $v/c \rightarrow 1$, shrinking the distribution from below, and the sextupole/octupole tune spread will decrease, as emittances become smaller, shrinking the distribution from above.

Sextupole tune footprint

The sextupoles used to zero chromaticity will produce an amplitude dependent tune shift proportional to the square of their excitation. Second order perturbation theory predicts, for the PD2 base configuration,

$$\Delta\nu_x = 0.120\ \epsilon_x/\pi + 0.114\ \epsilon_y/\pi$$
$$\Delta\nu_y = 0.114\ \epsilon_x/\pi + 0.230\ \epsilon_y/\pi \quad ,$$

where $\Delta\nu$ is given in units of 10^{-3} and ϵ is in mm-mr.[1] For the tuned configuration, the coefficients are somewhat larger.

$$\Delta\nu_x = 0.126\,\epsilon_x/\pi + 0.397\,\epsilon_y/\pi$$
$$\Delta\nu_y = 0.397\,\epsilon_x/\pi + 0.384\,\epsilon_y/\pi$$

The upper limit on transverse emittance is $\epsilon_{\rm inv} \leq 40\pi$ mm-mr, so that

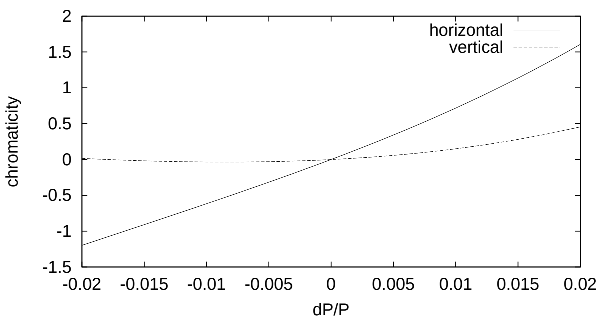
$$\epsilon/\pi = \frac{\epsilon_{\rm inv}/\pi}{\beta\gamma} \leq \frac{40}{9.47} = 4.22 \ \text{ mm-mr} \quad \text{at extraction}$$
$$\leq \frac{40}{1.30} = 30.8 \ \text{ mm-mr} \quad \text{at injection.}$$

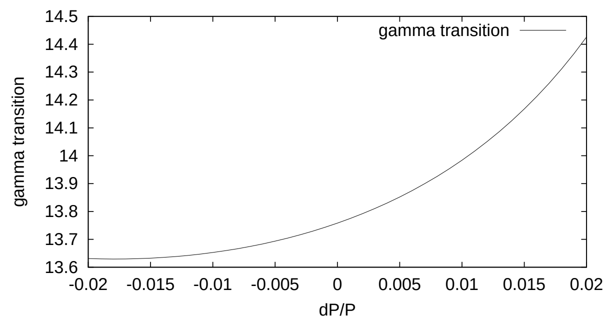
Even at injection into the tuned configuration, the vertical tune spread resulting from sextupole excitation will only be about 0.02. This can easily be overshadowed by space charge, which will lower the tunes of protons in the core.

[1] This is written using "emittance" notation, ϵ, but, since we are dealing with a single particle, ϵ is more properly interpreted as an action (or amplitude) coordinate, I, according to $\epsilon/\pi = 2I$.

Chromatic Properties

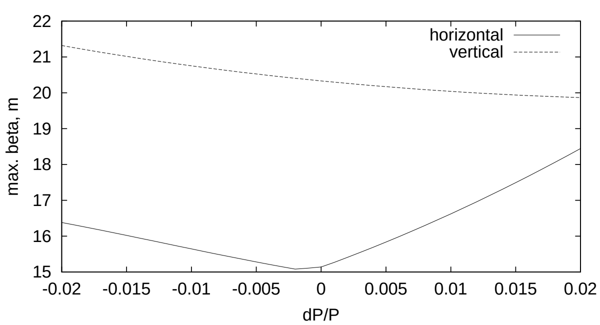
The chromaticities, ξ_x, ξ_y, are plotted below for the extended range $|\Delta p/p| \leq 0.02$. ξ_y is nearly flat for negative $\Delta p/p$, with a variation of less than 0.5 over the entire range. On the other hand, ξ_x increases monotonically, only slightly faster than linearly, by more than 2.5. The corresponding plot of γ_t vs. $\Delta p/p$ is the almost

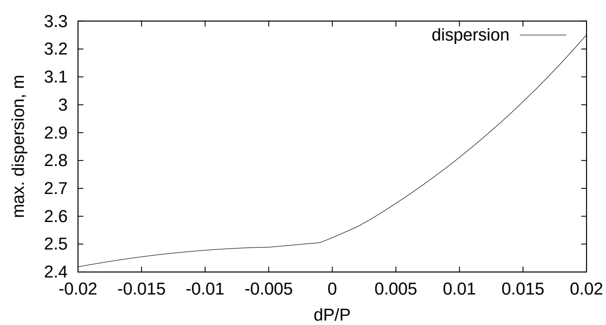




exponential looking curve displayed on the right. All values are larger than required.

