HALO FORMATION FROM WEAK SPACE CHARGE AND COLORED NOISE

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WARM-FLUID KV MODEL OF A CYLINDRICAL DC BEAM

S. Lund and R. C. Davidson, *Phys. Plasmas* **5**, 3028 (1998);
S. Strasburg and R. C. Davidson, *Phys. Rev. E* **61**, 5753 (2000).

Equation of motion for radial orbits :

$$\ddot{x} + [\eta^2 - \sqrt{\Gamma}(1-\eta^2)\cos\omega t]\,x = 0 \quad \text{for} \quad x < 1.0;$$

$$\ddot{x} + x - \frac{1-\eta^2}{x} = 0 \qquad \text{for } x \geq 1.0;$$

rms- matched with "$n = 1$" mode

η is the tune depression, Γ is the ratio of the electrostatic energy in the collective mode to the electrostatic energy in the equilibrium beam, and, for the lowest - order collective mode, $\omega = \sqrt{2(1+\eta^2)}$.

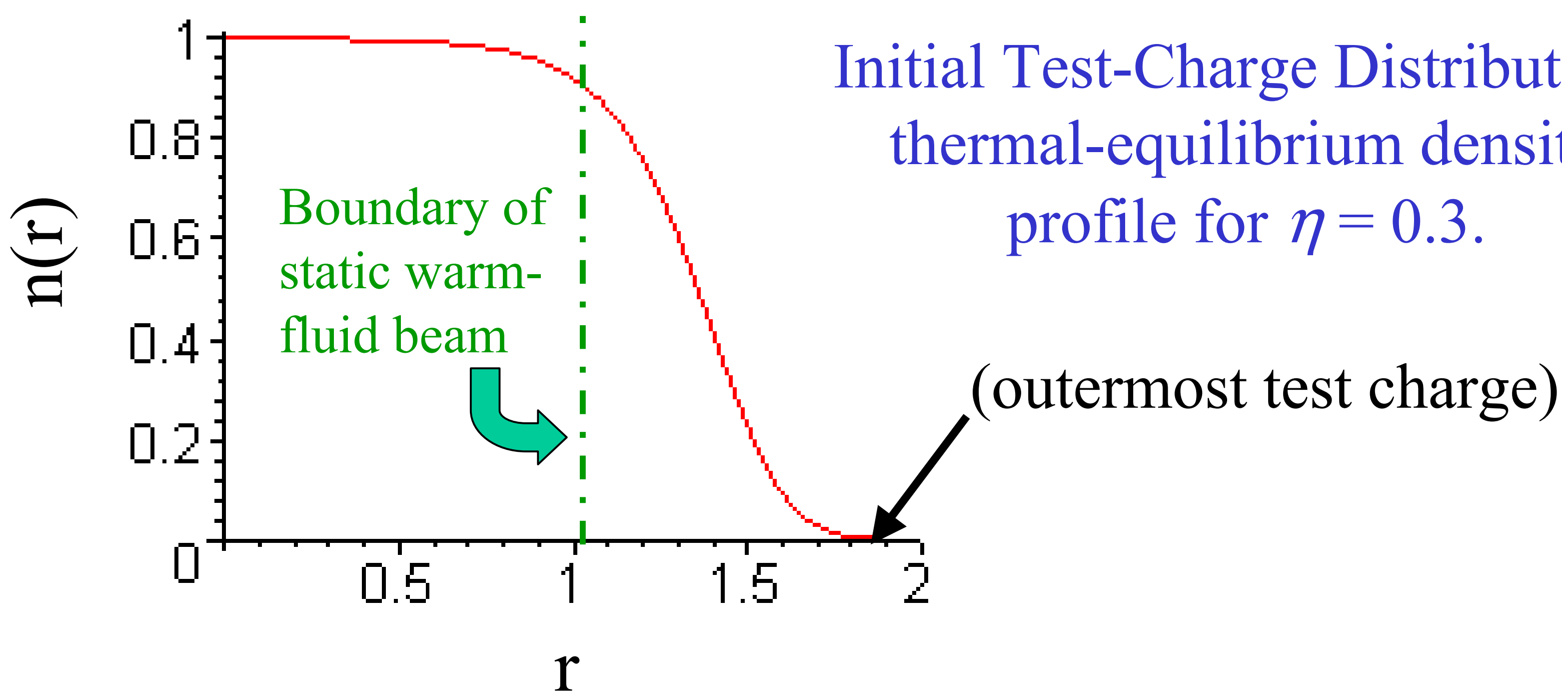
To include colored noise, we set $\omega \to \omega + \delta\omega$ and $\eta^2 \to \eta^2 + \omega\delta\omega$.

Consequently, noise now appears in *both* the net focusing and collective oscillation frequencies.

