

Dogleg Correction

W. Chou, February 27, 2003

1. Brainstorming:

- Enlarge the spacing between doglegs (Sasha)
- Relocate Dogleg-13 (Jim, Weiren)
- Lower the dogleg operation current (Ray)
- Use 3 legs instead of 4 legs (Milorad, Jim)
- Remove Dogleg-13 (Jim, Weiren)
- Use 45-degree legs (Jim, Weiren)
- Use correction wedge magnets (Weiren)
- Use correction quadrupoles (Weiren)
- Remove doglegs by employing horizontal extraction (Chuck)

2. first order perturbation theory:

A thin quadrupole at location 1 with strength:

$$\begin{aligned} q &= B'L/B\rho = 1/f & (\text{H}) \\ -q & & (\text{V}) \end{aligned}$$

(Recall that for a dogleg in L3, $q \equiv \theta^2/L = 0.02244 \text{ m}^{-1}$ in H-plane, $q = 0$ in V-plane)

- The tune change:

$$\begin{aligned}\Delta\nu(x) &= q \times \beta_1(x) / 4\pi \\ \Delta\nu(y) &= -q \times \beta_1(y) / 4\pi\end{aligned}$$

- The beta-wave at location 2:

$$\begin{aligned}\Delta\beta_2(x) &= -q \times \beta_1(x) \times \beta_2(x) \times \sin 2\psi(x) \\ \Delta\beta_2(y) &= q \times \beta_1(y) \times \beta_2(y) \times \sin 2\psi(y)\end{aligned}$$

β_1, β_2 = beta-functions of the unperturbed lattice

$\psi = \mu_2 - \mu_1$ = phase advance from 1 to 2

- The dispersion wave at location 2:

$$\Delta D_2 = -q \times D_1 \times \{\beta_1(x) \times \beta_2(x)\}^{1/2} \times \sin \psi(x)$$

D_1 = dispersion of the unperturbed lattice

(Note that the beta-wave frequency is twice that of the dispersion wave.)

3. Correction using quadrupoles:

- To correct the beta-wave:
 - Same sign of q , $(n+1/2)\pi$ apart, or
 - Opposite sign of q , $n\pi$ apart.
- To correct the dispersion wave:
 - Same sign of q , $(2n+1)\pi$ apart, or
 - Opposite sign of q , $2n\pi$ apart.
- To correct both:
 - Opposite sign of q , $2n\pi$ apart.

4. Difficulties in correcting the dogleg effect:

- The dogleg effect is in the H-plane only. But a correction quadrupole, while correcting the beta-wave in the H-plane, will generate a beta-wave in the V-plane.
- In a set of doglegs, there are 4 legs and 8 edges. The 8 edges cannot be approximated by a single focal lens, because the phase difference between the first and last edge is about 30° (in H-plane).
- The beta-wave and dispersion wave cannot be corrected inside the $2n\pi$ section.

5a. Effects of removing one dogleg:

Case	$\Delta v(x)$	$\beta_{\max}(x)$ (m)	$D_{\max}(x)$ (m)	Max $\epsilon(x)$ (mm-mrad)	Acceptance Increase
Two doglegs	0.064	46.5	5.89	8.5π	(base)
Dogleg-03 only	0.031	40.5	4.36	11.7π	38%
Dogleg-13 only	0.028	39.9	4.25	12.1π	42%
No dogleg	0	33.7	3.19	16.2π	91%

$$A = \{\beta_{\max} \times \epsilon_N / \beta\gamma\}^{-1/2} + D_{\max} \times \Delta p/p + \text{c.o.d.}$$

Parameters used in this calculation:

$$E = 400 \text{ MeV}$$

$$\beta\gamma = 1.0$$

$$\epsilon = \epsilon_N$$

$$\Delta p/p = \pm 0.13\%$$

$$\text{Closed orbit distortion (c.o.d.)} = 3 \text{ mm}$$

$$\text{Aperture} = \pm 1.2 \text{ inch}$$

Dogleg parameters: (using Lackey's calibration)

$$\text{Effective length} = 0.24722 \text{ m}$$

$$\text{Bending angle scaling: } 26 \text{ mm} / 446.047 \text{ A}$$

$$\text{Dogleg-03: } I = 480 \text{ A} \rightarrow \theta_3 = 62.51 \text{ mrad}$$

$$\text{Dogleg-13: } I = 460 \text{ A} \rightarrow \theta_{13} = 59.88 \text{ mrad}$$

5b. Effects of removing one dogleg: (higher Dogleg-03 current)

