Lifetime of Stark States Hydrogen Atom in Magnetic Field Calculation and Estimation of Losses at Stripping Injection

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The energy level of the hydrogen atom n is split in the uniform electric field into $n(n+1)/2$ sublevels [1]. Each sublevel can be characterized by a set of parabolic quantum numbers n_1, n_2, m such that the principal quantum number $n = n_1 + n_2 + m + 1$.

The Schrodinger equation for the hydrogen atom in a uniform electric field F parallel to z axis is of the form

$$\left(\Delta + \frac{2}{r} - 2Fz + 2E\right)\Psi = 0 \qquad (1)$$

where atomic units will be used. Equation is separable in parabolic coordinates, $x = (\xi\eta)^{1/2} cos\varphi, \quad y = (\xi\eta)^{1/2} sin\varphi, \quad z = 1/2(\xi - \eta)$

Defining the wave function Ψ as the product

$$\Psi = (\xi\eta)^{-1/2} V(\xi) U(\eta) e^{\pm im\varphi} \qquad (2)$$

Substituting 2 into 1, we obtain, for $V(\xi)$ and $U(\eta)$, the equations

$$\frac{d^2V}{d\xi^2} = \left(\frac{E}{2} + \frac{\beta_1}{\xi} + \frac{1-m^2}{4\xi^2} - \frac{F}{4}\xi\right) V = 0 \qquad (3)$$

$$\frac{d^2U}{d\eta^2} = \left(\frac{E}{2} + \frac{\beta_2}{\eta} + \frac{1-m^2}{4\eta^2} - \frac{F}{4}\eta\right) U = 0 \qquad (4)$$

where β_1 and β_2 are separation constants coupled by the requirement

$$\beta_1 + \beta_2 = 1$$

The functions $V(\xi)$ and $U(\eta)$ should be finite at the origin at $\xi \to 0$ and $\eta \to$

$$V(\xi) = \xi^{(m+1)/2}, \quad U(\eta) = \eta^{(m+1)/2}$$

The asymptotic solution for $V(\xi)$ at $\xi \to \infty$ falls off exponentially as

$$V(\xi) = \frac{A}{\eta^{1/4}} exp\left(-\frac{F^{1/2}}{3}\xi^{3/2} + \frac{E}{F^{1/2}}\xi^{1/2}\right) \qquad (5)$$

The asymptotic behavior of $U(\eta)$ at $\eta \to \infty$ shows that the problem as a whole is of an unbound character

$$U(\eta) = \frac{B}{\eta^{1/4}} sin\left(-\frac{F^{1/2}}{3}\eta^{3/2} + \frac{E}{F^{1/2}}\eta^{1/2} + \Phi\right) \qquad (6)$$

where A, B, Φ are constants for given F - electric field strength and E - energy level. A numerical solution of this equation is called as an ``exact numerical solution'' below. Using equations 3 and 4 the series in F for the Stark energy can be found:

$$\begin{aligned}
E = &-\frac{1}{2n^2} + \frac{3}{2}n(n_1 - n_2)F - \frac{n^4}{16}[17n^2 - 3(n_1 - n_2)^2 - 9m^2 + 19]F^2 \\
&+ \frac{3}{32}n^7(n_1 - n_2)[23n^2 - (n_1 - n_2)^2 + 11m^2 + 39]F^3 \\
&- \frac{n^{10}}{1024}[5487n^4 + 35182n^2 - 1134m^2(n_1 - n_2)^2 \\
&+ 1806n^2(n_1 - n_2)^2 - 3402n^2m^2 + 147(n_1 - n_2)^4 - 549m^4 \\
&+ 5754(n_1 - n_2)^2 - 8622m^2 + 16211]F^4 \\
&+ \frac{3}{1024}n^{13}(n_1 - n_2)[10563n^4 + 90708n^2 + 220m^2(n_1 - n_2)^2 \\
&+ 98n^2(n_1 - n_2)^2 + 772n^2m^2 - 21(n_1 - n_2)^4 + 725m^4 \\
&+ 780(n_1 - n_2)^2 + 830m^2 + 59293]F^5 + 0(F^6)
\end{aligned} \qquad (7)$$

The value of Γ is inversely proportional to the lifetime of the atomic state. Asymptotic formulae for Γ can be obtained by different ways:

$$\Gamma = \frac{(4R)^{2n_2+m+1} e^{-\frac{2R}{3}}}{n^3 n_2!(n_2+m)!}$$

$$\times \left(1 - \frac{n^3}{4}\left(34n_2^2 + 34n_2 m + 46n_2 + 7m^2 + 23m + \frac{53}{3}\right)F + 0(F^2)\right) \qquad (8)$$

where $$R = \frac{(-2E_o)^{3/2}}{F} \qquad (9)$$

Expanding R in power series of F, we obtain

$$\Gamma = \left(\frac{4}{Fn^3}\right)^{2n_2+m+1} exp\left[3(n_1-n_2) - \frac{2}{3Fn^3}\right]\frac{1}{n^3 n_2!(n_2+m)!}$$

$$\times(1 - \frac{n^3}{8}[36n(n_1-n_2) - 21(n_1-n_2)^2 + 17n^2 + 68n_2^2 + 68n_2 m$$