INVESTIGATIVE STRATEGY

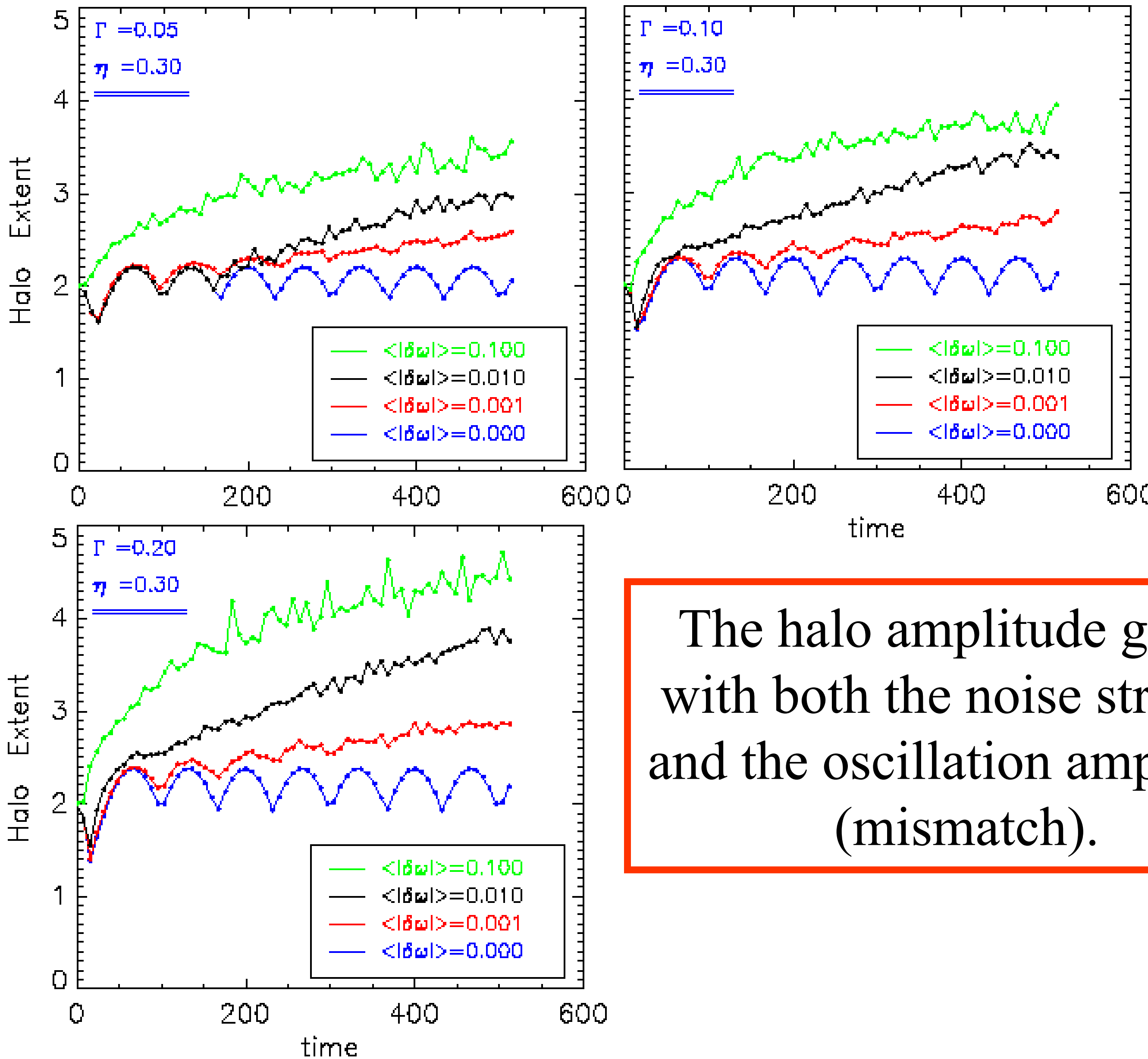
[I. V. Sideris and C. L. Bohn, *Phys. Rev. ST Accel. Beams* **7**, 104202 (2004)]

Populate the oscillating warm-fluid beam with 10^6 test charges distributed according to the thermal-equilibrium density profile. Radial orbits have zero initial velocity. Assign each orbit its own noise. Integrate and track.



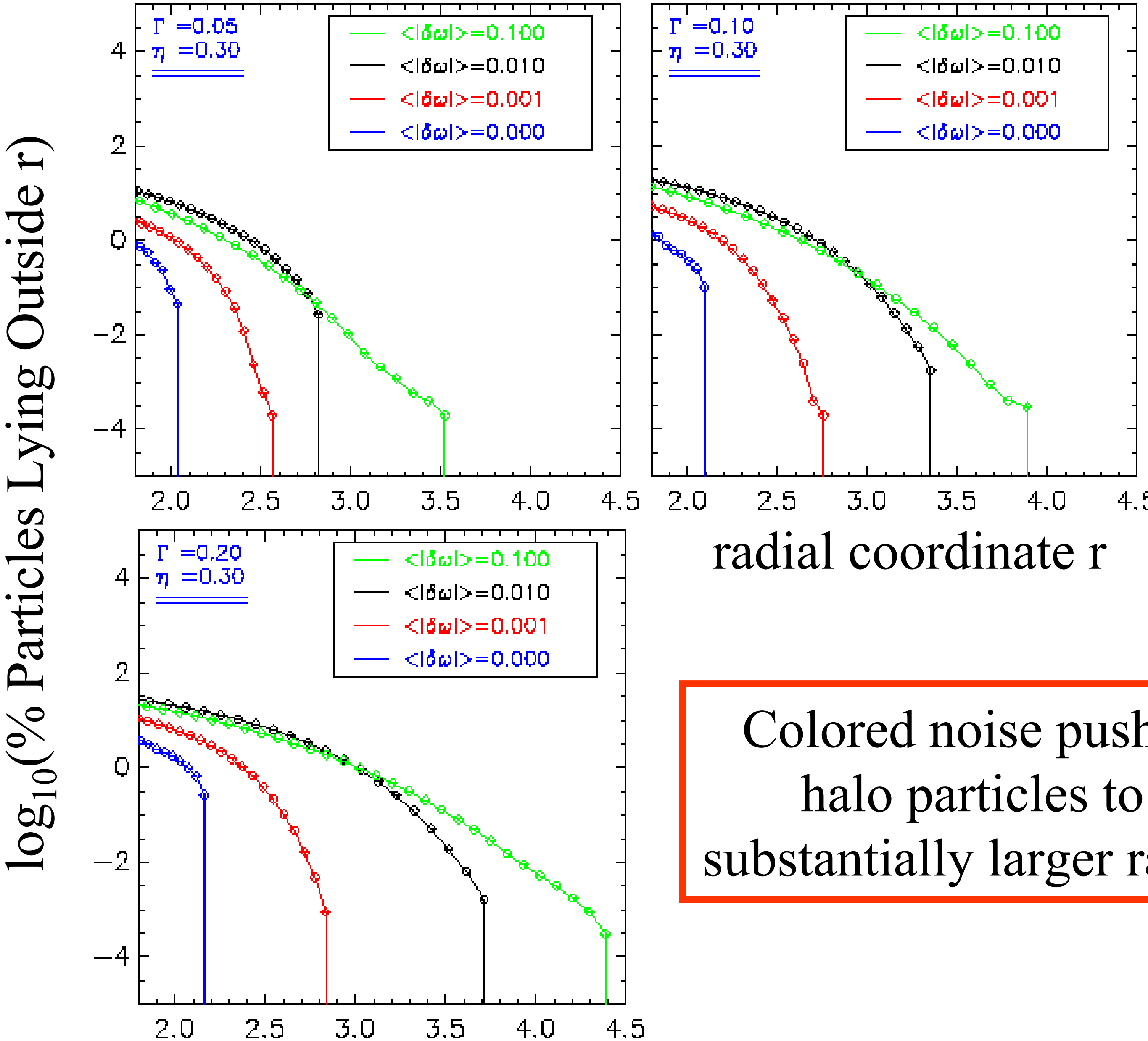
Initial Test-Charge Distribution: thermal-equilibrium density profile for $\eta = 0.3$.

