Halo-formation and Equilibrium in High Intensity Hadron Rings
: A Role of Nonlinear Parametric Resonances Excited by Intrinsic Beam-Core Oscillations

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Agenda

1. Introduction / Purpose / Calculation condition
2. Derivation of Isolated Resonance Hamiltonian (IRH)
3. Justification of IRH
4. TIME VARYING of IRH
5. Conclusion

[INTRODUCTION]

- Activation of environment due to beam loss → Big Problem

  Halo causes beam loss.
→ Important to understand mechanisms of halo formation.

- Halo formation has been studied by

  - simulations such as particle-in-cell (PIC)
  - theoretical analysis such as particle-core-model (PCM).

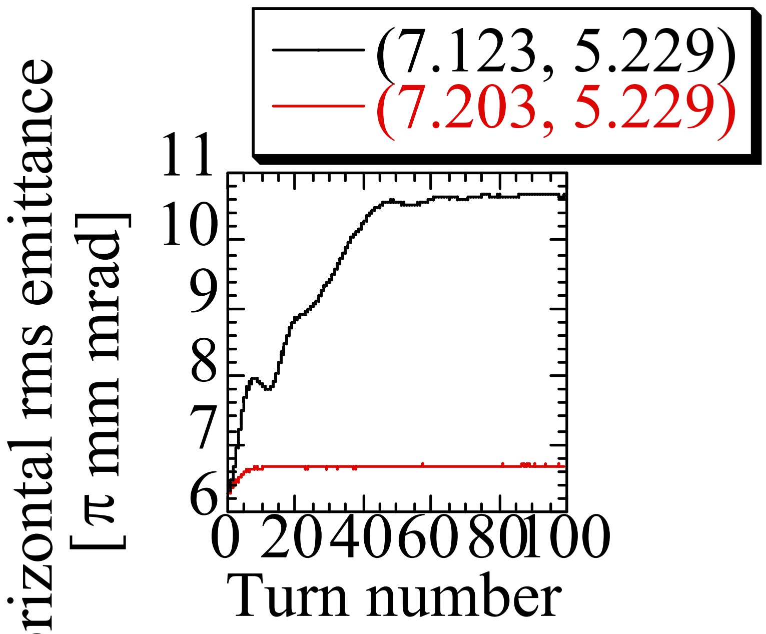
- Speculation for halo formation mechanism given by existing analyses

  → Parametric resonance excited between
  betatron oscillation and oscillating space charge force.

● Existing analyses can not be applied to halo formation under non-equilibrium state because

1. rms emittance grows up → ×PCM analysis

2. non-equilibrium state finishes generally less than a few tens turns
→ × Poincaré map analysis (simulation)
× Frequency analysis (simulation)



Emittance growth
Simulation result

A novel analytic technique is important, which can be applied to halo formation under non-equilibrium state.

[PURPOSE]

For circular accelerators, understand

1. mechanisms of halo formation
2. mechanisms for a particle distribution to achieve an equilibrium

through a non-equilibrium state.

[MY WORK]

A novel analytic technique was developed by using classical nonlinear theory : Isolated Resonance Hamiltonian (IRH).

Halo's location of a 2D Gaussian beam under non-equilibrium state in FODO lattice was analyzed by using IRH.

[CALCULATION CONDITION 1]

(1) Typical FODO lattice (KEK 12Gev PS).

(2) 500MeV, $\Delta p/p = 0$

(3) bare tune (ν_x, ν_y) = (7.123, 5.229), (7.203, 5.229)