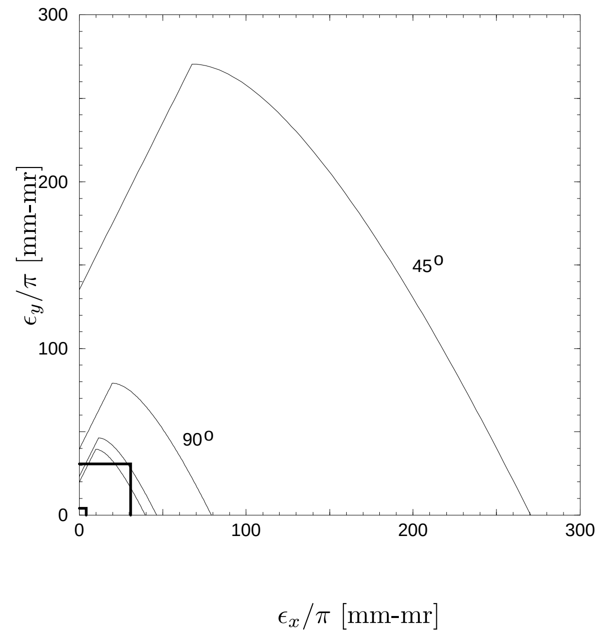
Maxima of β_x, β_y, and D are plotted below as a function of $\Delta p/p$. The variation of $\beta_{y,\max}$ and $D_{\max}$ are monotonic, while that of





$\beta_{x,\max}$ goes through a minimum near $\Delta p/p = 0$. As in the previous figures, there is much more variation for positive than negative $\Delta p/p$. Estimates of the closed orbit based on the value $D|_{\Delta p/p=0}$ should be increased by $\approx 12\%$ at the momentum acceptance limit, $\Delta p/p = 1\%$.

Sextupole resonance: $\nu_x + 2\nu_y = 29$



A (moderately) "safe region" for the $\nu_x + 2\nu_y = 29$ resonance is bounded by the curves

$$\frac{1}{8}\left(\frac{\delta}{g}\right)^2 = \frac{1}{4}\frac{\epsilon_y}{\pi} + 2\left(\sqrt{\frac{\epsilon_x}{2\pi}} - \frac{1}{4}\frac{|\delta|}{g}\right)^2 \quad ,$$

$$\text{and} \quad -\frac{1}{4}\left(\frac{\delta}{g}\right)^2 = 2\,\frac{\epsilon_x}{\pi} - \frac{\epsilon_y}{\pi} \quad ,$$

$$\text{where} \quad g = \frac{\sqrt{2}}{8\pi}\left|\sum \frac{B''l}{B\rho}\sqrt{\beta_x}\beta_y e^{i(\psi_x + 2\psi_y - \delta\cdot\theta)}\right| \quad ,$$

and $\delta = \nu_x + 2\nu_y - 29$. If a superperiodicity breaking phase error between arcs is $\Phi \equiv \Delta\psi_x + 2\Delta\psi_y \pmod{2\pi}$, then

$$g_{\text{racetrack}} = 2\,|\,\sin(\Phi/2)\,|\,g_{\text{arc}} \quad .$$

The "safe regions" are plotted above for $\Phi \in \{\,45°, 90°, 135°, 180°\,\}$. Two squares in the lower left corner show the beam emittances at injection and extraction.

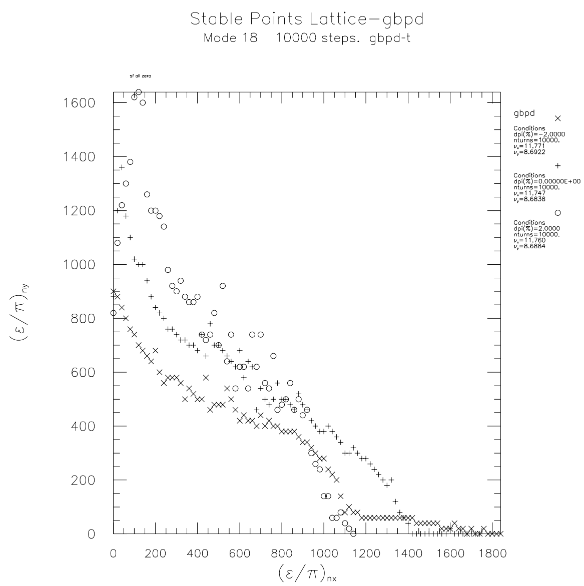
Sextupole resonance (cont.)