HALO EXTENT vs. OSCILLATION AMPLITUDE



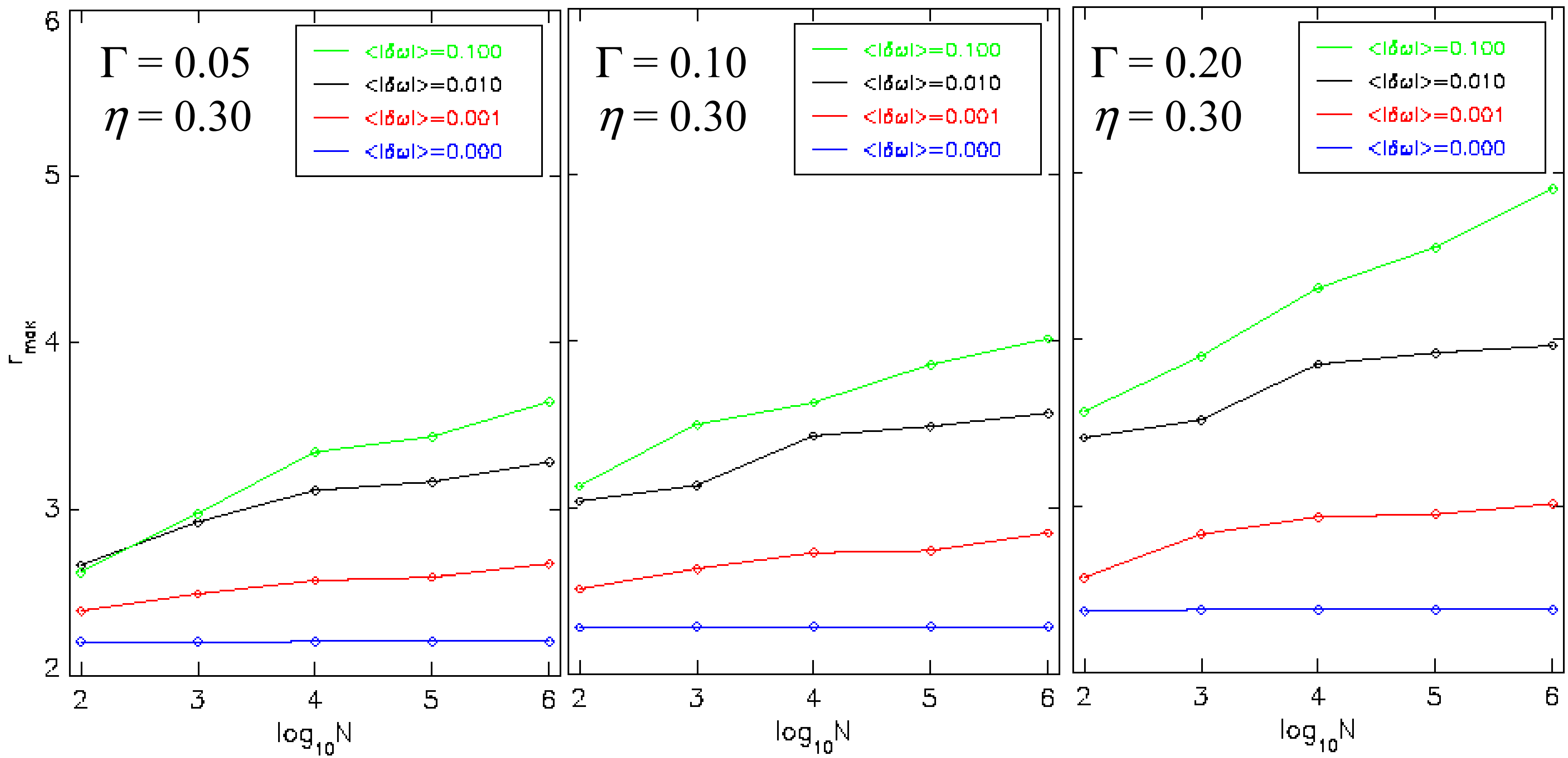
The halo amplitude grows with both the noise strength and the oscillation amplitude (mismatch).

HALO DENSITY PROFILE



Colored noise pushes halo particles to substantially larger radii!