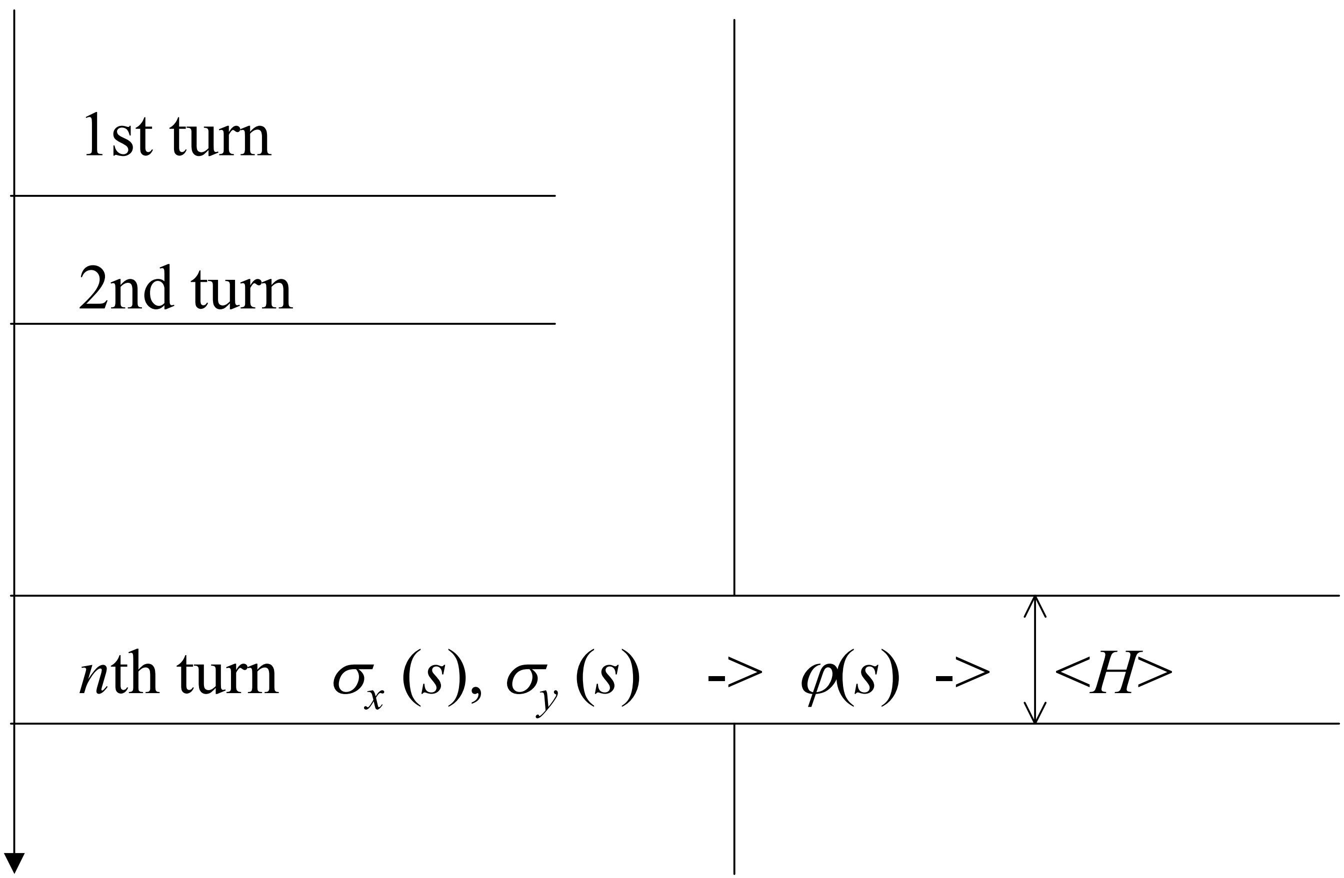
(4) 2-D (x and y direction) Gaussian beam.

(5) Time varying of RMS beam size is obtained from the simulation results.

[CALCULATION CONDITION 2]

Simulation | Analysis

Tracking



[DERIVATION of IRH]

(1) Single particle Hamiltonian for Betatron oscillation

$$H(\psi_x,\psi_y,I_x,I_y;\theta)=\nu_x I_x+\nu_y I_y+\frac{eR}{\gamma^2 pv}\varphi(\psi_x,\psi_y,I_x,I_y;\theta)$$

(ψ_x,ψ_y,I_x,I_y) : Action - angle variable

θ : Dependent variable

φ : Space charge potential

(2) φ oscillates with θ because of intrinsic beam-core oscillation.

