# Resonant Beam Response in the PSR Accumulator Ring


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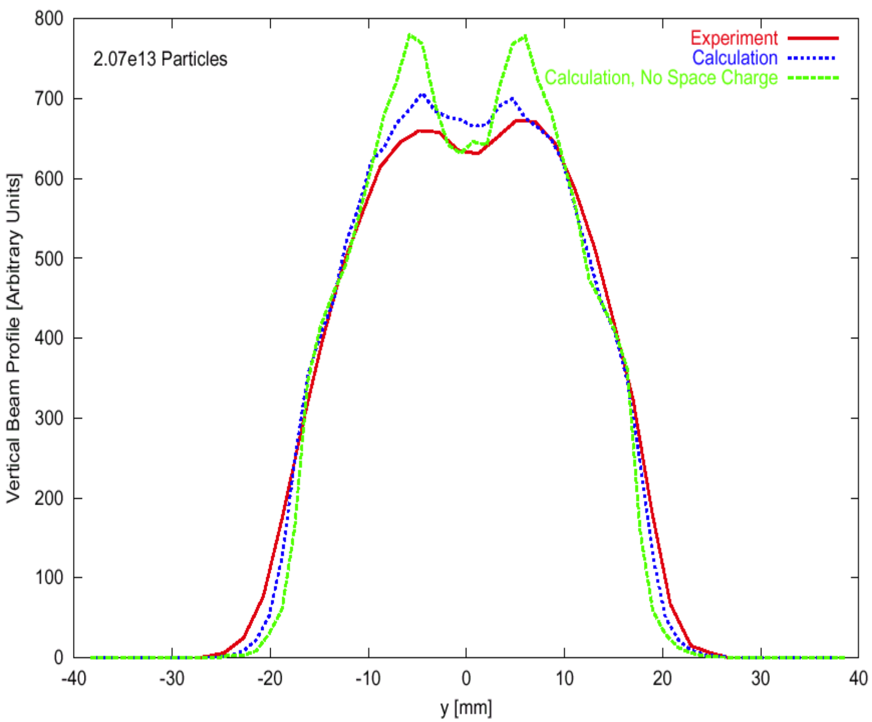
# Previous Results:

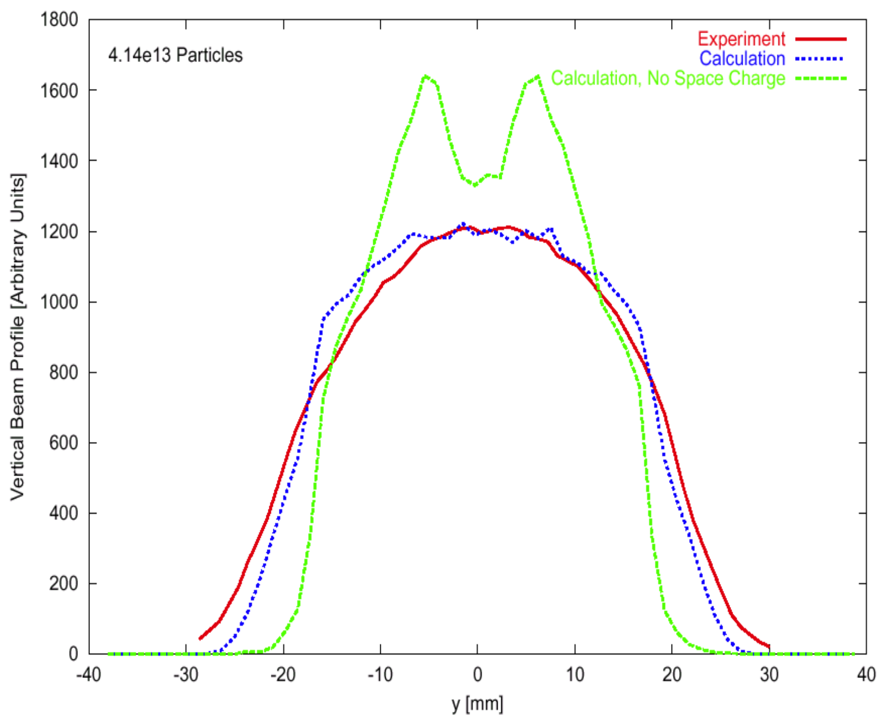
- Beam profile measurements were performed in the extraction line for a range of intensities at PSR.
- The experiment was carefully simulated using ORBIT, including lattice settings, injection, painting, and RF scenarios.
- The calculated and measured beam profiles were compared, with reasonably good agreement, particularly regarding systematic behavior.
- The calculations were then analyzed to extract the physics responsible for the observed behavior.