HALO AMPLITUDE vs. NUMBER OF PARTICLES



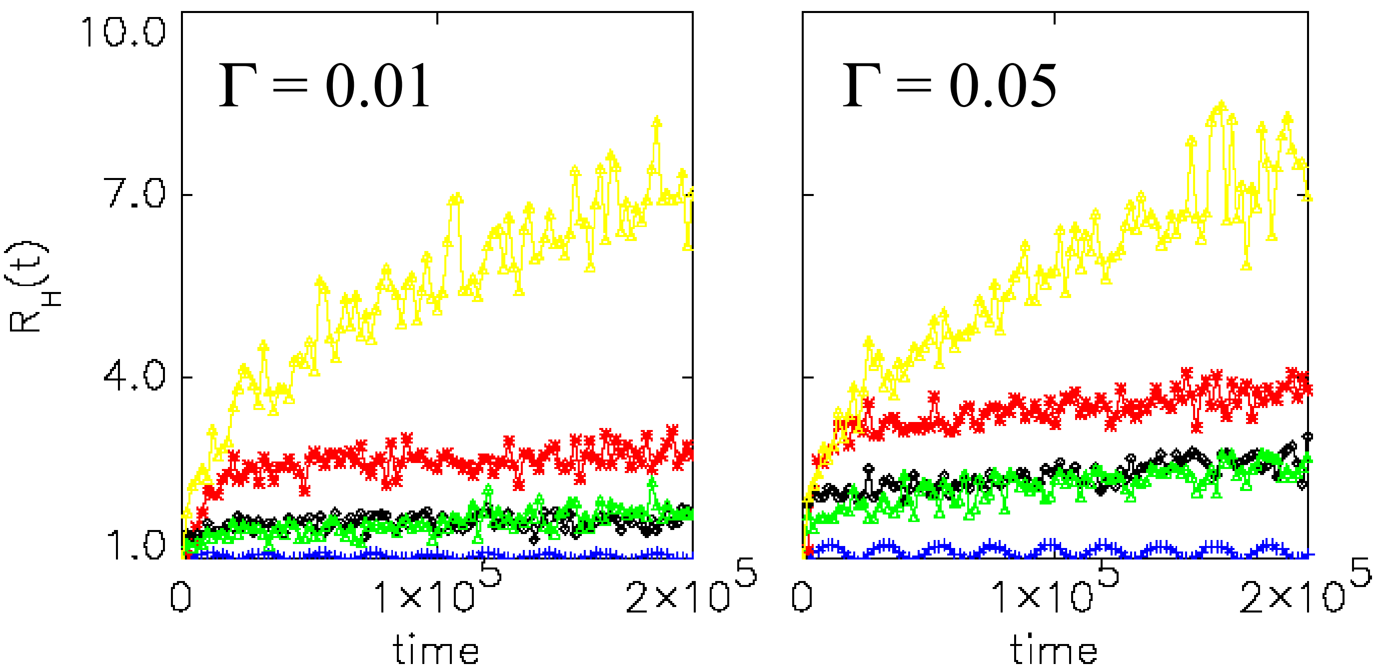
Regardless of noise strength or mismatch, the halo amplitude scales as $\log_{10}N$.

SMALL SPACE CHARGE ACTING OVER A LONG TIME

Duration of Run: $t = 2{\times}10^5$ units ~3,000 Booster ring transits

1,000 Test Particles per Run, Space-Charge-Depressed Tune $\eta = 0.95$,

Autocorrelation Time $t_c = 80$ (~8 typical orbital periods)



Blue: $\langle|\delta\omega|\rangle = 0$

Green: $\langle|\delta\omega|\rangle = 10^{-4}$

Black: $\langle|\delta\omega|\rangle = 10^{-3}$

Red: $\langle|\delta\omega|\rangle = 10^{-2}$

Gold: $\langle|\delta\omega|\rangle = 10^{-1}$

WHY WORRY ABOUT BEAM HALO?

- Proton machines such as spallation-neutron-source drivers:
  - Need □1 nA m^{-1} GeV^{-1} beam loss for hands-on maintenance.
  - For 1 mA, 1 GeV beam, this is just □1 particle in 10^6 per meter.

- Electron machines such as energy-recovery linacs:
  - Need □1 μA beam loss for machine and electronics protection.
  - For 100 mA beam (high-*P* FELs), this is just □1 particle in 10^5.

Comprehensive understanding of beam-halo formation is imperative!

- Standard Picture: Parametric resonance
  - Viewed as *the* fundamental mechanism of halo formation.
  - Predicts hard upper bound to halo amplitude.

Question of the Hour: Is parametric resonance *really* everything?

SUMMARY AND CONCLUSIONS

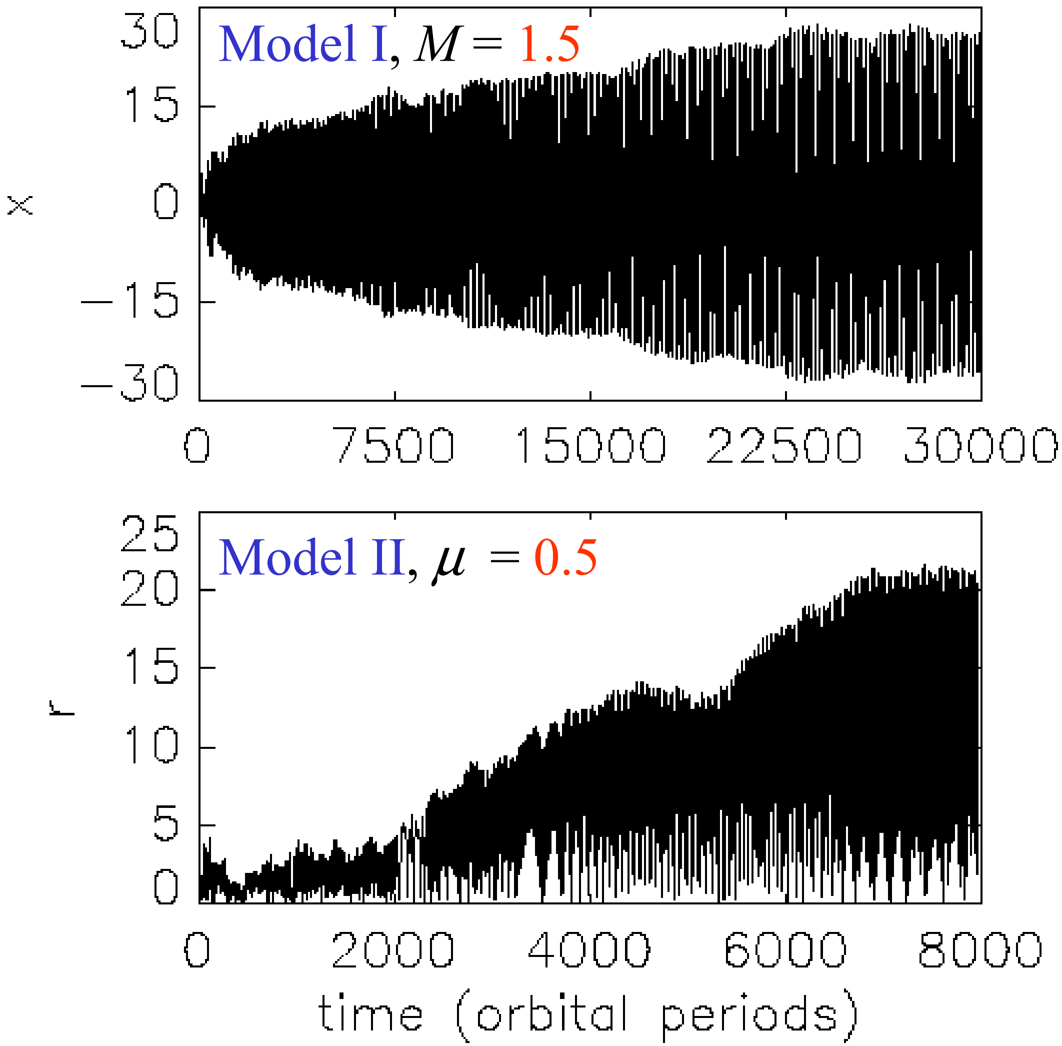
- Noise, an *unavoidable* phenomenon, can have ***major*** effects
  - expands the phase space
  - redistributes particles through phase space
  - affects Coulomb systems in general (e.g. galaxies, too!)