(= mismatching and lattice-structure)

$\rightarrow$ φ is separated into rapid oscillation term and slow oscillation term.

- Rapid oscillation term can be removed by time average $<\varphi>$.
- Slow oscillation term is just resonance term

(3) <*H*> is called as Isolated Resonance Hamiltonian.

IRH for ”nonlinear resonance between Betatron oscillation

and oscillating space charge forces of 2D Gaussian beam

if $\boxed{i\left(2\delta\nu_{s.c.,x}-\kappa\right)\approx 0}$ is satisfied“ is

$$\boxed{H\left(\Psi_x, I_x, I_y\right)=\left(\nu_x-\frac{\kappa}{2\delta}\right)I_x+\frac{eR}{\gamma^2 pv}\left\langle\varphi\left(\Psi_x, I_x, I_y\right)\right\rangle}\ .$$

$\nu_{s.c.}$:Depressed tune

$i,\ \delta$: Integer

κ : Oscillating number of rms beam size per 1 turn

$$\left\langle \varphi\left(\Psi_x, I_x, I_y\right)\right\rangle = \frac{eN}{4\pi\varepsilon_0}\sum_{n=1}^{n_{\max}} \frac{1}{n!}\binom{2n}{n} G(0)_n^{(0)}\left(-\frac{I_x}{2}\right)^n$$

$$+\frac{eN}{4\pi\varepsilon_0}\sum_{n=2}^{n_{\max}} \frac{1}{n!}\sum_{r=1}^{n-1}\binom{n}{r}\binom{2n-2r}{n-r}\binom{2r}{r} G(0)_n^{(6)}\left(-\frac{I_x}{2}\right)^{n-r}\left(-\frac{I_y}{2}\right)^r$$

$$+\frac{eN}{4\pi\varepsilon_0}\sum_{i=1}^{\infty}\sum_{n=i\delta}^{n_{\max}} \frac{1}{n!}\left(-\frac{I_x}{2}\right)^n\binom{2n}{n-i\delta} S_1(i,n)\cos(2i\Psi_x)$$

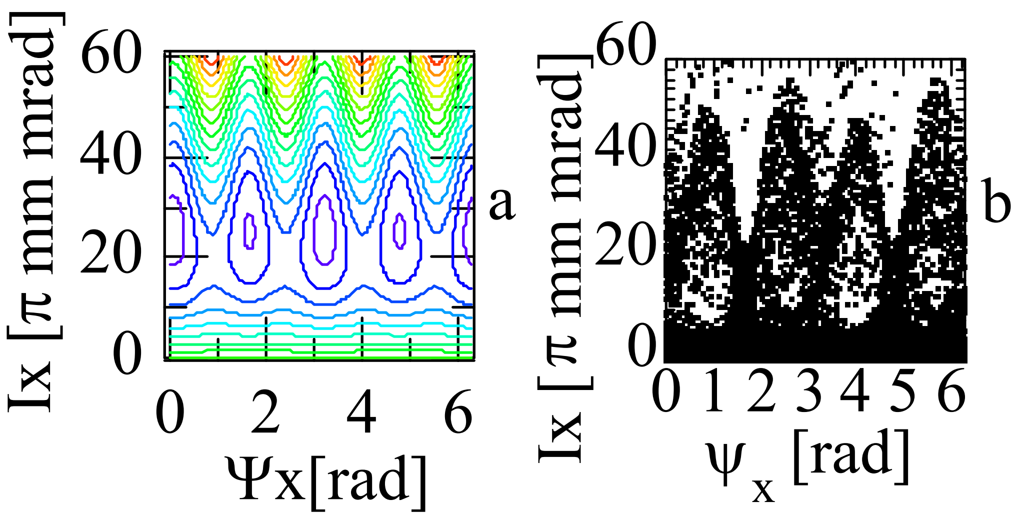
$$+\frac{eN}{4\pi\varepsilon_0}\sum_{i=1}^{\infty}\sum_{n=i\delta}^{n_{\max}} \frac{1}{n!}\left(-\frac{I_x}{2}\right)^n\binom{2n}{n-i\delta} S_2(i,n)\sin(2i\Psi_x)$$

$$+\frac{eN}{4\pi\varepsilon_0}\sum_{i=1}^{\infty}\sum_{n=1+i\delta}^{n_{\max}} \frac{(-1)^n}{n!}\sum_{r=1}^{n-i\delta}\binom{n}{r}\binom{2r}{r}\binom{2n-2r}{n-r-i\delta}\left(\frac{I_x}{2}\right)^{n-r}\left(\frac{I_y}{2}\right)^r S_3(i,n,r)\cos(2i\Psi_x)$$