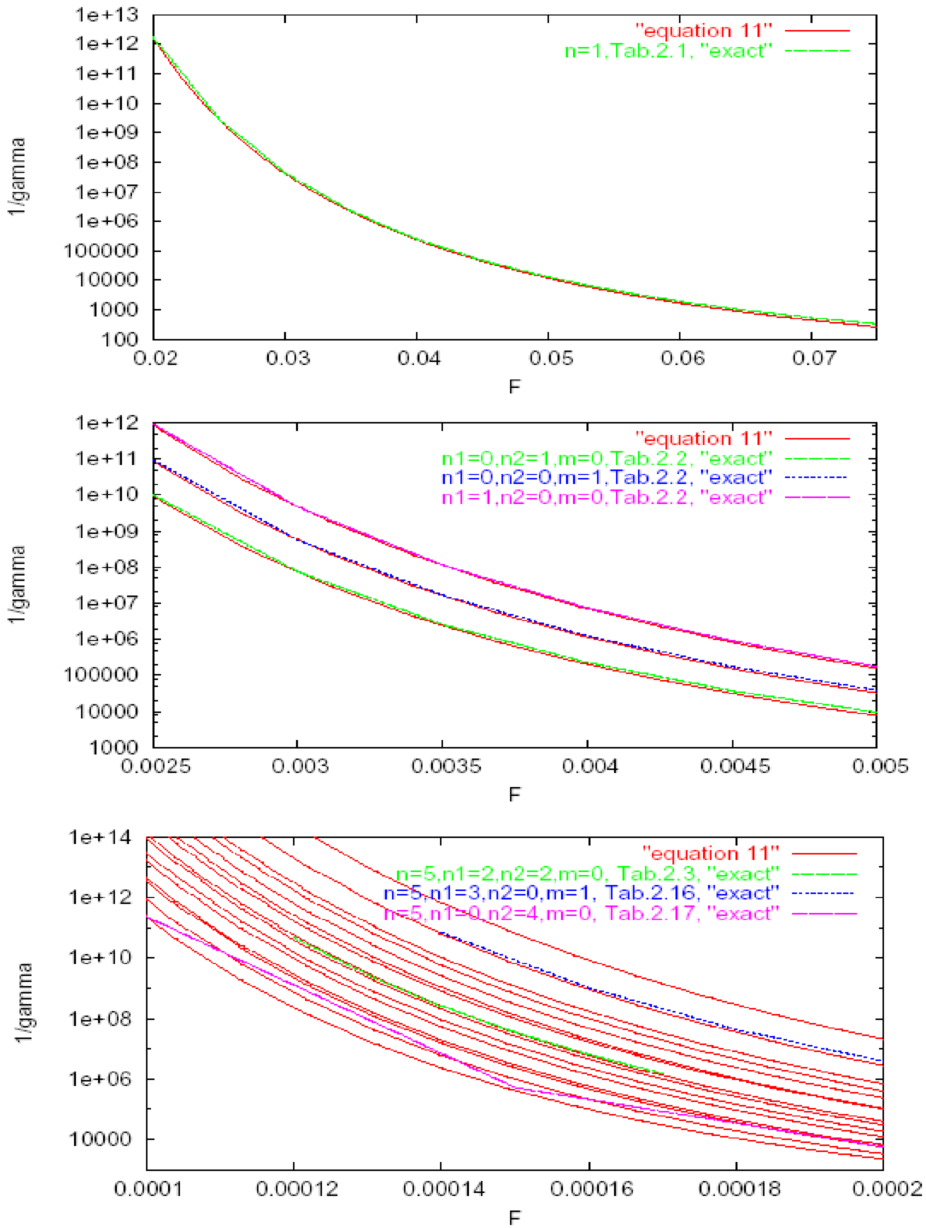
$$+92n_2 + 5m^2 + 46m + \frac{163}{3}]F + 0(F^2)) \qquad (10)$$

Semiempirical formula for Γ

$$\Gamma = \frac{(4R)^{2n_2+m+1}}{n^3 n_2!(n_2+m)!}$$
$$\times \ exp\left(-\frac{2}{3}R - \frac{n^3F}{4}\left(34n_2^2 + 34n_2 m + 46n_2 + 7m^2 + 23m + \frac{53}{3}\right)\right) \qquad (11)$$