- Details ***do*** matter (halo being just one example)
  - control of rms properties is necessary but *not* sufficient
  - simulation codes must accommodate 'modes' at *all* scales
  - initial conditions are *critical* and must be specified accurately

- Collective modes affect dynamics ***differently*** from rms mismatch
  - phase-space tori are *much* more fragile
  - phase mixing is *much* faster and more voluminous

LONG-TIME EVOLUTION OF LARGE-AMPLITUDE ORBITS

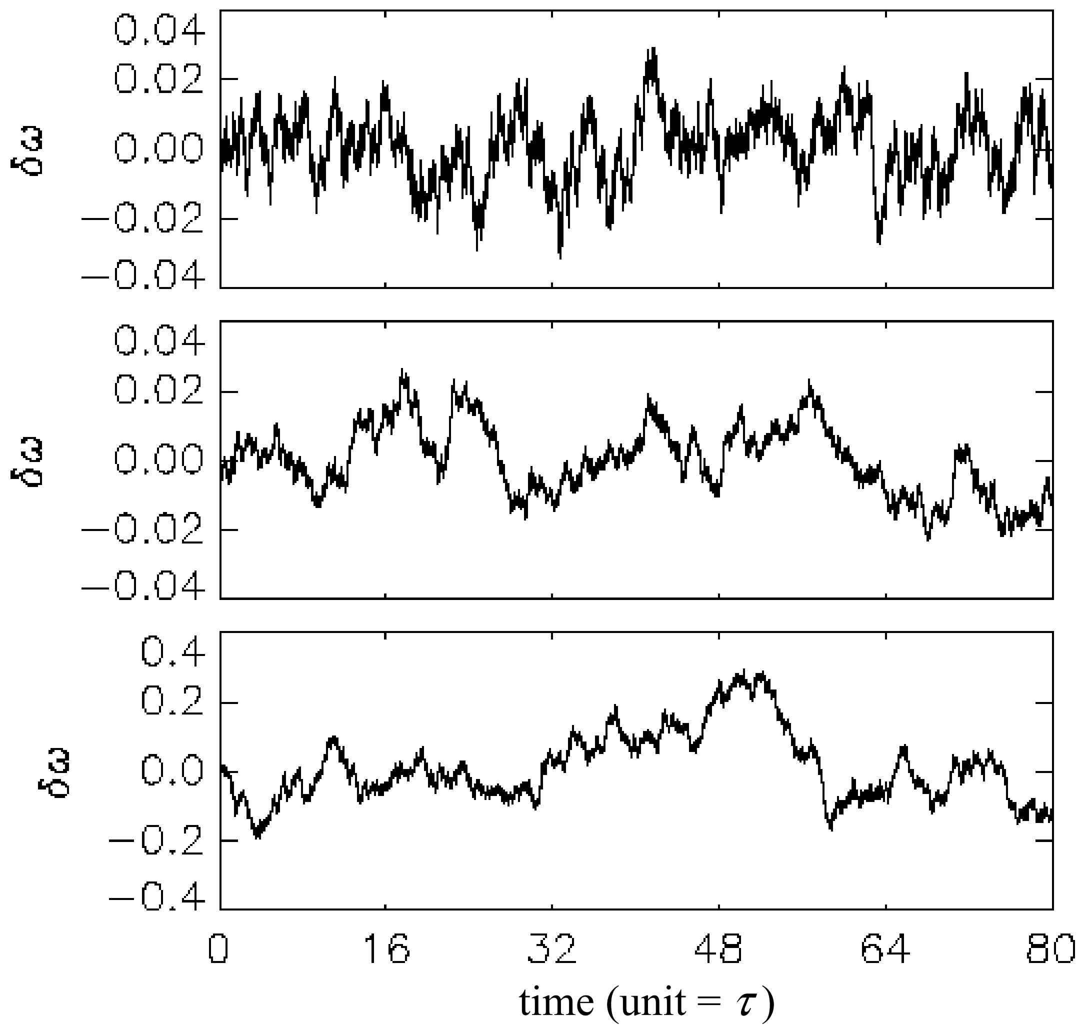
$$\langle|\delta\omega|\rangle = 0.01\,,\ t_c = 12\tau$$



Colored noise REMOVES the hard upper bound to the halo amplitude! This is important in, e.g., storage rings that include time-dependent noise.

EXAMPLE MANIFESTATIONS OF COLORED NOISE ALONG AN ORBIT

$$\langle \delta\omega(t) \rangle = 0, \;\; \langle \delta\omega(t)\delta\omega(t_1) \rangle \propto \exp(-|t - t_1|/t_c),$$