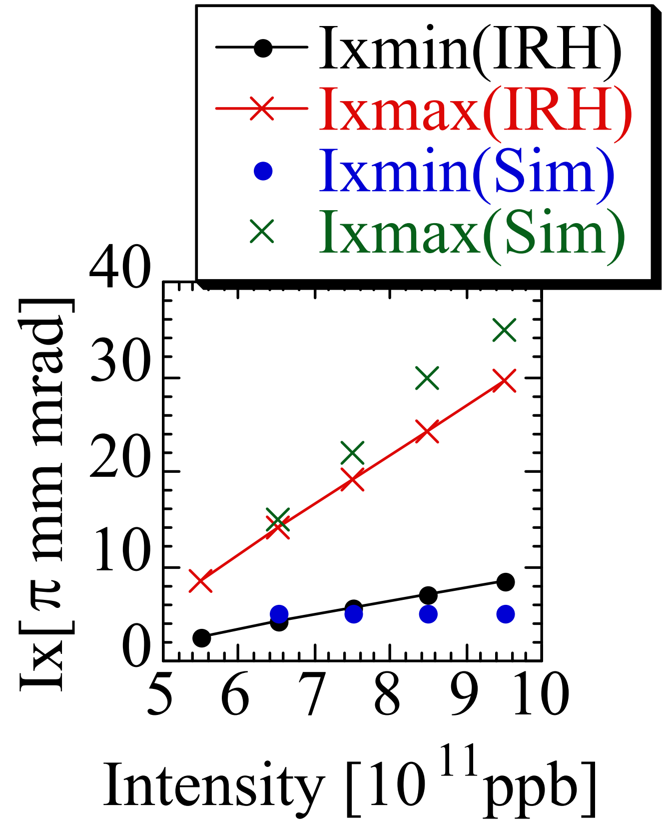
$$+\frac{eN}{4\pi\varepsilon_0}\sum_{i=1}^{\infty}\sum_{n=1+i\delta}^{n_{\max}} \frac{(-1)^n}{n!}\sum_{r=1}^{n-i\delta}\binom{n}{r}\binom{2r}{r}\binom{2n-2r}{n-r-i\delta}\left(\frac{I_x}{2}\right)^{n-r}\left(\frac{I_y}{2}\right)^r \gamma_4(i,n,r)\sin(2i\Psi_x)$$

[JUSTIFICATION of IRH]

(Example)
Structure resonance ($4\nu_x = 28$)
Bare tune $\nu_x = (7.123, 5.229)$
10th turn



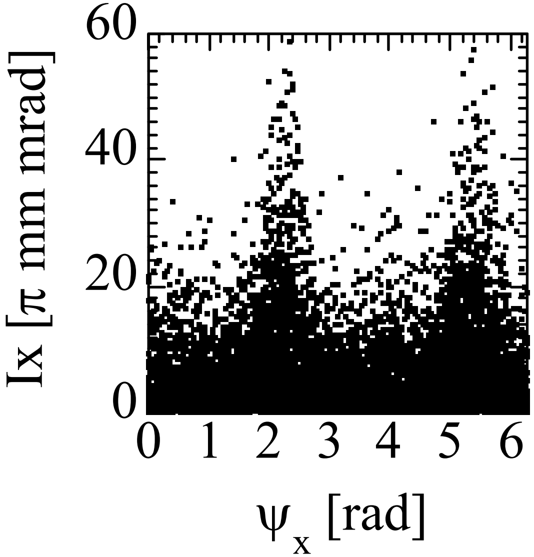
Comparison of phase space map between (a) IRH and (b) Simulation. (8.5e11[ppb])