# Previous Results:

**Beam profile measurements were performed in the extraction line and subsequently simulated in detail for a range of intensities at PSR.**

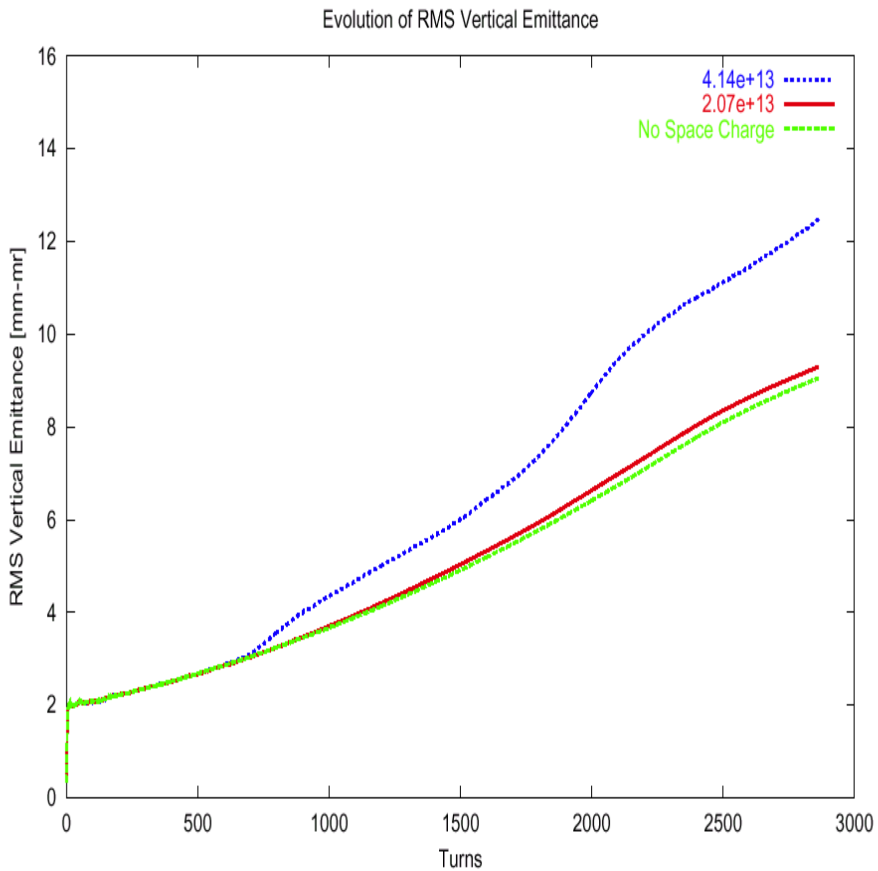
**Measurements and simulations both yield vertical beam broadening at high intensity and reasonable systematic detailed agreement.**