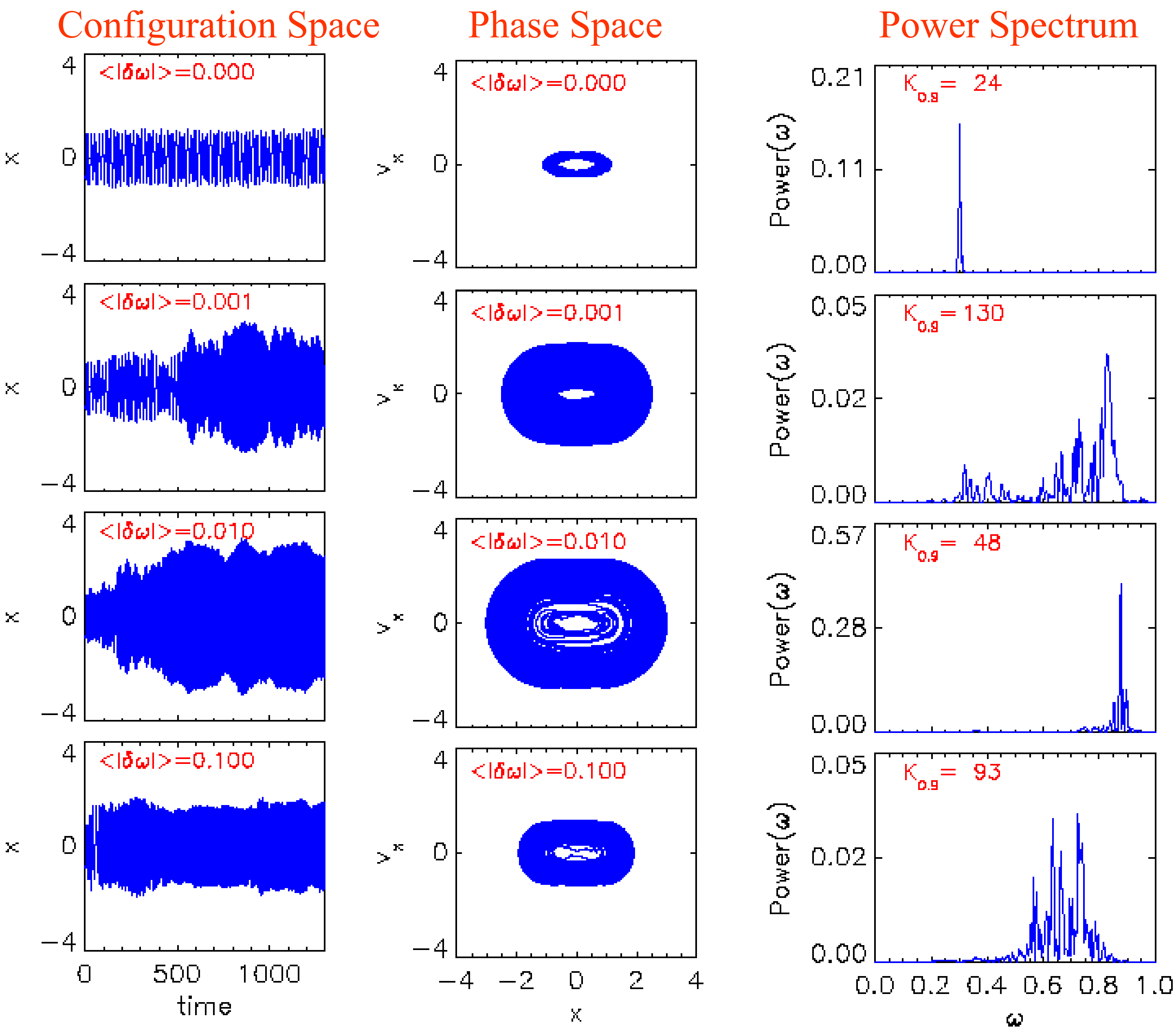
$$\langle |\delta\omega| \rangle = 0.01 \qquad t_c = 1.5\tau$$

$$\langle |\delta\omega| \rangle = 0.01 \qquad t_c = 12\tau$$

$$\langle |\delta\omega| \rangle = 0.1 \qquad t_c = 12\tau$$

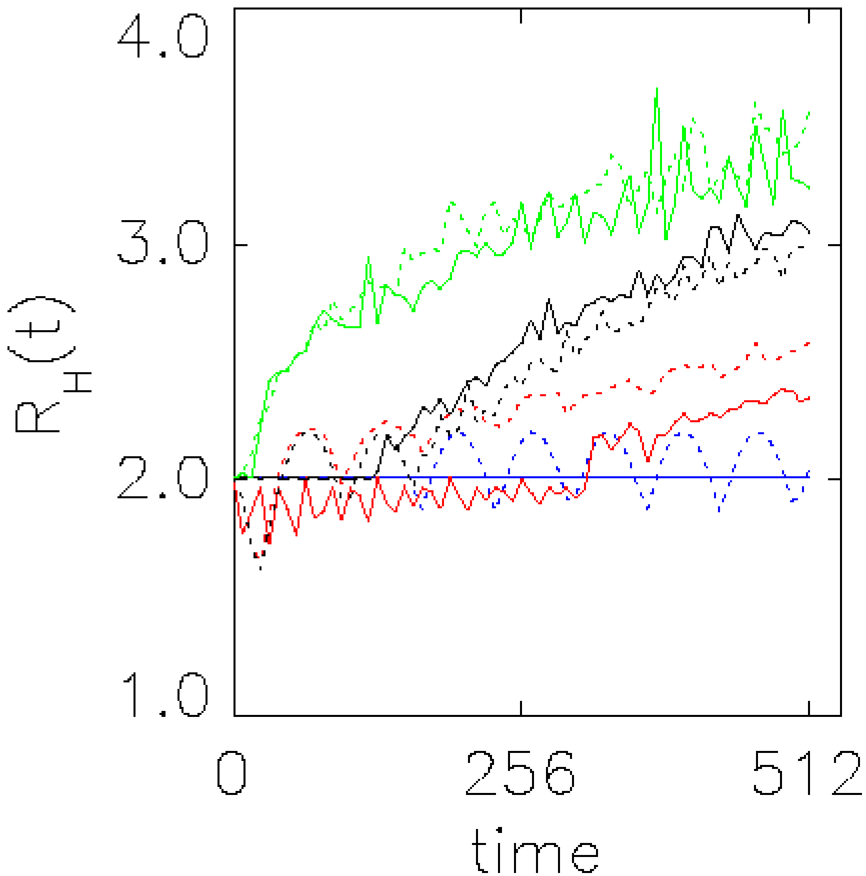
(τ = orbital period of a typical particle)

ORBITAL CHAOTICITY [Γ=0.10, $x(0)$=-0.733407]



INFLUENCE OF ANGULAR MOMENTUM (Γ=0.05, t_c=80)