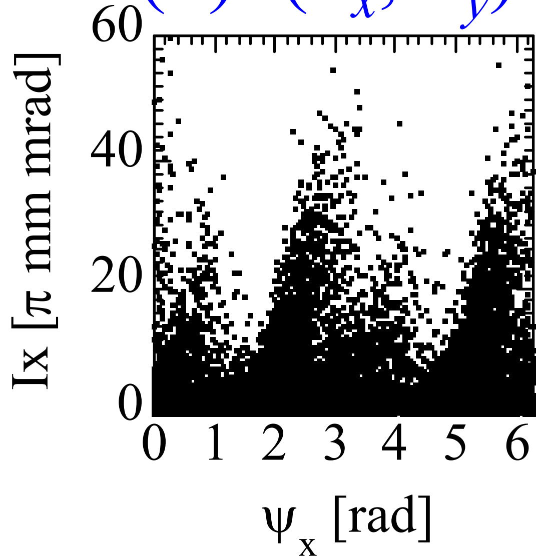
Intensity dependence of resonance width between IRH and Simulation results (10th turn)

Good agreement !!

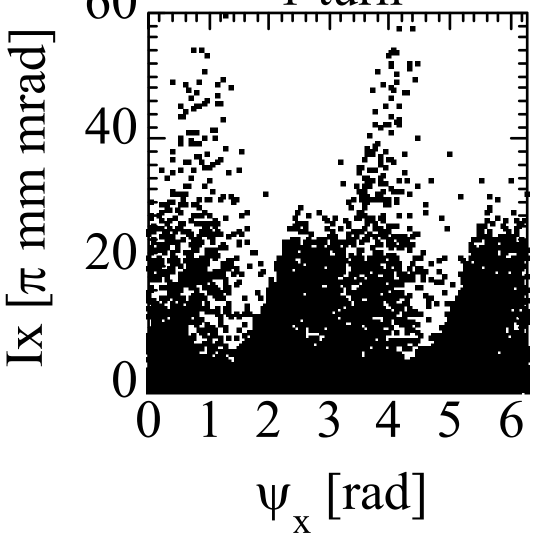
[Simulation Results (1) $(\nu_x, \nu_y) = (7.123, 5.229)$]