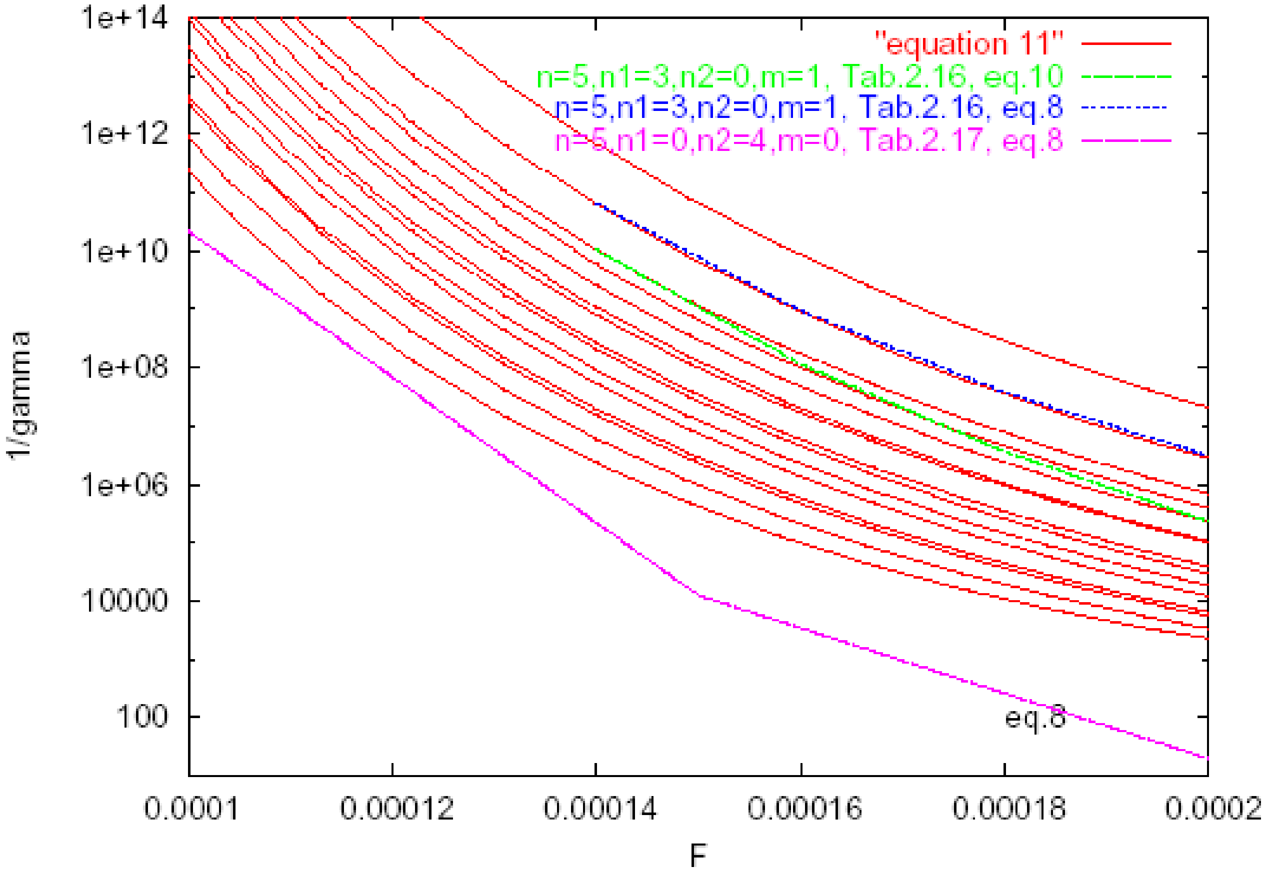
References:

[1] Rydberg states of atoms and molecules, Editors: R.F.Stebbings and F.B.Dunning, Department of Space Physics and Astronomy Rice University, Cambridge University Press 1983, pp.31-71. R.J.Damburg and V.V.Kolosov, "Theoretical studies of hydrogen Rydberg atoms in electric fields".



Comparison of calculated lifetime $T=1/\Gamma$ of Stark states hydrogen atoms using "exact numerical solution" and using equation 11 for n=1 (top), n=2 (middle) and n=5 (bottom). Atomic units are used. An "exact numerical solution" lifetime is taken from Tables 2.1 – 2.3 and 2.16, 2.17 of [1].

[1] Rydberg states of atoms and molecu-les, Editors: R.F.Stebbings and F.B. Dun-ning, Department of Space Physics and Astronomy Rice University, Cambridge University Press 1983, pp.31-71. R.J.Damburg and V.V.Kolosov, ``Theoretical studies of hydrogen Rydberg atoms in electric fields"



Comparison of calculated lifetime T=1/ Γ of Stark states hydrogen atoms using equation 8, 10 and 11 for n=5 (Table 2.16 and 2.17 [1]). Atomic units are used. The equation 8 gives one-two order of magnitude different results compared to equations 10 and 11.

