Third order resonances correction in the Booster using harmonic sextupoles

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A third order resonances correction system consists of four normal and four skew sextupoles for correction of horizontal $3\nu_x = N$, vertical $3\nu_y = M$ and coupling resonances $\nu_x + 2\nu_y = K$ and $2\nu_x + \nu_y = L$. Sextupoles are located at the long straight sections number 4, 5, 7 and 8 (Fig. 1). An "ideal" system for correction of horizontal $3\nu_x = N$ resonance should include pair of sextupoles located at phase advance of $\psi_1 - \psi_2 = \pi/6$ such a way that $3\Delta\psi = \pi/2$ for excitation of resonance harmonic with amplitude $A = sqrt[(Acos(\gamma))^2 + (Asin(\gamma))^2]$ and phase γ in a range of $0 - 360^o$. Here

$Acos(\gamma) = \Sigma[(l \cdot \partial^2 B/\partial x^2 \cdot \beta_x^{3/2} \cdot c_o^{1/2} cos(3\psi_x)]/(2\ B\rho)$,

$Asin(\gamma) = \Sigma[(l \cdot \partial^2 B/\partial x^2 \cdot \beta_x^{3/2} \cdot c_o^{1/2} sin(3\psi_x)]/(2\ B\rho)$

here $c_o = 1\ m$

To prevent excitation of additional chromaticity another pair of sextupoles located at phase advance of 180^o with respect to the first one must be used with currents of excitation opposite to the currents of the first one. The β-functions and dispersion in the location of corresponding sextuples must be identical. This provides chromaticity:

$\xi_{x/y} = \Sigma[D \cdot \beta_{x/y} \cdot l \cdot \partial^2 B/\partial x^2]/(4\ \pi\ B\ \rho) = 0$

Unfortunately it is practically impossible to provide these conditions in a real machine. As shown in Fig. 1 in the Fermilab Booster the sextupole correctors are located not exactly in the same positions of the long straight sections. This effects difference in the β and dispersion functions and change phase advances between sextupoles.

The edge focusing of the injection and extraction orbit bumps in the Booster additionally produces β and dispersion functions beating with large amplitudes. The maximum horizontal β_x at injection is increased with respect to linear optics by 36% from 33.7 m to 46.0 m and dispersion by 94% from 3.2 m to 6.2 m; in the vertical plane, the maximum β_y is increased by 25% from 20.5m to 25.6 m. The lattice functions in an "ideal" system, in the Booster without injection and extraction orbit bumps (version V1.4), with bump magnets (version V1.7a) and with increased distance between bump magnets (version V1.7e, after shutdown) are presented in Table 1.

name	β_x	ψ_x	$3\psi_x$	D	β_y	ψ_y	$3\psi_y$
	m	2π	2π	m	m	2π	2π
ideal system							
SEXL44	6.2510	5.9942	17.9825	1.8420	20.0120	6.2832	18.8496
SEXL55	6.2510	7.5650	22.6949	1.8420	20.0120	7.8540	23.5619
SEXL47	6.2510	11.2281	33.6842	1.8420	20.0120	11.5171	34.5512
SEXL58	6.2510	12.7988	38.3965	1.8420	20.0120	13.0879	39.2636
version V1.4							
SEXL44	6.2690	5.9910	17.9731	1.8575	19.9785	6.1952	18.5857
SEXL55	6.8735	7.5304	22.5912	1.8600	20.1950	7.9011	23.7033
SEXL47	6.6685	11.6145	34.8434	1.8555	19.9825	11.6522	34.9565
SEXL58	6.3000	12.9465	38.8395	1.8590	20.1680	13.3046	39.9139
version V1.7a							
SEXL44	6.9510	6.2800	18.8401	1.0735	24.5825	6.3712	19.1135
SEXL55	7.1835	7.5115	22.5346	1.1935	16.5975	8.1273	24.3819
SEXL47	5.0950	11.7464	35.2392	1.6845	20.4060	11.9412	35.8236
SEXL58	8.6165	13.0502	39.1505	0.8590	19.8345	13.3832	40.1496
version V1.7e							
SEXL44	5.6635	5.9690	17.9071	1.5470	24.7215	6.4654	19.3962
SEXL55	8.0590	7.3388	22.0163	1.3765	16.2830	8.1713	24.5139
SEXL47	5.5730	11.4228	34.2685	1.9175	19.4400	12.0166	36.0498
SEXL58	7.3145	12.8774	38.6322	1.3350	20.9020	13.4523	40.3569

Table 1: Lattice functions in an "ideal" system, in the Booster without injection and extraction orbit bumps (version V1.4), with bump magnets (version V1.7a) and with increased distance between bump magnets (version V1.7e, after shutdown). Vertical betatrone tune $\nu_x = 6.66667$ (resonance $3\nu_x = N$).