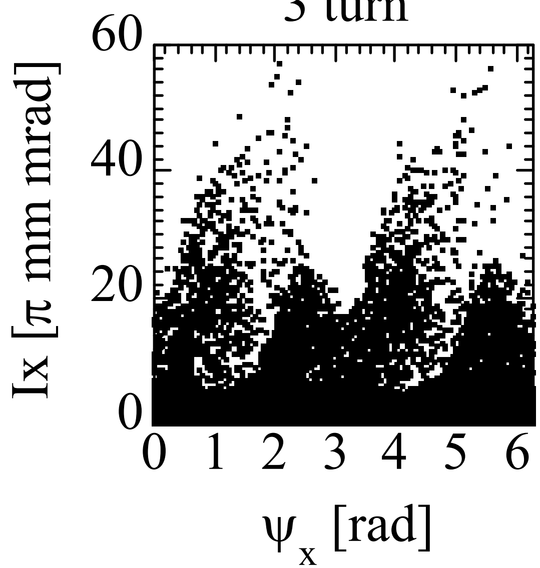
1 turn



3 turn



5 turn

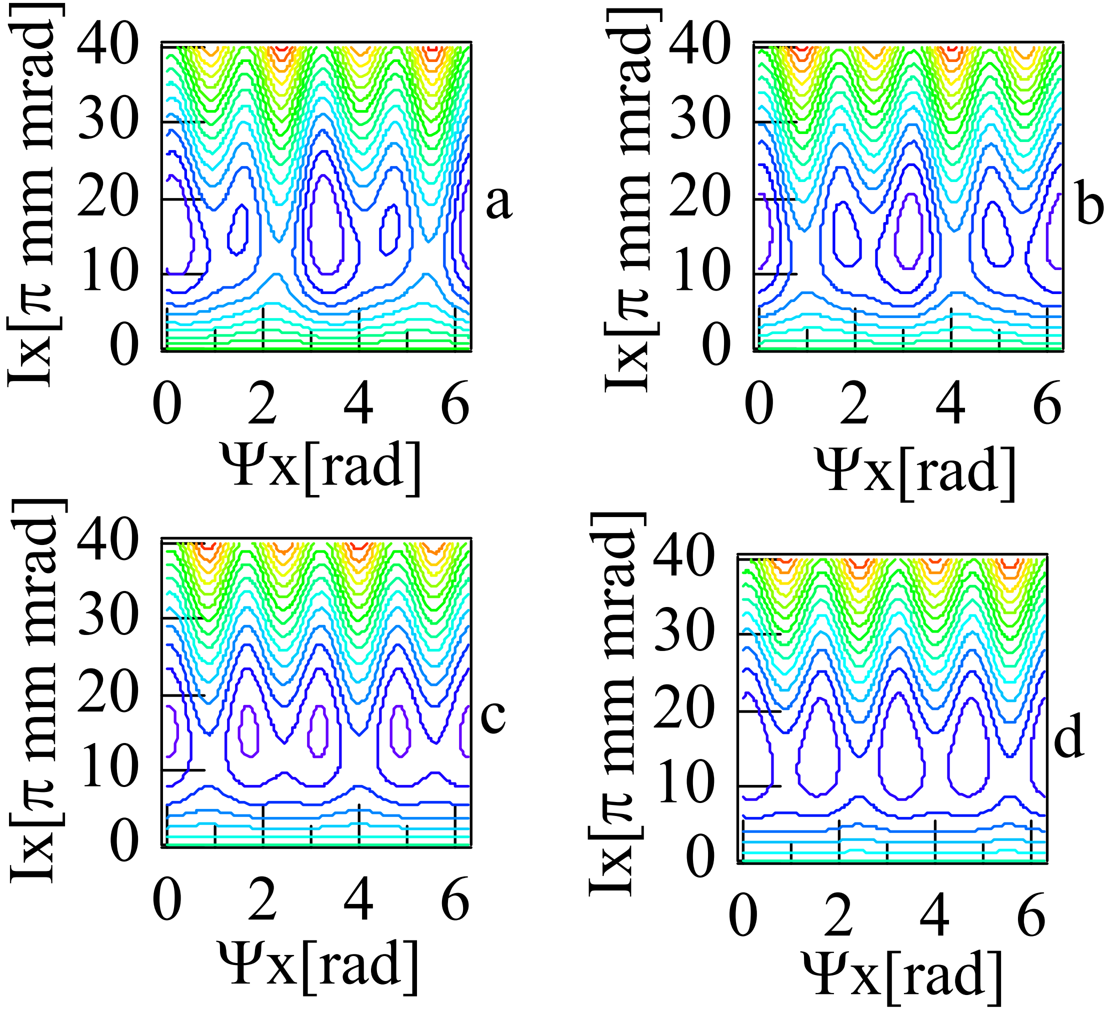


7 turn

○ What happens under non-equilibrium state in simulation results ?

○ What causes complicated particle distribution?

[TIME VARYING of IRH (1) (ν_x, ν_y) = (7.123, 5.229)]



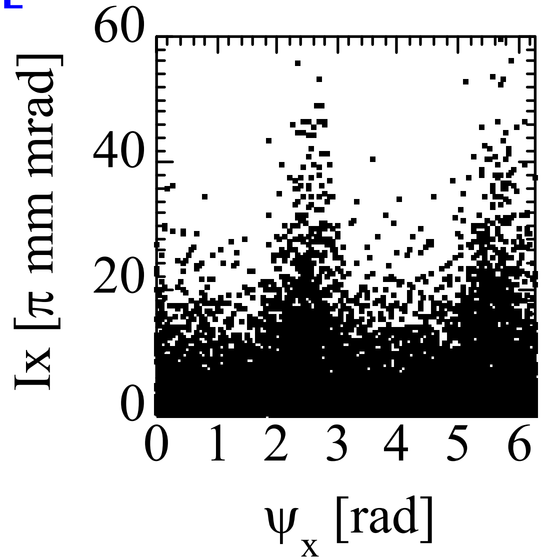
IRH of (ν_x, ν_y) = (7.123, 5.229)
(a) 1st turn, (b) 3rd turn
(c) 5th turn, (d) 7th turn