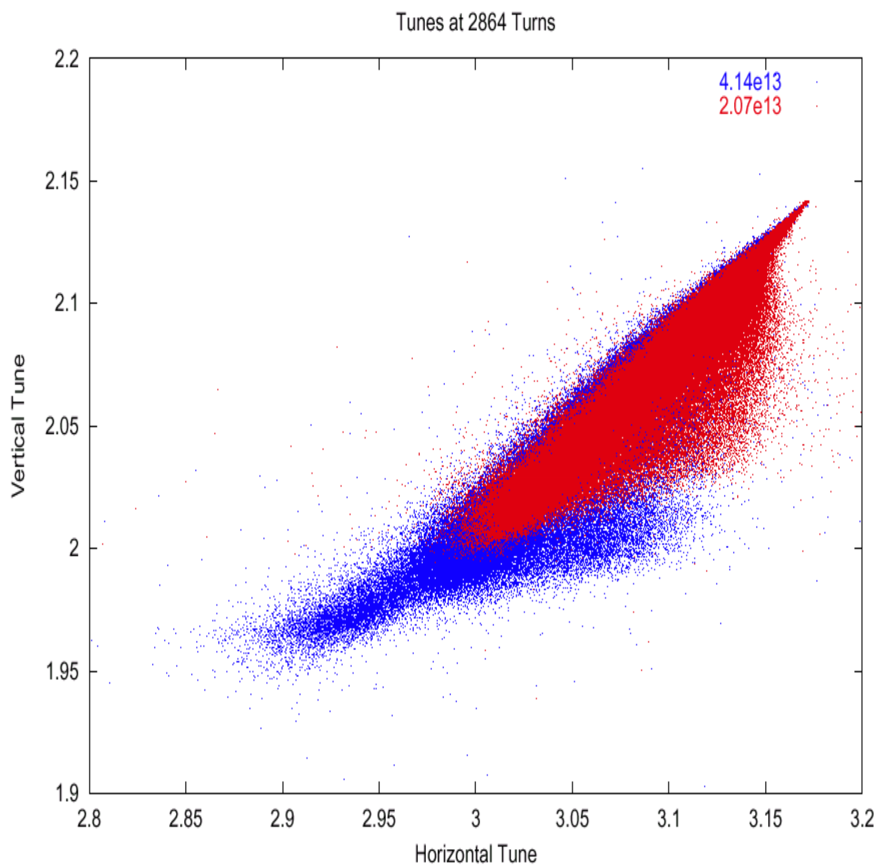


# Previous Results:

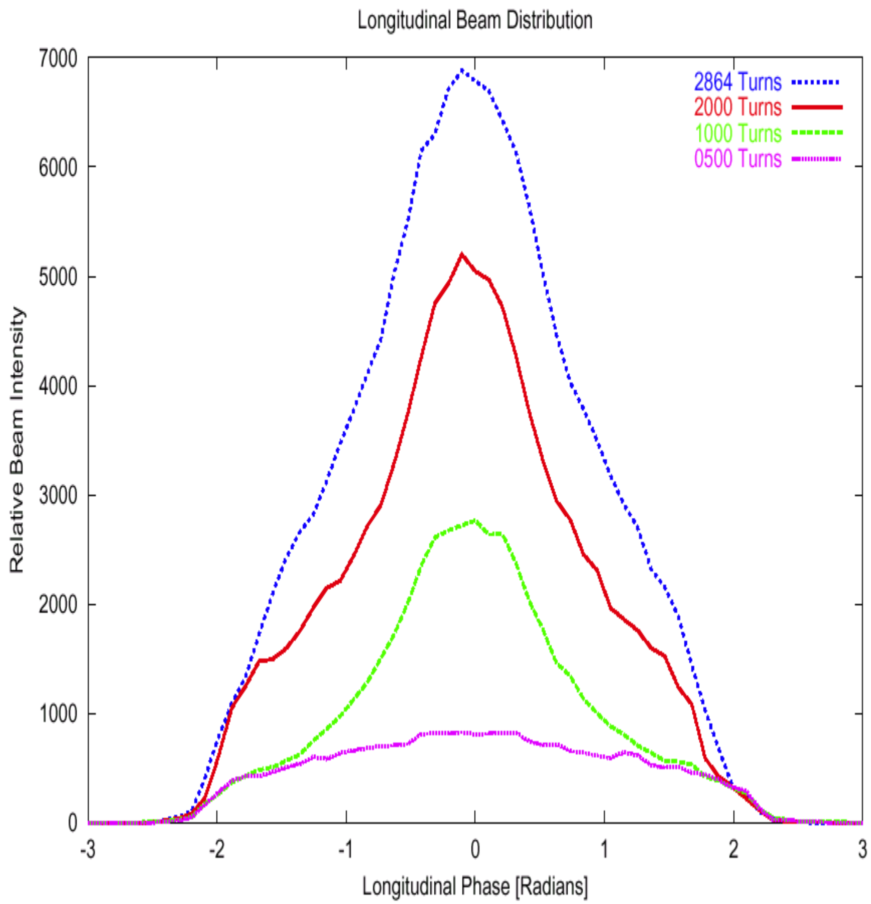
**Analysis of the simulations indicates beam broadening at high intensity. This is associated with the incoherent tunes of a substantial fraction of the beam crossing $Q_y = 2$.**