Case	$\Delta v(x)$	$\beta_{\max}(x)$ (m)	$D_{\max}(x)$ (m)	Max $\epsilon(x)$ (mm-mrad)	Acceptance Increase
Two doglegs	0.082	48.7	6.84	7.1π	(base)
Dogleg-03 only	0.045	42.7	4.93	10.4π	46%
Dogleg-13 only	0.030	39.2	4.23	12.3π	73%
No dogleg	0	33.7	3.19	16.2π	128%

$$A = \{\beta_{\max} \times \epsilon_N / \beta\gamma\}^{-1/2} + D_{\max} \times \Delta p/p + \text{c.o.d.}$$

Parameters used in this calculation:

$$E = 400 \text{ MeV}$$

$$\beta\gamma = 1.0$$

$$\epsilon = \epsilon_N$$

$$\Delta p/p = \pm 0.13\%$$

$$\text{Closed orbit distortion (c.o.d.)} = 3 \text{ mm}$$

$$\text{Aperture} = \pm 1.2 \text{ inch}$$

Dogleg parameters: (using Lackey's calibration)

$$\text{Effective length} = 0.24722 \text{ m}$$

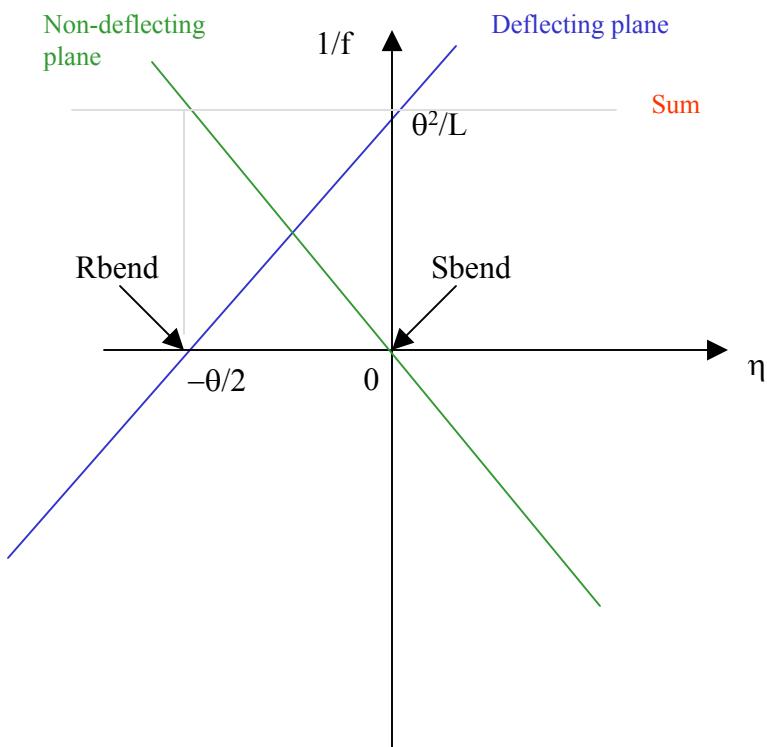
$$\text{Bending angle scaling: } 26 \text{ mm} / 446.047 \text{ A}$$

$$\text{Dogleg-03: } I = 572 \text{ A} \rightarrow \theta_3 = 74.49 \text{ mrad}$$

$$\text{Dogleg-13: } I = 459.8 \text{ A} \rightarrow \theta_{13} = 59.88 \text{ mrad}$$

Edge Focusing of a Wedge Magnet

Focusing strength as a function of edge angle η :



L = magnet length

θ = bend angle

η = edge angle

Sector magnet (S_{bend}): $\eta = 0$

Rectangular magnet (R_{bend}): $\eta = -\theta/2$

Focusing strength: (> 0 focusing)

Deflecting plane:

$$1/f = 2\eta\theta/L + \theta^2/L$$

Non-deflecting plane:

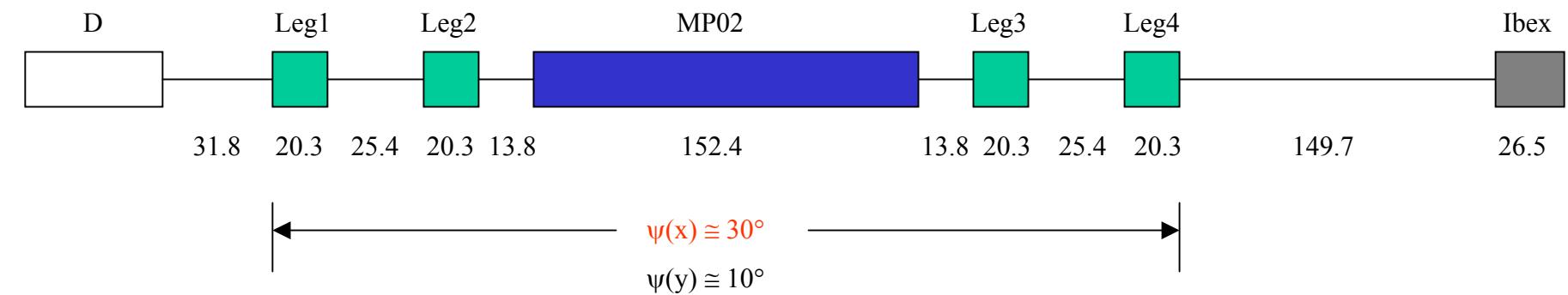
$$1/f = -2\eta\theta/L$$

Sum = θ^2/L (always focusing!)

To be more precise, this sum is from the body focusing in the deflecting plane. The sum of the edges in the two planes is zero.

L3 Dogleg Layout

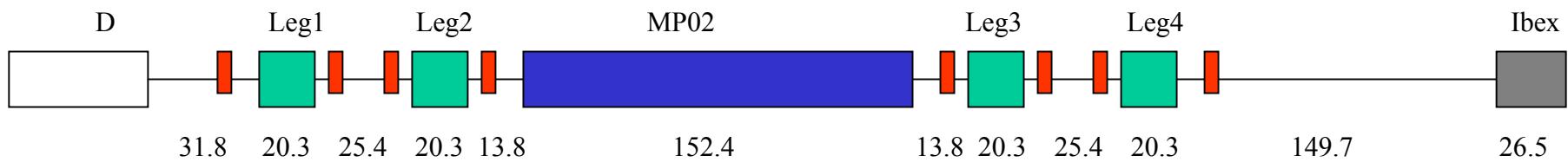
(unit in cm)



L3 Dogleg Correction Using 8 Quads

(unit in cm)

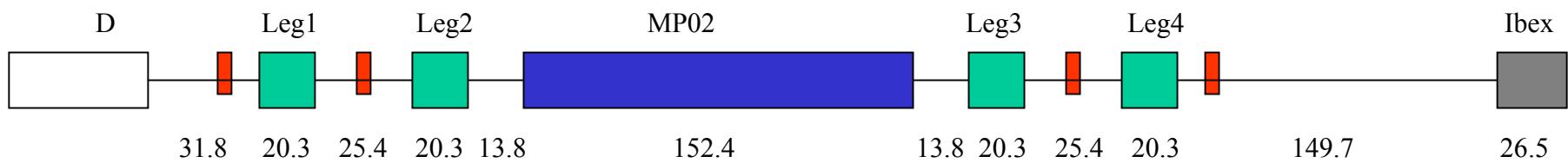
█ Correction quad (QD)



L3 Dogleg Correction Using 4 Quads

(unit in cm)

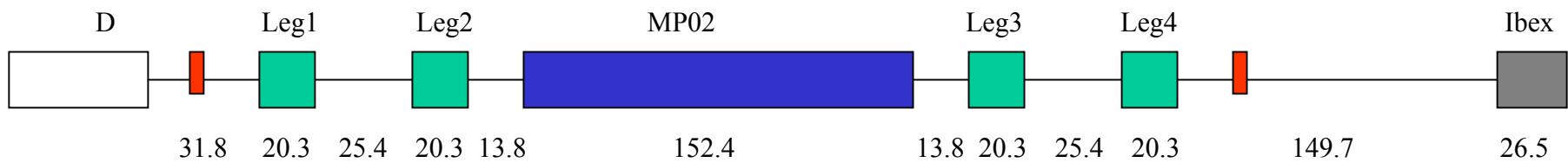
■ Correction quad (QD)



L3 Dogleg Correction Using 2 Quads

(unit in cm)

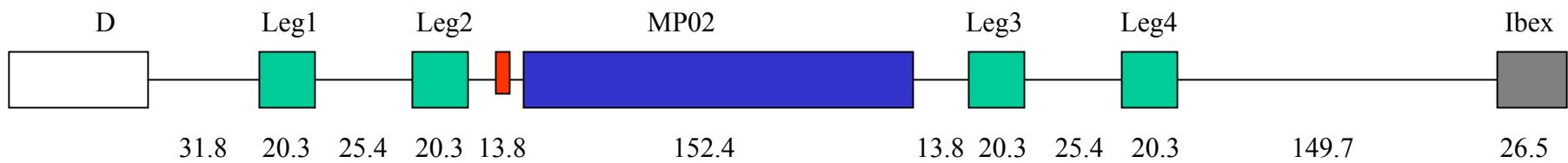
■ Correction quad (QD)