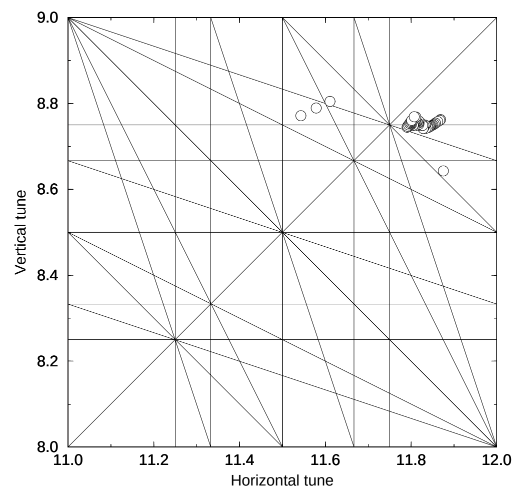
- This optimistic statement does not take into account the effects of space charge and (to second order) sextupoles in distributing tunes throughout the beam. How small, for example, must Φ be in order to reduce the "tune width" to $\delta \leq \pm 0.01$ for all particles at injection? Answering this is complicated by the fact that the largest tune shifts will occur near the core of the beam. For the sake of argument, let us say that 10% of the beam (emittance) gets shifted within reach of the resonance line. Then, we require that superperiodicity be preserved at the level $\Phi < 24°$ or better, significantly more restrictive than the optical value of 105°. However, it is possible that space charge detuning could limit the instability produced by the resonance. If the PD2 base configuration is seriously considered, this issue should be studied thoroughly. Near extraction, where particle tunes will be closer to the reference point, $\Phi < 21°$ should be sufficient to allow moving the entire beam within ± 0.01 of the resonance line, although there is no reason to do so.

- Or, because $\langle \Delta(\psi_x + 2\psi_y)|_{\text{cell}} \rangle = (2/3) \cdot 2\pi$, the resonance driving term can be (almost) zeroed across each arc module individually by putting sextupoles in all three cells instead of only two. This would require putting quadrupole and sextupole into the same trim element.

Dynamic Aperture

The dynamic aperture of the PD2 lattice has been estimated by tracking. Only the dipoles, quadrupoles, and chromaticity sextupoles have been included. The results, calculated at the





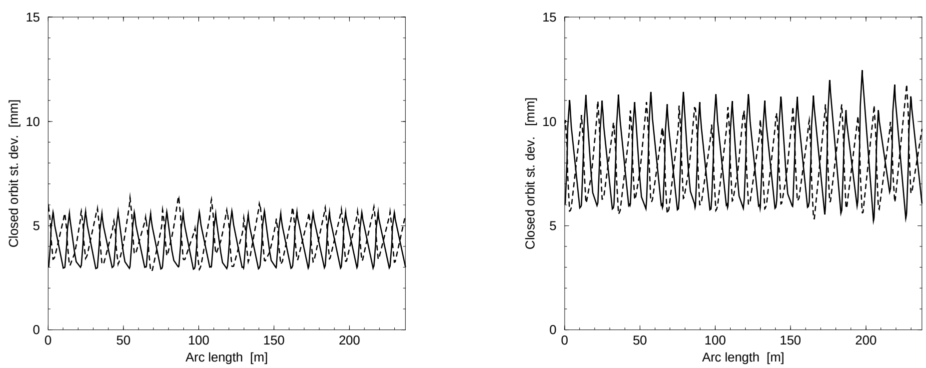
injection energy of 600 MeV, where dynamic aperture is smallest, are shown above. Along the diagonal, the dynamic aperture is at $\epsilon_{\text{inv}} \approx 10 \times 40\pi$ mm-mr. For purely horizontal orbits it increases to $\epsilon_{\text{inv}} \approx 25 \times 40\pi$ mm-mr, and for mostly vertical orbits it is slightly less, $\epsilon_{\text{inv}} \approx 20 \times 40\pi$ mm-mr.

Peaks of the tune spectra were calculated for all orbits just inside the dynamic aperture. A scatterplot is shown above. There is a clustering about the line $4\nu_y = 35$, a resonance line excited at second order in the strength of sextupoles. The chromaticity sextupoles both excite this resonance and provide the necessary tune spread to put it within the reach of very large amplitude orbits.

And so forth ...

Other topics in the Proton Driver Study (Chapter 3) include:

• **Closed orbit error.** Due to misaligned quads, dipole roll, and dipole field error: $\sigma_{\text{c.o.}} \approx 5$ mm, base configuration, and $\sigma_{\text{c.o.}} \approx 10$ mm, tuned configuration. This is fixed with steering



magnets and special dipole windings. Maximum deflections: 50 mm and 77 mm. Windings: $\approx 1.8\%\ \theta$ and $8.4\%\ \theta$.

• **Linear coupling.** Due to rolled quads, coupling into orthogonal plane is $\approx 9\%$: if the horizontal excursion of the beam is 1 cm, the estimated (r.m.s.) excursion in the vertical direction generated by coupling will be 0.9 mm.

• **Chromaticity error.** Due to sextupolar error field in the dipoles: negligible.

• $3\nu_y = 26$ **resonance.** Due to roll misalignment of chromaticity sextupoles: negligible.

• **Alternatives.** Especially (a) different phase advance across arc module, (b) missing magnet lattice (racetrack), (c) transitionless lattice (triangle).