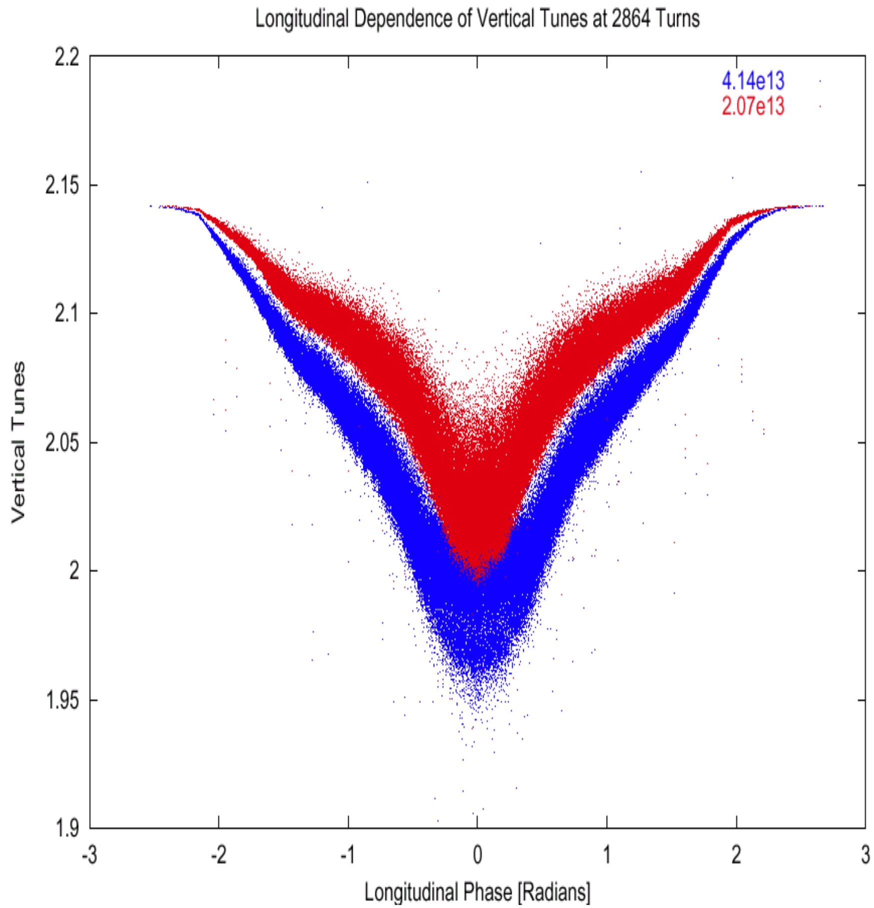


# Previous Results:

**In the simulations, longitudinal peaking (low bunch factor) increases the local beam intensity and tune depression.**

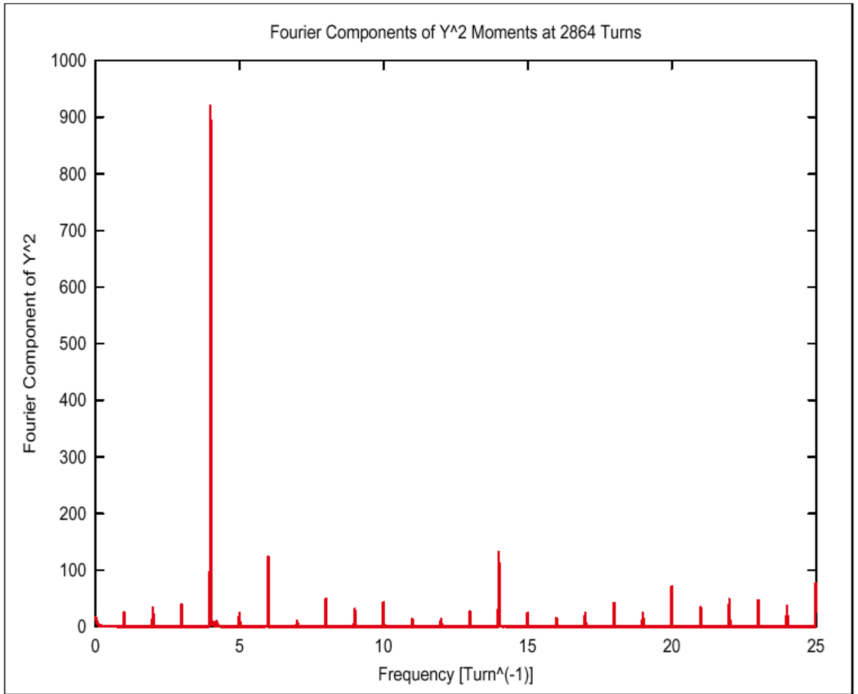


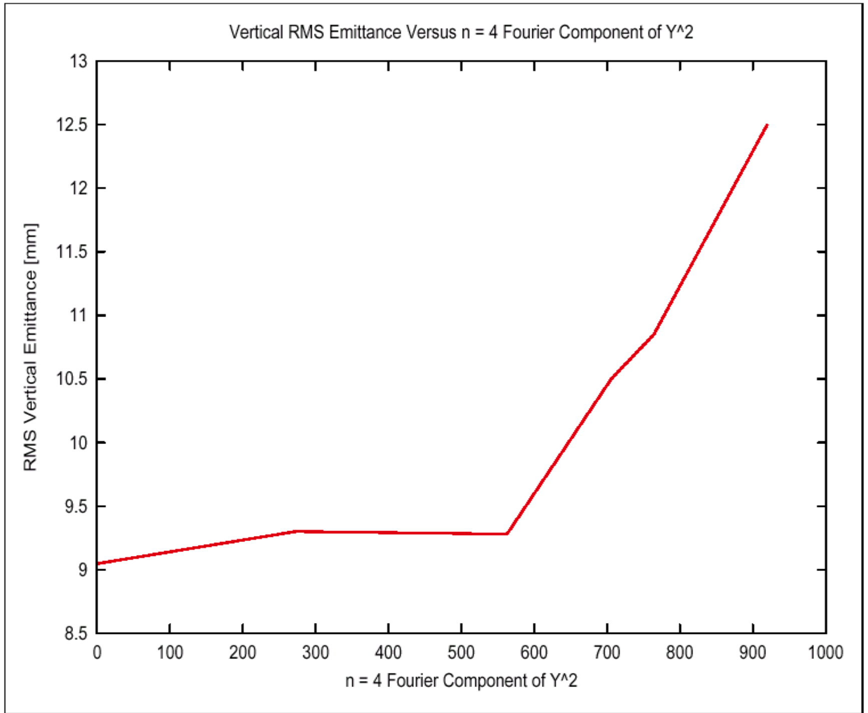
# Previous Results:

**Spectrum analysis of the beam second moments in the simulations shows a large N=4 component at high intensity, consistent with a coherent half integer resonance at $Q_y = 2$.**

**For a number of cases, the final RMS emittance increases linearly with the N=4 amplitude above a threshold value.**




**Spectral content of vertical second moment oscillations of final beam for the high intensity default case.**




**Vertical rms emittance versus amplitude of n = 4 Fourier component of vertical second moment oscillations for the cases presented here.**

# Previous Results: Summary

- Beam profile measurements and simulations show reasonable systematic and detailed agreement, and both yield vertical beam broadening at high intensity.