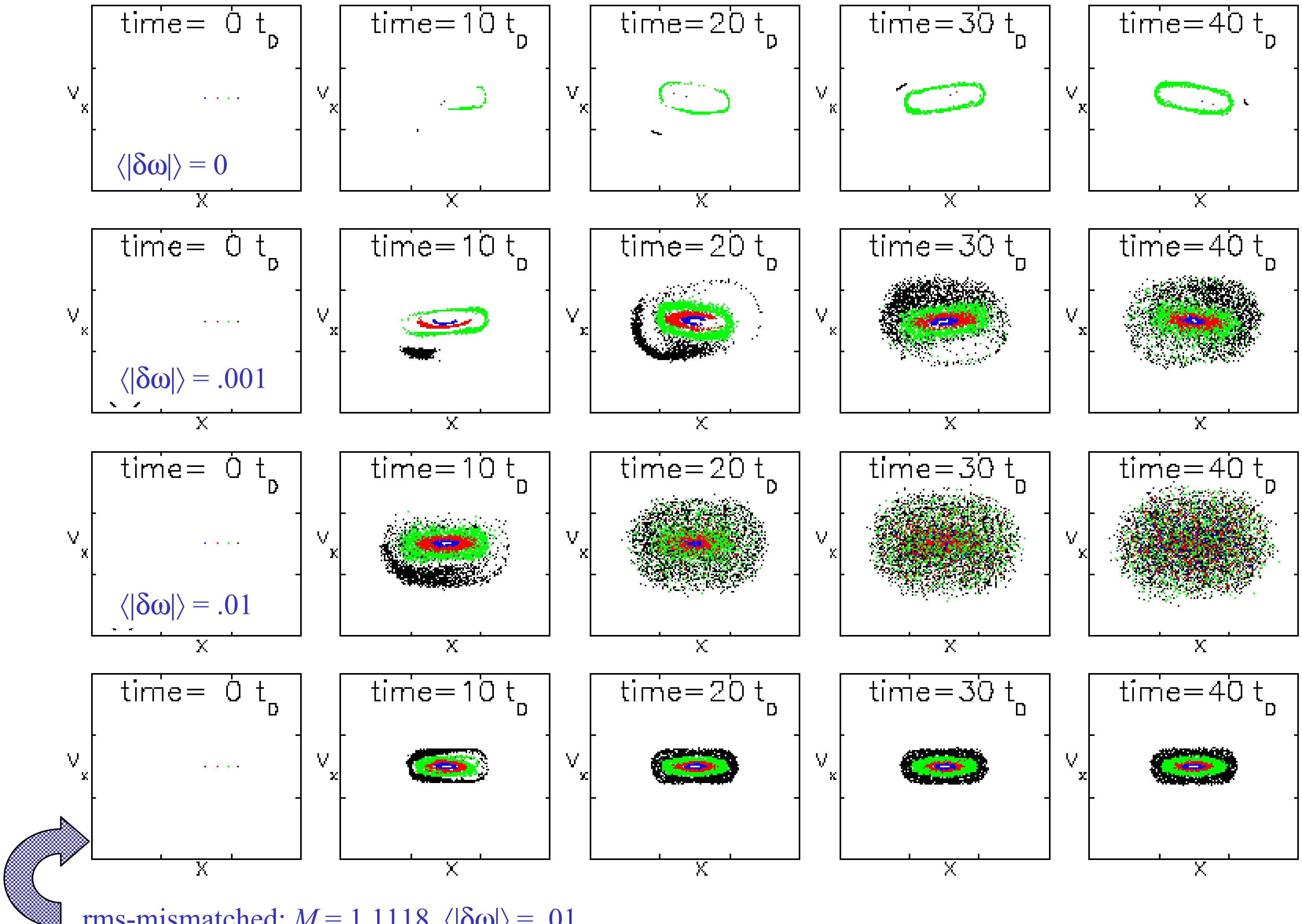
Radial (- - - - -) vs. Initially Circular (————) Orbits



Blue: $\langle|\delta\omega|\rangle = 0$
Red: $\langle|\delta\omega|\rangle = 0.001$
Black: $\langle|\delta\omega|\rangle = 0.01$
Green: $\langle|\delta\omega|\rangle = 0.1$

Angular momemtum has little impact.
Reason: Large halo orbits originate from the interior.

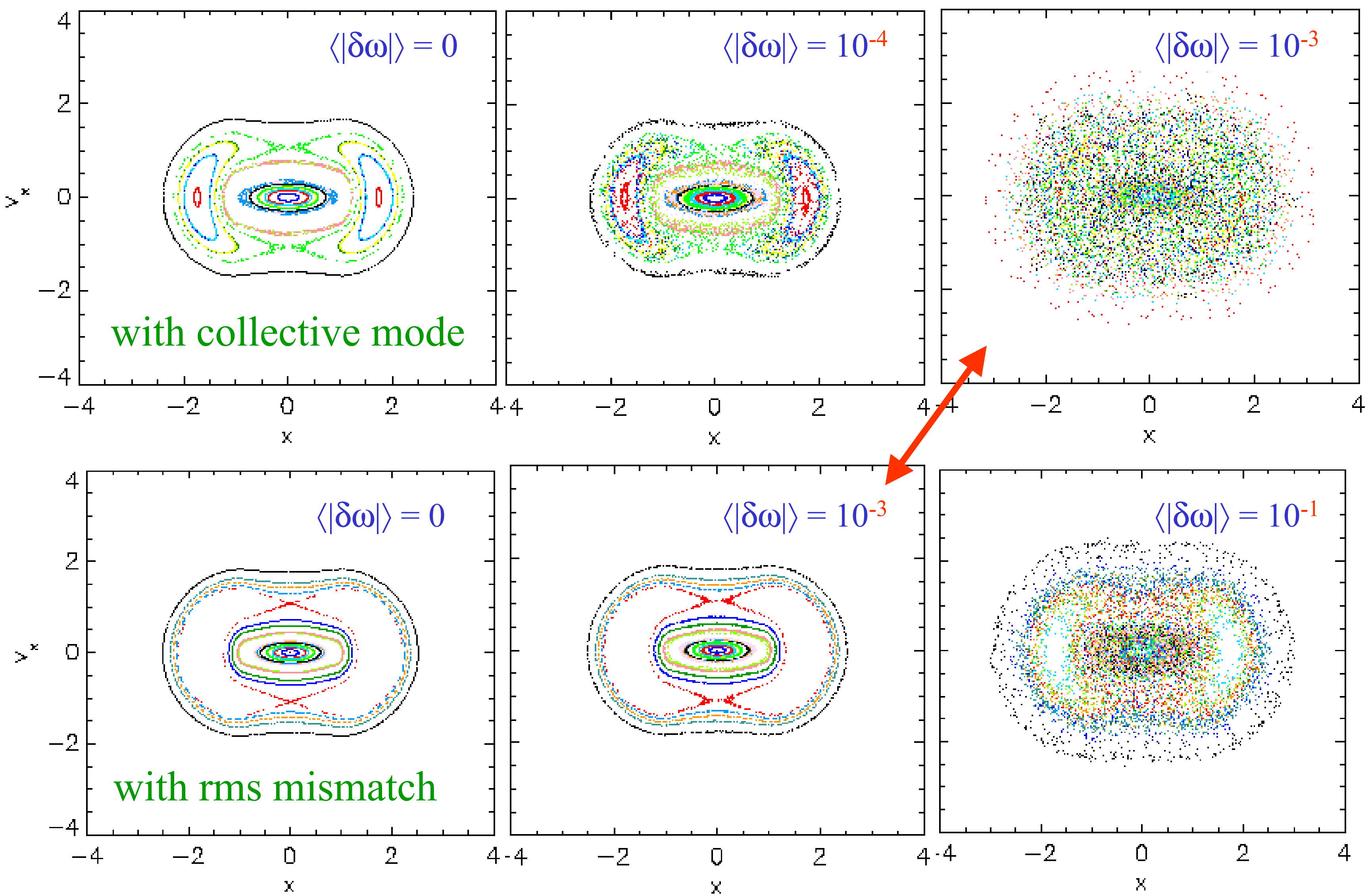
COLORED NOISE AND PHASE MIXING (Γ=0.05, t_c=80)