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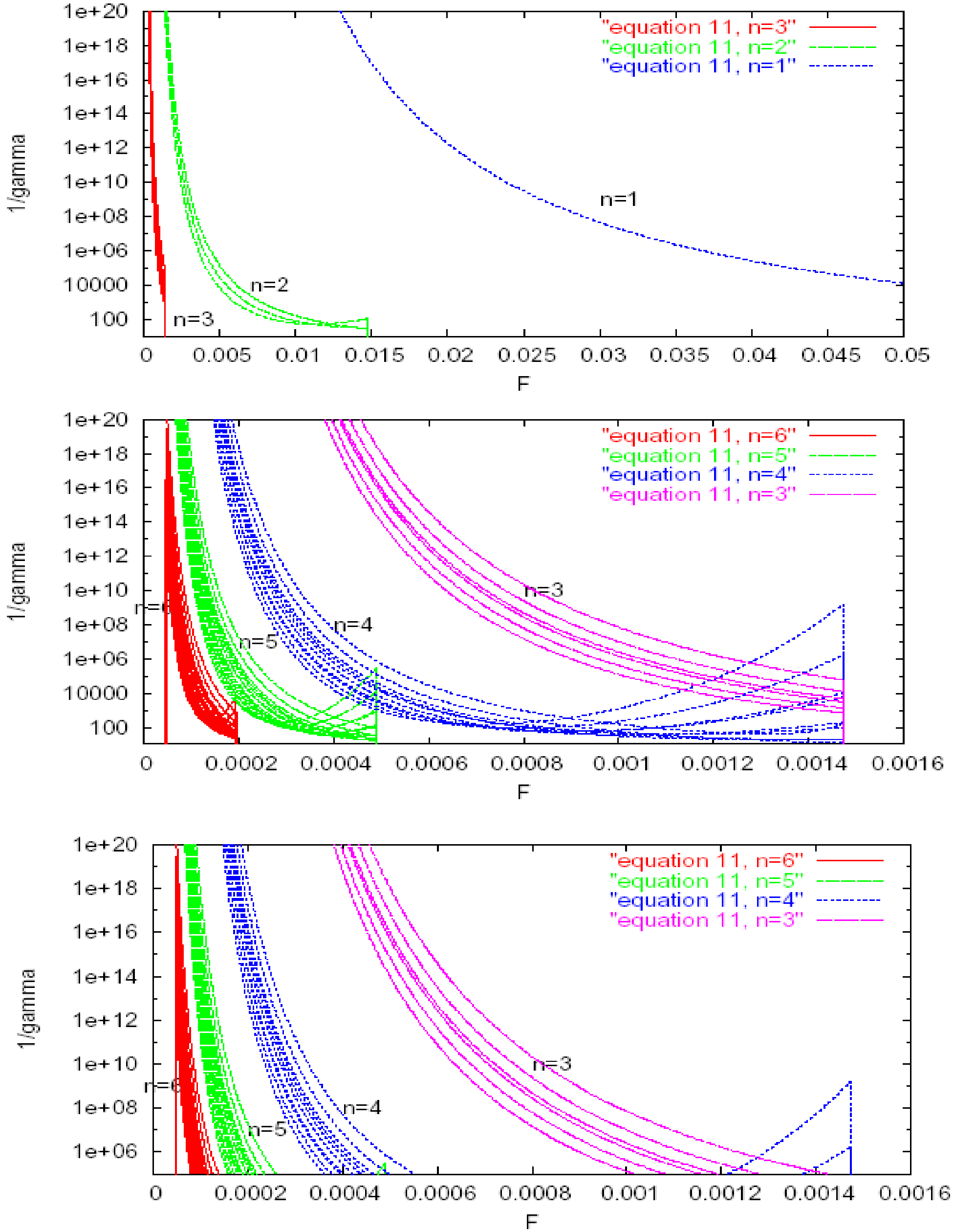
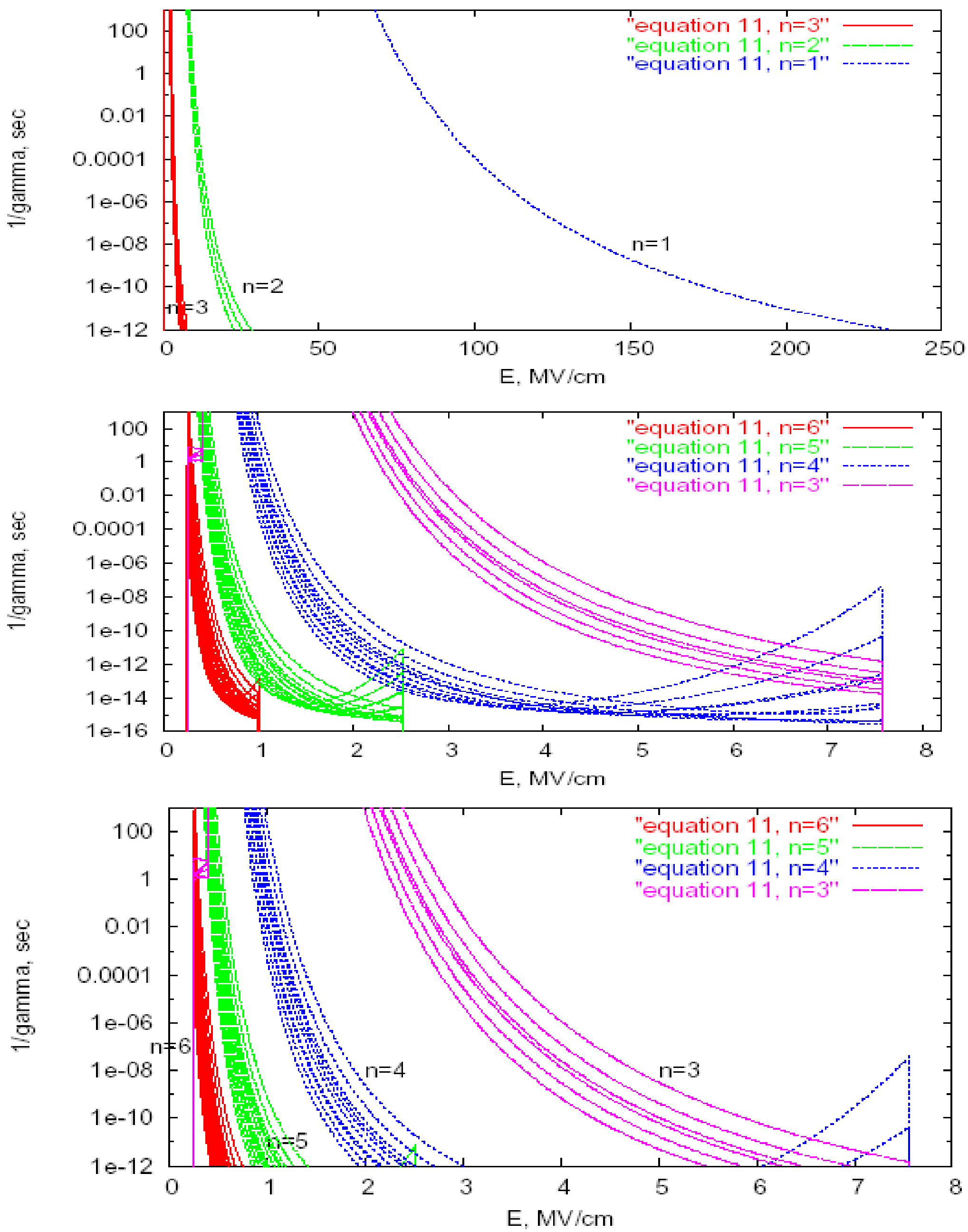