- Analysis of the simulations indicates that the beam broadening at high intensity is accompanied by the incoherent tunes of a substantial fraction of the beam crossing $Q_y = 2$.

- In the simulations, longitudinal peaking (low bunch factor) increases the local beam intensity and tune depression.

- Spectrum analysis of the beam second moments in the simulations shows a large N=4 component at high intensity, consistent with a coherent half integer resonance at $Q_y = 2$.

- For a number of cases, the final RMS emittance increases linearly with the N=4 amplitude above a threshold value, also consistent with a coherent half integer resonance at $Q_y = 2$.

# PSR Half Integer Resonance: An Envelope Equation Analysis

- **A detailed analysis of half integer resonances with space charge and magnet errors was presented using the envelope equations in the thesis of Sacherer (1968).**

- **Envelope Equation for y (similar equation for x):**

$$\frac{d^2y}{d\phi^2} + [\nu_y^2 + 2\nu_y \cdot \Delta\nu_{ey} \cos n\phi]y - \frac{\nu_y^2}{y^3} - \frac{4r_0\lambda}{\gamma^3\beta^2}\frac{\nu_y^2\beta_y^{3/2}}{\sqrt{\varepsilon_y}}\frac{1}{\sqrt{\beta_x\varepsilon_x}\,x + \sqrt{\beta_y\varepsilon_y}\,y} = 0$$

where

$(\varepsilon_x, \varepsilon_y)$ = emittances, $(\beta_x, \beta_y)$ = beta functions, $(\nu_x, \nu_y)$ = tunes

$$x \to \frac{x}{\sqrt{\beta_x\varepsilon_x}}, \quad y \to \frac{y}{\sqrt{\beta_y\varepsilon_y}}, \quad d\phi = \frac{ds}{\nu_y\beta_y}$$

$\Delta\nu_{ey}$ = strength of magnet error, $\lambda$ = longitudinal particle density

# PSR Half Integer Resonance: An Envelope Equation Analysis

- Sacherer's analysis simplifies the space charge term by replacing the oscillating beta functions by their average values. The resulting equations are then solved perturbatively for periodic solutions, assuming that the magnet error and space charge terms are small perturbations.

- Sacherer's results that pertain to the present situation are:
  - Because the tunes are separated ($\nu_x$= 3.17, $\nu_y$= 2.14), x-y coupling is weak, and a 1-D envelope analysis in y is appropriate.
    - Strong x-y coupling occurs only when the tunes are nearly equal.
    - For circular beams this occurs only when

$$\left|\nu_y - \nu_x\right| \le \frac{\Delta\nu_{sc}}{4} \ (\approx 0.05 \text{ for PSR})$$

  - Because magnet errors are ignored here, the periodic solutions to the envelope perturbation equation are constants of unit amplitude except for a very narrow resonance at

$$\Delta\nu_{sc} = \frac{8}{5}\cdot\left|\nu_y - 2\right| \rightarrow \nu_y = 1.916$$

# PSR Half Integer Resonance: An Envelope Equation Analysis

- To apply the results of Sacherer's analysis to PSR, we calculate periodic solutions of the full, coupled, envelope equations under the following assumptions:
  - We use the same detailed description of the lattice as was used in the ORBIT particle simulations.
  - We present the results in normalized form as shown above.