○ Dominant resonance

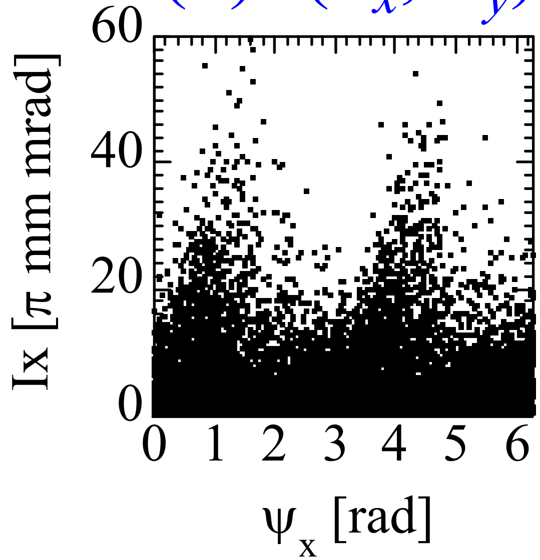
Mismatching resonance
(initial a few turns)
↓
Structure resonance
(after decay of mismatch)

○ Particles moved to outer
edge of resonance island
□□□□□↓
□□□Halo formation

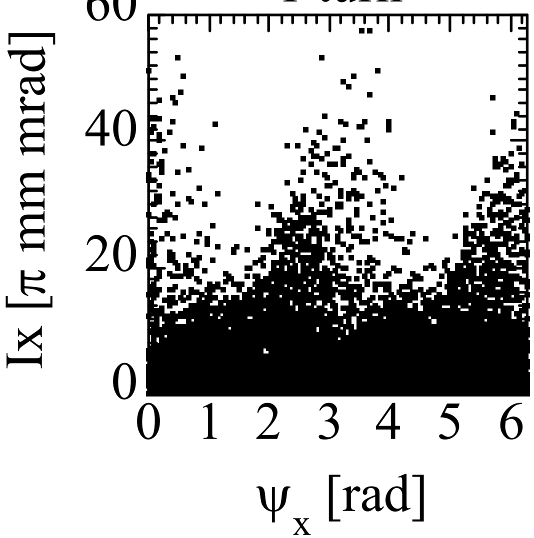
[Simulation Results (2) (ν_x, ν_y) = (7.203, 5.229)]