Calculated lifetime T=1/ Γ of Stark states hydrogen atoms in electric field using equation 11. Top - n=1-3, middle and bottom n=3-6. Atomic units are used.



Calculated lifetime $T=1/\Gamma$ of Stark states hydrogen atoms in electric field using equation 11. Top – n=1-3, middle and bottom n=3-6. Lifetime is in a particle own rest frame, electric field is in MV/cm. The semi-empirical formula 11gives wrong results for lifetime below 1.e-14 sec with minimum instead of asymptotic falling to zero curve. Lifetime of 1.e-14 sec corresponds to the mean decay length of (0.03-0.3)mm for Pc=(10-100) GeV hydrogen atoms, which in most practical cases may be assumed equal to zero. In the region of an order of magnitude higher lifetime (>1.e+14 sec) the equation 11gives good results.

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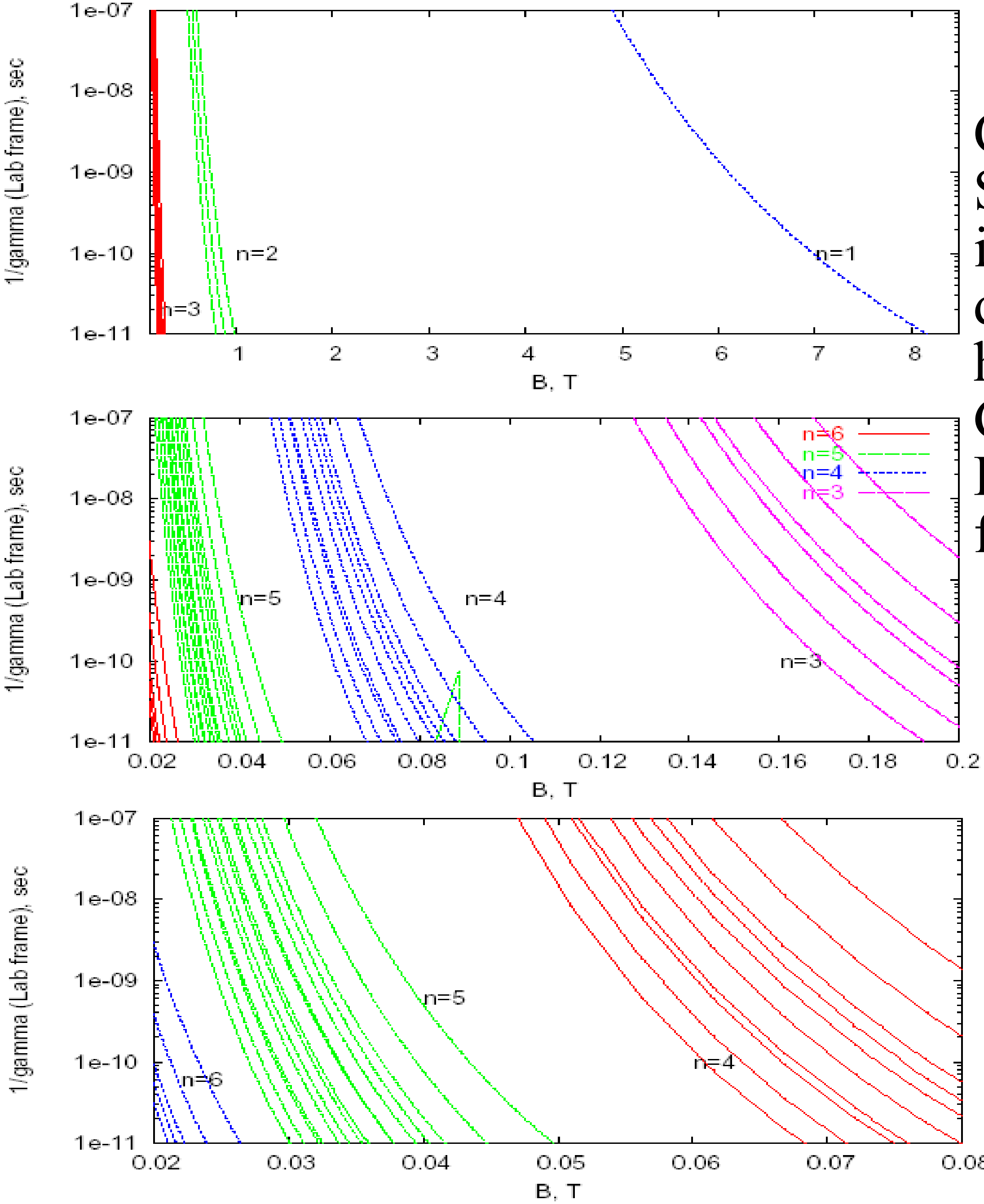