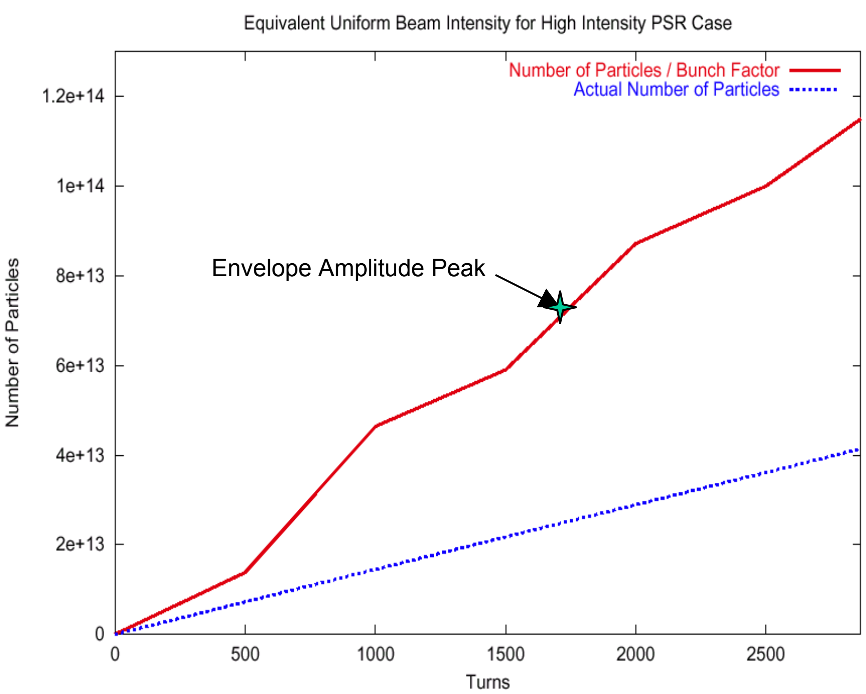
- Because of the simplifying assumptions made by Sacherer in his analysis, we expect some differences in the details of our results. Sacherer
  - neglects the rapid oscillations of the lattice functions in the space charge term, and
  - treats the magnet errors and space charge as perturbations.

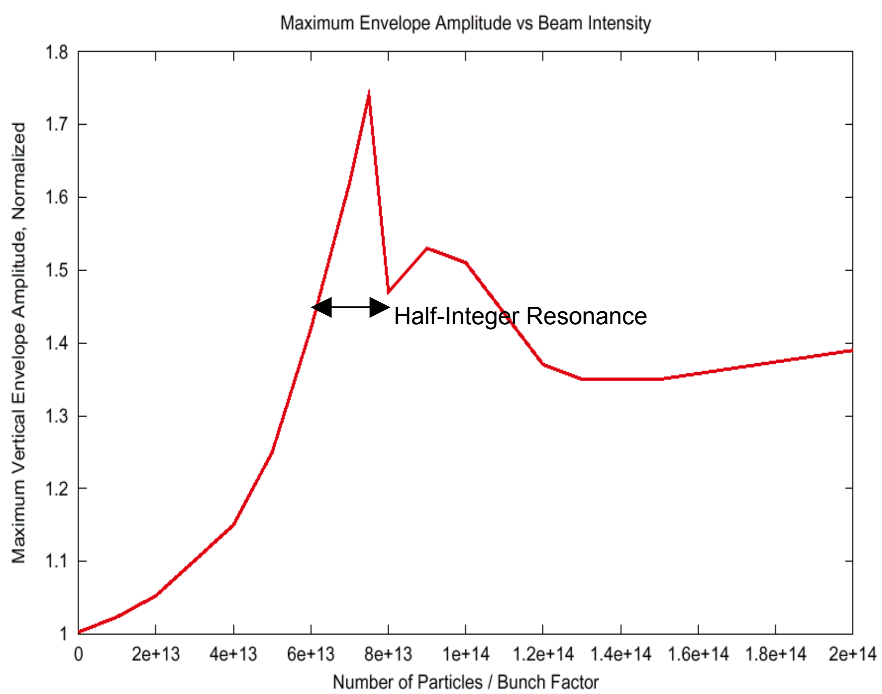
- In light of these similarities and differences, the full envelope equation solutions
  - will show essentially independent, decoupled x and y motion,
  - will reveal resonant behavior for incoherent tunes near $Q_y$ = 1.916, but
  - will give non-constant solutions off the resonance, particularly at high intensities where Sacherer's assumptions are violated.

# PSR Half Integer Resonance: An Envelope Equation Analysis

**Because our picture of PSR beam broadening suggests sensitivity to peak beam intensity, it is this quantity which should be compared to the beam intensity in the envelope equations. This is obtained from the simulations by dividing the average intensity by the bunch factor.**