L3 Dogleg Correction Using 1 Quads

(unit in cm)

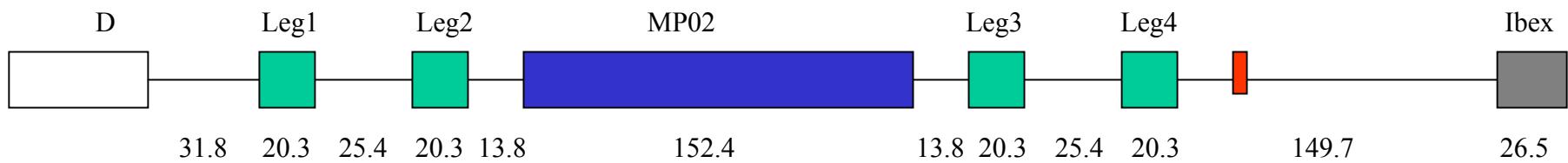
■ Correction quad (QD)



L3 Dogleg Correction Using 1 Quads

(unit in cm)

■ Correction quad (QD)



Correction with Different Number and Layout of Quadrupoles

NO DOGLEGS:

```
-----  
total length = 474.202754 Qx = 6.699721 Qy = 6.799965  
delta(s) = 0.000000 mm Qx' = 3.671194 Qy' = -11.119681  
alfa = 0.337183E-01 betax(max) = 33.679540 betay(max) = 20.460394  
gamma(tr) = 5.445873 Dx(max) = 3.189910 Dy(max) = 0.071312  
Dx(r.m.s.) = 2.480809 Dy(r.m.s.) = 0.023090  
-----
```

DOGLEG03 WITH NO CORRECTION QUADS:

```
-----  
total length = 474.202754 Qx = 6.730798 Qy = 6.799964  
delta(s) = 0.000000 mm Qx' = 2.582392 Qy' = -7.069763  
alfa = 0.332114E-01 betax(max) = 40.449277 betay(max) = 20.460649  
gamma(tr) = 5.487273 Dx(max) = 4.353720 Dy(max) = 0.077498  
Dx(r.m.s.) = 2.509874 Dy(r.m.s.) = 0.025280  
-----
```

DOGLEG03 WITH 8 CORRECTION QUADS (ONE FOR EACH EDGE):

```
-----  
total length = 474.210754 Qx = 6.699839 Qy = 6.923242  
delta(s) = 0.000000 mm Qx' = 3.635760 Qy' = -24.910214  
alfa = 0.337121E-01 betax(max) = 33.694484 betay(max) = 60.142373  
gamma(tr) = 5.446369 Dx(max) = 3.190579 Dy(max) = 0.095541  
Dx(r.m.s.) = 2.475334 Dy(r.m.s.) = 0.038359  
-----
```

DOGLEG03 WITH 4 CORRECTION QUADS (ONE FOR EACH LEG):

```
-----  
total length = 474.206754 Qx = 6.699624 Qy = 6.923497  
delta(s) = 0.000000 mm Qx' = 3.605126 Qy' = -25.202650  
alfa = 0.337120E-01 betax(max) = 33.898100 betay(max) = 60.358225  
gamma(tr) = 5.446375 Dx(max) = 3.206717 Dy(max) = 0.095311  
Dx(r.m.s.) = 2.478046 Dy(r.m.s.) = 0.038136  
-----
```

DOGLEG03 WITH 2 CORRECTION QUADS (BEFORE #1, AFTER #4):

```
-----  
total length = 474.204754 Qx = 6.699264 Qy = 6.922219  
delta(s) = 0.000000 mm Qx' = 3.637774 Qy' = -24.496359  
alfa = 0.337099E-01 betax(max) = 33.865929 betay(max) = 59.156691  
gamma(tr) = 5.446547 Dx(max) = 3.194984 Dy(max) = 0.067170  
Dx(r.m.s.) = 2.479314 Dy(r.m.s.) = 0.028055  
-----
```