Calculated lifetime $T=1/\Gamma$ of Stark states hydrogen atoms in magnetic field correspon-ding to electric field for hydrogen atoms of $E_{kinetic} = 8$ GeV using equation 11. Lifetime is in a laboratory frame.

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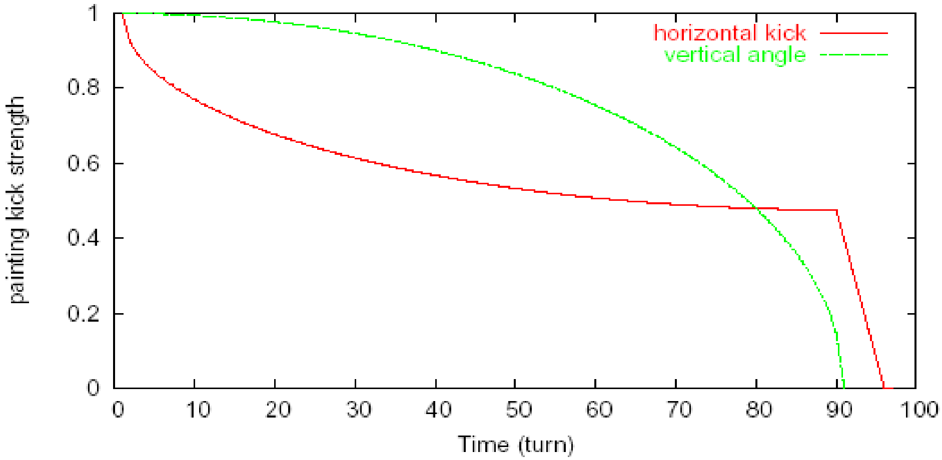

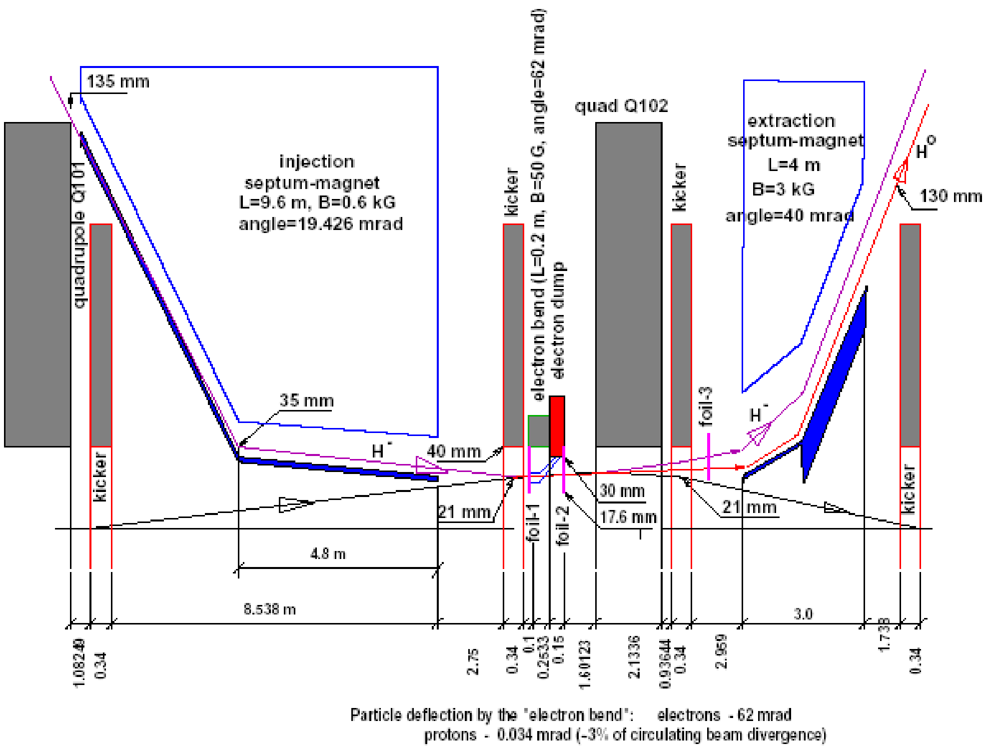

Vertical kicker-magnet strength and horizontal angle of the beam in the foil during injection (top). The kicker strength decreases fast to ~60% of maximum during 20 turns, and then slowly drops to 50% during another 70 turns. An unstripped part of the beam after interaction with the foil - the Ho Stark states hydrogen atoms - may be stripped to protons by a magnetic field of accelerator elements.