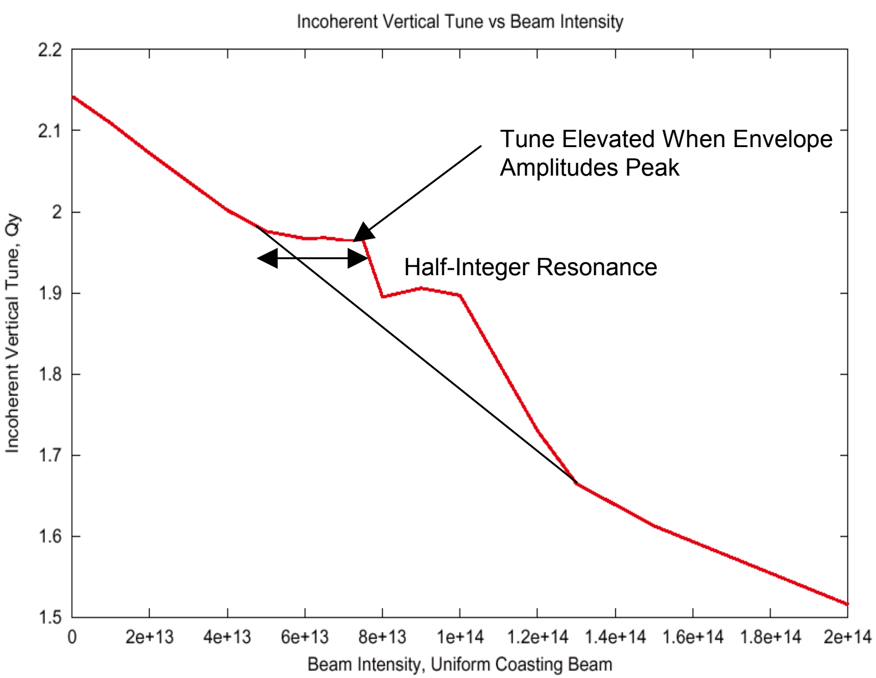
**The amplitude of the periodic solution of the envelope equations peaks strongly at a beam intensity corresponding to that at about 1750 turns in the high intensity particle simulation. This corresponds to the half-integer coherent resonance.**