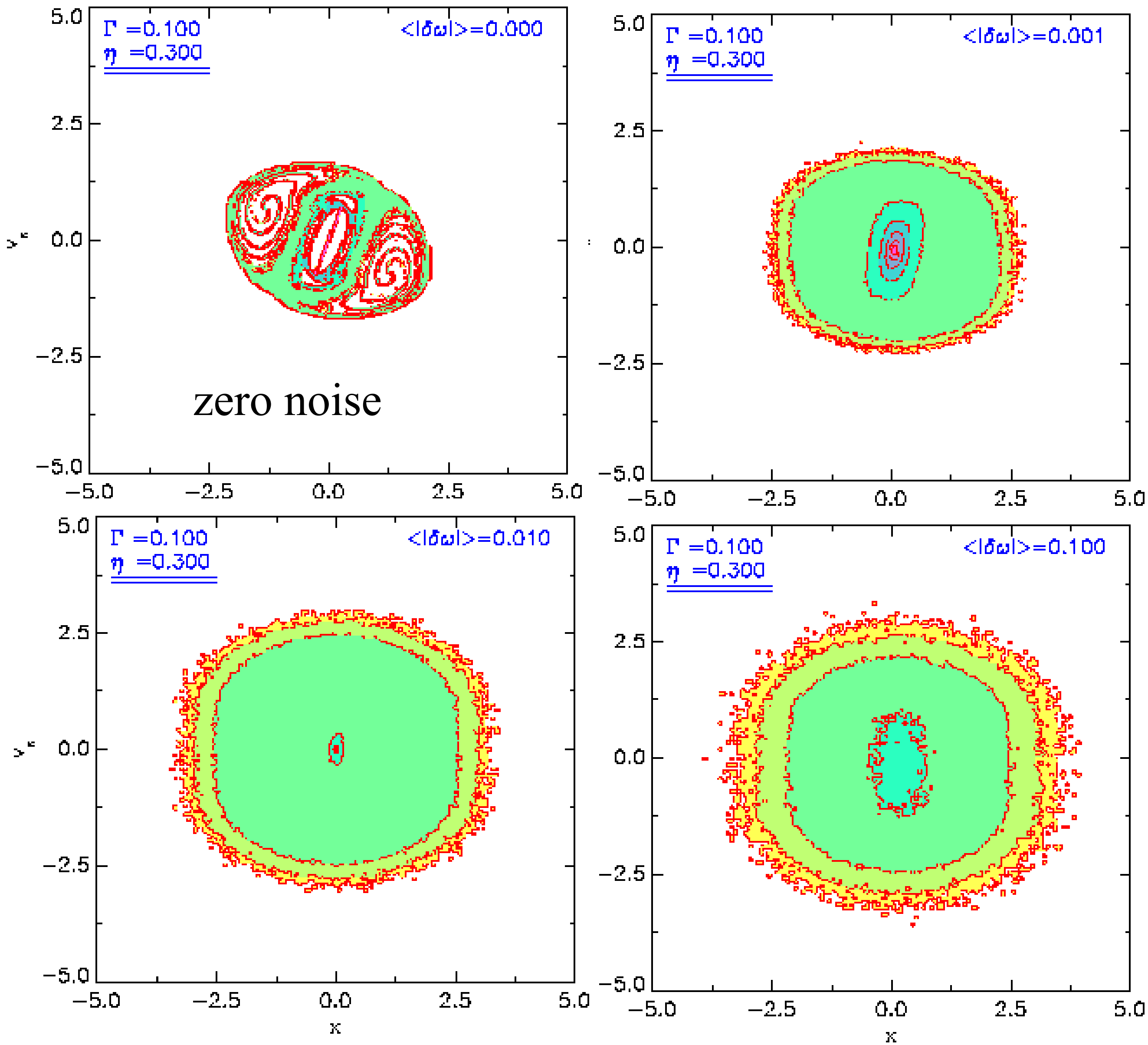
rms-mismatched: M = 1.1118, ⟨|δω|⟩ = .01

COLLECTIVE MODES ≠ RMS MISMATCH!

POINCARÉ SECTIONS (Γ=0.05, t_c=80, 18 orbits over ~250 t_D)



(x,v_x) PHASE SPACE: $\eta = 0.3$, $\Gamma = 0.1$



$t_c \cong 6\tau;\ t_{run} = 40\tau$

10^6 test particles

Noise *expands* the phase space and "*smears*" particles through phase space.

TINY OSCILLATION, TINY SPACE CHARGE, LONG RUN