The stripping foil is located at the exit of painting kicker number 2, very close to the kicker edge in the fringe field of the magnet.

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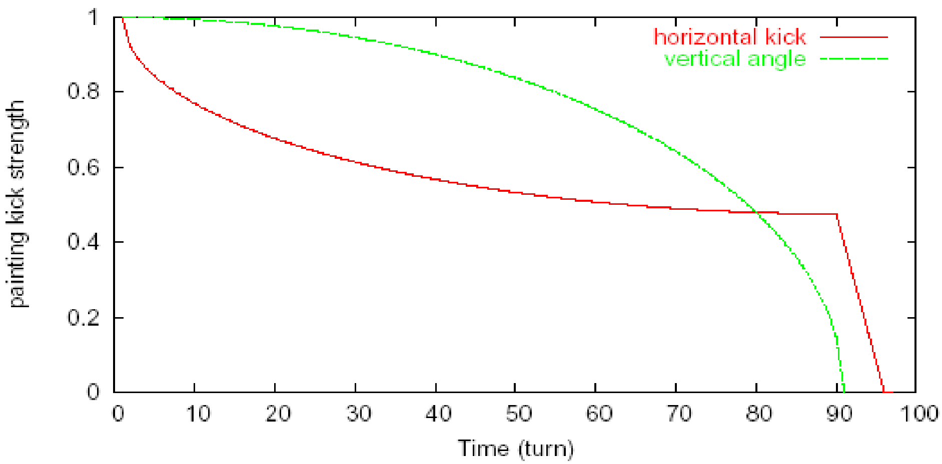

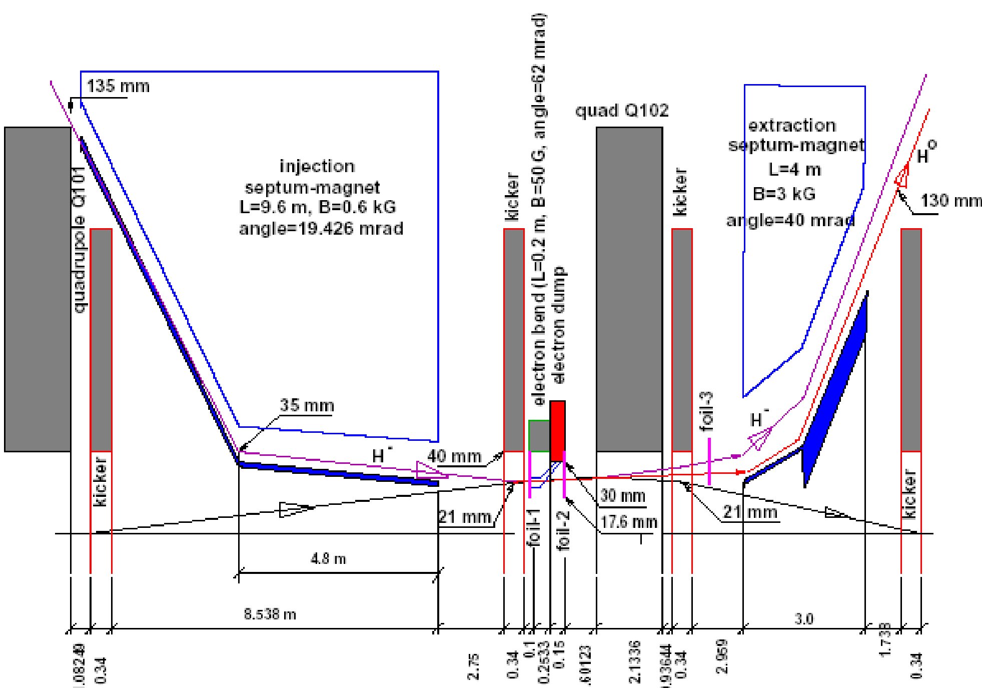

Particle deflection by the "electron bend": electrons - 62 mrad
protons - 0.034 mrad (~3% of circulating beam divergence)

The kicker magnet field is chosen such a way that during injection the magnetic field provides stripping of Stark states hydrogen atoms with principal quantum number n=5 to protons. This corresponds to kicker length of 0.34 m and maximum field of 0.1 T. At these parameters of magnet, the magnetic field during ~80% of injection cycle is in the range of (0.05- 0.06)T.

The probability of H(-) stripping by magnetic field of kicker magnet number 2 during the first turn of injection (B=0.1T) is 0.002. It drops to 0.00005 during five turns (B= 0.08T).This gives stripping of 5.e-05 of injected beam (7.5e+09ppp) or 7W of power lost in the injection region.