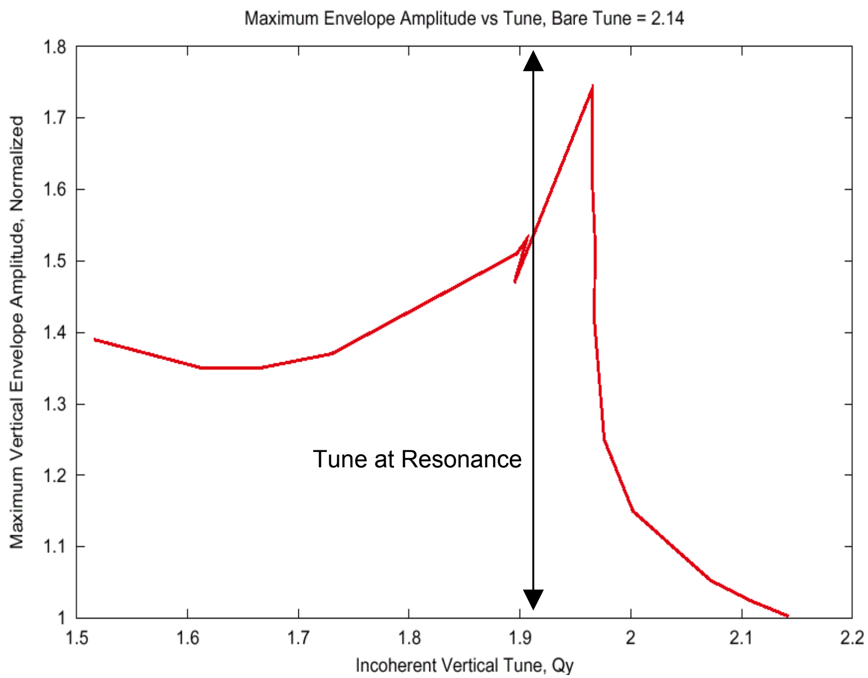


# PSR Half Integer Resonance: An Envelope Equation Analysis

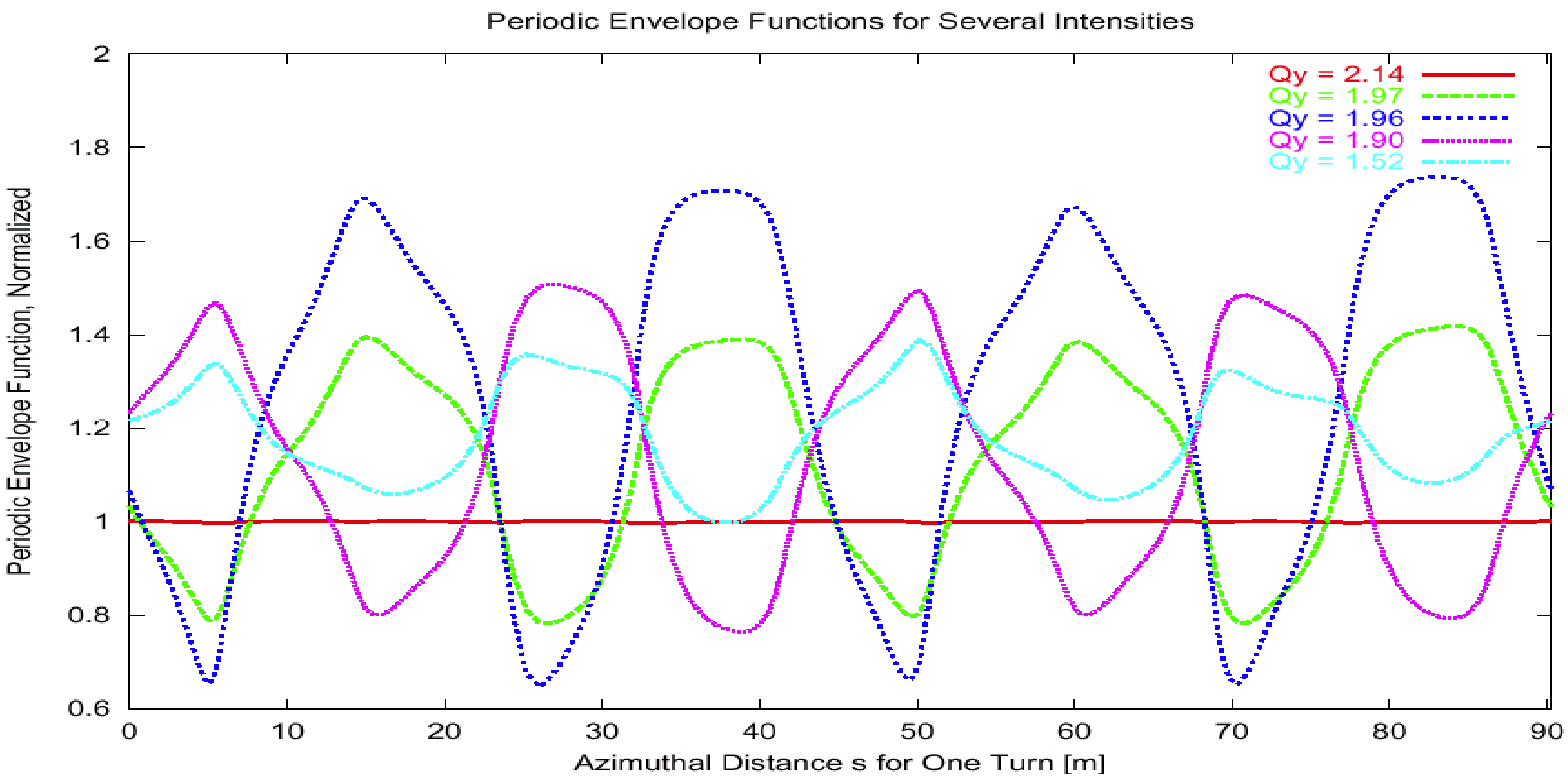
The incoherent tune of test particles in the envelope potential decreases with increasing intensity. However, rather than a uniform decrease, the tune is elevated in the region where the envelope amplitudes peak. This is a nonlinear effect: the large amplitudes weaken the space charge force, thus reducing the tune shift. Because of this, plotting the envelope amplitude versus the incoherent tune places the apparent location of the resonance above the predicted value. However, "removing the bump" in the tune plot places Sacherer's prediction of 1.916 just above $6*10^{13}$ protons, in the center of the resonance. The peak at $7.5*10^{13}$ protons occurs just before dropping out of resonance on the high intensity side. Even so, strong nonlinearities persist at intensities above the resonance.



# PSR Half Integer Resonance: An Envelope Equation Analysis

**Periodic envelope functions are shown for a number of different beam intensities, parameterized by the incoherent tunes. As expected, at low intensity ($Q_y$ = 2.14), the envelope function is nearly constant. After passing through the resonance peak ($Q_y$ < 1.916) the phase of the envelope function oscillations shifts by 180 degrees.**



# PSR Half Integer Resonance: Conclusions

- Periodic solutions to the full, coupled, envelope equations using the PSR ring lattice with tunes $(Q_x, Q_y) = (3.17, 2.14)$
  - display essentially independent, decoupled x and y motion,
  - reveal resonant behavior in the y equation amplitudes for incoherent tunes near $Q_y = 1.916$ and envelope intensities of $(5\text{-}7.5)*10^{13}$, which correspond to PSR peak beam intensities from 1000 to 1750 turns in the simulation of a high intensity case,
  - give non-constant solutions off the resonance, particularly at high intensities where the sign of the oscillations flips.

- These results
  - further corroborate the half integer resonance at $Q_y = 2$ as cause of the observed beam broadening in PSR,
  - agree with Sacherer's analysis in the independence of the x and y solutions and the location of the resonance in incoherent tune,
  - disagree with Sacherer's analysis in the amplitude and variation of the envelope functions off resonance, particularly at higher intensities. This is understandable in terms of the simplifying assumptions used in Sacherer's analysis.

# PSR Half Integer Resonance: Acknowledgments


- We wish to acknowledge Bob Macek and the PSR staff for encouraging this work and for providing access and assistance in carrying out high intensity measurements at PSR.

- For theoretical guidance and interpretation we used the following PhD thesis:

  Transverse Space-Charge Effects in Circular Accelerators
  by F. J. Sacherer
  Lawrence Radiation Laboratory
  University of California, Berkeley, 1968