DOGLEGO3 WITH 1 CORRECTION QUAD (AFTER #2):

```
-----  
total length = 474.203754 Qx = 6.700177 Qy = 6.933350  
delta(s) = 0.000000 mm Qx' = 3.376615 Qy' = -30.246601  
alfa = 0.336961E-01 betax(max) = 35.348193 betay(max) = 69.871017  
gamma(tr) = 5.447658 Dx(max) = 3.315170 Dy(max) = 0.204508  
Dx(r.m.s.) = 2.479651 Dy(r.m.s.) = 0.112906  
-----
```

DOGLEGO3 WITH 1 CORRECTION QUAD (AFTER #4):

```
-----  
total length = 474.203754 Qx = 6.703247 Qy = 6.931417  
delta(s) = 0.000000 mm Qx' = 4.034909 Qy' = -23.689408  
alfa = 0.336837E-01 betax(max) = 37.393908 betay(max) = 67.944646  
gamma(tr) = 5.448663 Dx(max) = 3.468042 Dy(max) = 0.067902  
Dx(r.m.s.) = 2.481822 Dy(r.m.s.) = 0.024100  
-----
```

Comparison of 90-degree and 45-degree Doglegs

NO DOGLEGS:

```
-----  
total length = 474.202754 Qx = 6.699721 Qy = 6.799965  
delta(s) = 0.000000 mm Qx' = 3.671194 Qy' = -11.119681  
alfa = 0.337183E-01 betax(max) = 33.679540 betay(max) = 20.460394  
gamma(tr) = 5.445873 Dx(max) = 3.189910 Dy(max) = 0.071312  
Dx(r.m.s.) = 2.480809 Dy(r.m.s.) = 0.023090  
xco(max) = 0.000000 yco(max) = 0.000000  
xco(r.m.s.) = 0.000000 yco(r.m.s.) = 0.000000  
-----
```

DOGLEGO3 WITH ONE 90-DEGREE LEG (#1):

```
-----  
total length = 474.202754 Qx = 6.708678 Qy = 6.799964  
delta(s) = 0.000000 mm Qx' = 3.564105 Qy' = -10.098851  
alfa = 0.335071E-01 betax(max) = 35.701317 betay(max) = 20.460437  
gamma(tr) = 5.463006 Dx(max) = 3.459935 Dy(max) = 1.017283  
Dx(r.m.s.) = 2.475067 Dy(r.m.s.) = 0.576976  
xco(max) = 0.000000 yco(max) = 0.000000  
xco(r.m.s.) = 0.000000 yco(r.m.s.) = 0.000000  
-----
```

DOGLEGO3 WITH TWO 90-DEGREE LEGS (#1, #2):

total length =	474.202754	Qx	=	6.717015	Qy	=	6.799964
delta(s) =	0.000000 mm	Qx'	=	3.354231	Qy'	=	-8.890775
alfa =	0.334717E-01	betax(max)	=	37.650486	betay(max)	=	20.460477
gamma(tr) =	5.465891	Dx(max)	=	3.780531	Dy(max)	=	0.094832
		Dx(r.m.s.)	=	2.479282	Dy(r.m.s.)	=	0.033662
		xco(max)	=	0.000000	yco(max)	=	0.000000
		xco(r.m.s.)	=	0.000000	yco(r.m.s.)	=	0.000000

DOGLEGO3 WITH ONE 45-DEGREE LEG (#1):

total length =	474.202754	Qx	=	6.704183	Qy	=	6.812854
delta(s) =	0.000000 mm	Qx'	=	4.319430	Qy'	=	-12.518876
alfa =	0.338704E-01	betax(max)	=	34.671178	betay(max)	=	22.328975
gamma(tr) =	5.433625	Dx(max)	=	3.401872	Dy(max)	=	1.096908
		Dx(r.m.s.)	=	2.489210	Dy(r.m.s.)	=	0.624357
		xco(max)	=	0.000000	yco(max)	=	0.000000
		xco(r.m.s.)	=	0.000000	yco(r.m.s.)	=	0.000000

DOGLEGO3 WITH TWO 45-DEGREE LEGS (#1, #2):

total length =	474.202754	Qx	=	6.708365	Qy	=	6.826078
delta(s) =	0.000000 mm	Qx'	=	3.669581	Qy'	=	-11.046799
alfa =	0.335649E-01	betax(max)	=	35.617577	betay(max)	=	24.497145
gamma(tr) =	5.458297	Dx(max)	=	3.533421	Dy(max)	=	0.708466
		Dx(r.m.s.)	=	2.474793	Dy(r.m.s.)	=	0.396466
		xco(max)	=	0.000000	yco(max)	=	0.000000
		xco(r.m.s.)	=	0.000000	yco(r.m.s.)	=	0.000000