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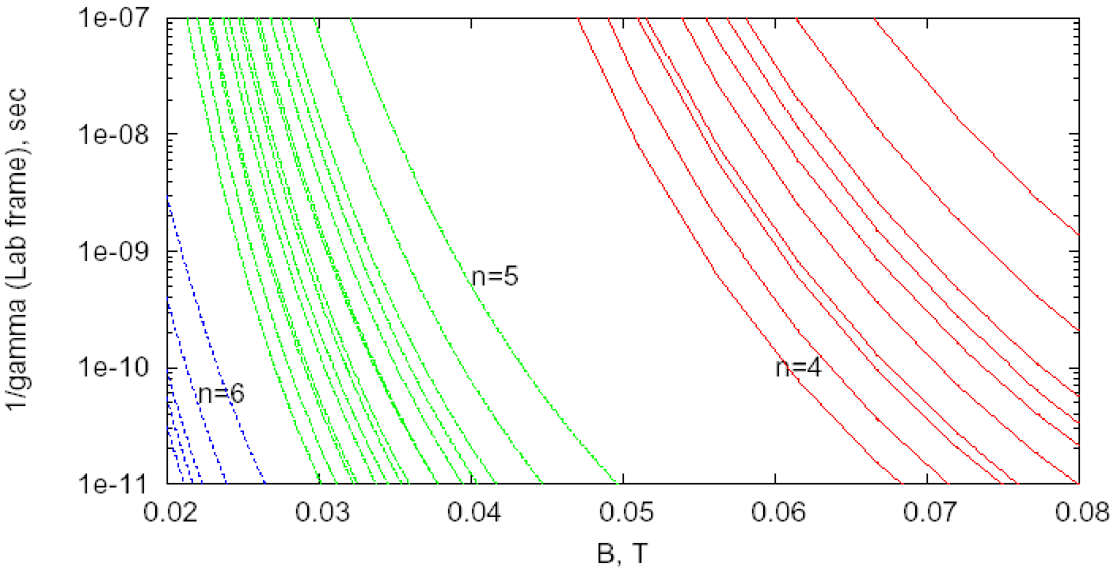
n	B	lifetime	mean decay length		
	T	sec	m		
injection kicker No.2, $L_{field} = (1-2)\ cm$					
4	0.1	>1.e-11	0.003	stripped	
4	0.06	>1.e-10	0.03	unstripped	
5	0.06	<1.e-12	0.0003	stripped	go to circulating beam
4	0.05	>1.e-08	3.0	unstripped	
5	0.05	<1.e-11	0.003	stripped	go to circulating beam
quadrupole Q102, $L_{field} = 2.1\ m$, no particles with n>4					
4	0.025	>1.e+02		unstripped	
injection kicker No.3, $L_{field} = 0.34\ m$, no particles with n>4					
4	0.1	<1.e-10	0.03	stripped	go to circulating beam
4	0.06	1.e-10 - 1.e-06	0.03 - 300	some are stripped	go to beam halo
4	0.05	1.e-08 - 1.e-04	3 - 30000	unstripped	go to beam dump



A stripping probability of E=8 GeV H(o) Stark states hydrogen atoms in the kicker magnets and quadrupole Q102. We assumed here that H(o) atoms pass a distance of (1-2) cm in a maximum fringe field of the kicker magnet number 2. This distance is enough for H(o) atoms with n=5 to be stripped. All atoms with n=5 are stripped to protons and go to the circulating beam without changing emittance of the beam, some atoms with n=4 are left unstripped and go to the beam dump and, unfortunately, some fraction of them is stripped along the kicker number 3. These protons will contribute to the circulating beam halo and cause losses behind the kicker.

Experimental data on H(o) yields produced by foil stripping of 800-MeV H(-) at 100 $\mu g/cm^2$ graphite foil [2].

n=1, 2	93.3%
n=3	3.6%
n=4	1.5%
n=5	0.7%
n=6	0.3%
n>6	0.6%
total	100%

Assuming that distribution of yields of different states n almost does not depend on the foil thickness, one may expect ~97% (n=1, 2 and 3) of the total amount of H(o) to end at the external beam dump, ~1% (n=4) contribute beam halo, and ~2% go to the circulating beam without emittance increase.

Foil number 3 is used for a final stripping of atoms (n=1-4) behind the kicker number 3 to reduce field stripping and losses along the extraction magnet and beam line.

[2] Measurement of H(-), H(o), and H(+) yields produced by foil stripping of 800-MeV H(-) ions, M.S.Gulley, P.B.Keating, H.C.Bryant, E.P.MacKerrow, W.A.Miller, D.C.Rislove, S.Cohen, J.B.Donahue, D.H.Fitzgerald, S.C.Frankle, D.J.Funk, R.L.Hutson, R.J.Macek, M.A.Plum, N.G.Stanciu, O.B.van Dyck, C.A.Wilkinson, C.W.Planner, Physical Review A, Volume 53, Number 5, May 1996.

Conclusions

The reason for this calculation was to show the gap between different n states (in a region of n=1-6) that would allow an external B field to separate them. States n=4 have big lifetime to end at the dump (at B=0.05-0.06T, 1.e-08sec, s=3m), but states n=5 have very small lifetime (1.e-11sec, s=3mm) and are stripped very shortly after the foil in the fringe fields of injection kicker-magnet.

It was shown that all atoms with n=5 are stripped to protons and go to circulating beam without changing emittance of the beam, some atoms with n=4 are left unstripped and go to the beam dump. Unfortunately, some fraction of atoms with n=4 is stripped along the injection kicker number 3. These protons will contribute to the circulating beam halo and cause losses behind the kicker.