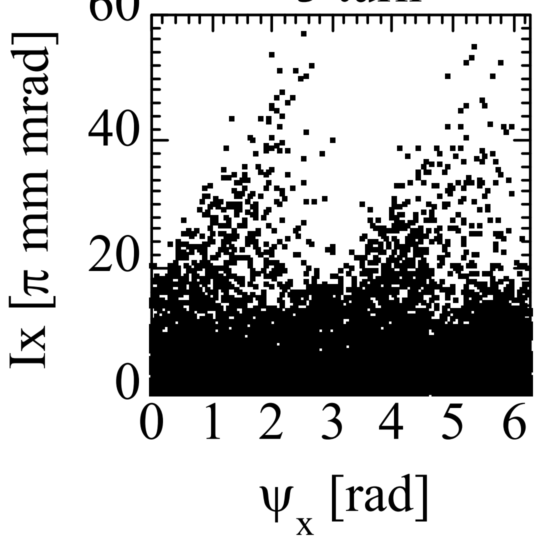
1 turn



3 turn

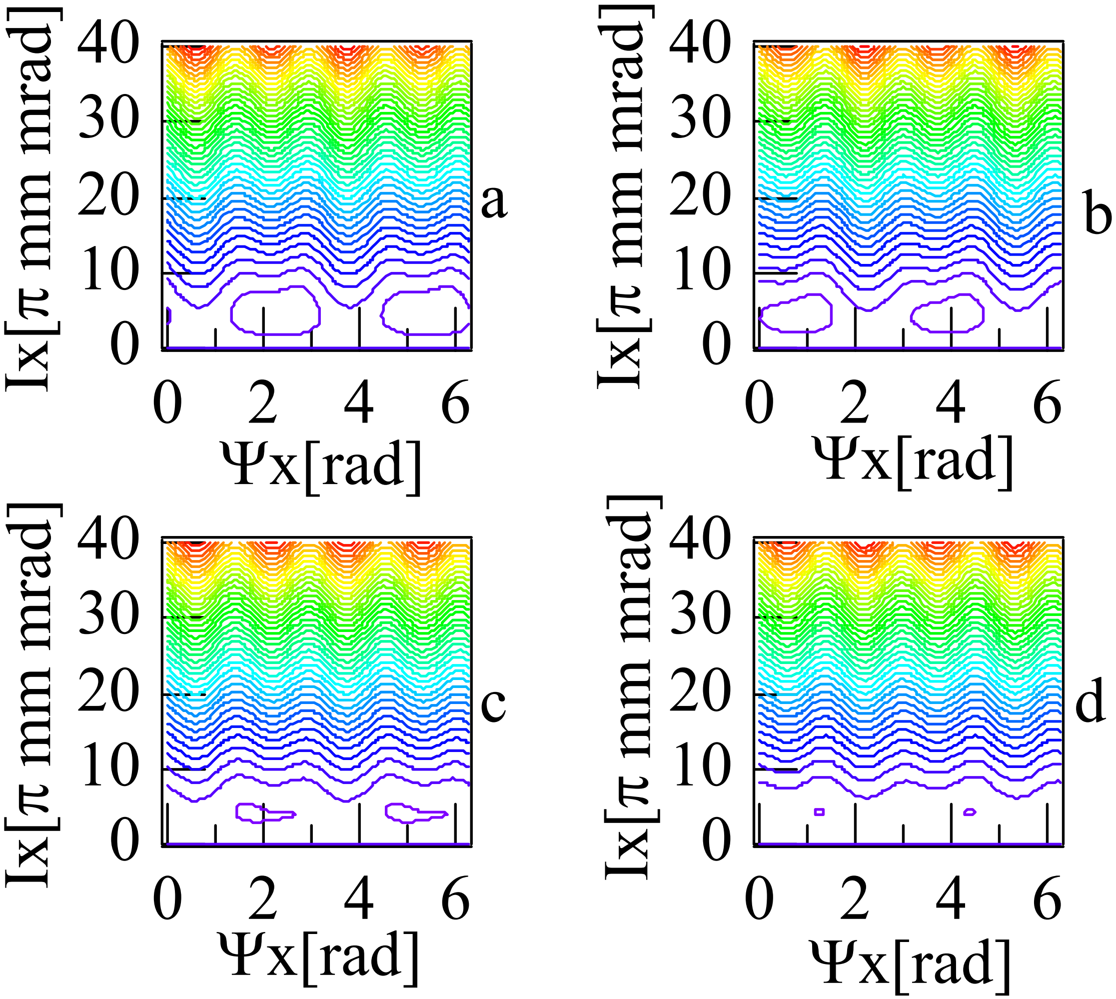


5 turn



7 turn

[TIME VARYING of IRH (2) (ν_x, ν_y) = (7.203, 5.229)]



IRH of (ν_x, ν_y) = (7.203, 5.229)
(a) 1st turn, (b) 3rd turn
(c) 5th turn, (d) 7th turn

○ Dominant resonance

Mismatching resonance
(initial a few turns)

↓

VANISH
(after decay of mismatch)

○ Father resonance does not occur

□□□□□↓

Arrive at equilibrium state

[Conclusion]

1. Nonlinear resonances excited by intrinsic beam core oscillation at non-equilibrium state remarkably contribute to halo generation.

2. Beam distribution achieves an equilibrium state through decay process of nonlinear resonances.

3. It turns out that the present analytic approach technique is quite useful to understand

what happens in simulation results

and

what causes complicated particle distribution.