As an example a correction of horizontal $3\nu_x = N$ resonance is presented here, but correction of vertical $3\nu_y = M$, coupling resonances $\nu_x + 2\nu_y = K$ and $2\nu_x + \nu_y = L$ can be done using this system.

A vector $\beta_x^{3/2} cos(3\psi_x) + \beta_x^{3/2} sin(3\psi_x)$ which represents a phase of harmonic correction sextupoles in "ideal" system, in the Booster without bump magnets and in the Booster with bump magnets are shown in Fig. 2. In the "ideal" system all four vectors are equivalent, and are located at 90^o and 180^o. In the V1.4

version vectors are approximately equivalent, but phases are not optimal. In a real machine (V1.7a) vectors are different very much, and phases are not optimal. The system with increased distance between bump magnets (version V1.7e, after shutdown) is slightly better compared to existing one.

If $A = sqrt(A^2_{cos} + A^2_{sin})$ is a strength of horizontal resonance $3\nu_x = N$, and emittance of the circulating beam is ϵ, than at the distance from the resonance line of $\delta\nu_{res} \cong 0.2 \cdot A \cdot sqrt(\epsilon/c_o)$, the large amplitude particles of the beam come to the resonance conditions. Saying another words, the $\delta\nu_{res}$ is a half-width of resonance for particles with amplitude, defined by emittance ϵ. $\delta\nu_{res} = 0.0008$ for large amplitude particles ($3\sigma_x$) in the circulating beam at injection (95% normalized emittance is $12\pi\ mm.mrad$) if $A = sqrt(A^2_{cos} + A^2_{sin}) = 0.95$.

Currents in the harmonic correction sextupoles required for excitation of harmonic amplitude of $A = sqrt(A^2_{cos} + A^2_{sin}) = 0.95$ are shown in Fig. 3, Tables 2 and 3 as a function of phase of resonance harmonic. These currents are a solution of system of two equations:

$\Sigma[(i(k) \cdot 0.003717 \cdot \beta_x^{3/2} cos(3\psi_x)]/(2\ B\ \rho) = 0.95 \cdot cos(\gamma)$
$\Sigma[(i(k) \cdot 0.003717 \cdot \beta_x^{3/2} sin(3\psi_x)]/(2\ B\ \rho) = 0.95 \cdot sin(\gamma)$
here $i(k) \cdot 0.003717 = l \cdot \partial^2 B/\partial x^2$,
$i(44) = -i47, \quad i(55) = -i58$,
and $\nu_x = 6.66667$ (resonance $3\nu_x = N$).

The currents of sextupole excitation are minimal in the "ideal" system, and they are a factor of 2 bigger in the real system. As shown in Fig. 4 chromaticity excited by the harmonics sextupoles is equal to zero in the "ideal" system, and reach $\Delta\xi_x = 0.003$ and $\Delta\xi_y = 0.025$ in the real machine.

Currents for correction of vertical $3\nu_y = M$ resonance may be found from equations:

$\Sigma[(i(k) \cdot 0.003717 \cdot \beta_y^{3/2} cos(3\psi_y)]/(2\ B\ \rho) = A \cdot cos(\gamma)$
$\Sigma[(i(k) \cdot 0.003717 \cdot \beta_y^{3/2} sin(3\psi_y)]/(2\ B\ \rho) = A \cdot sin(\gamma)$
here $i(k) \cdot 0.003717 = l \cdot \partial^2 B/\partial x^2$,
$i(44) = -i47, \quad i(55) = -i58$,
and $\nu_y = 6.66667$ (resonance $3\nu_y = M$).

Currents for correction of coupling resonance $\nu_x + 2\nu_y = K$ may be found from equations:

$\Sigma[(i(k) \cdot 0.003717 \cdot \beta_x^{1/2} \cdot \beta_y cos(\psi_x + 2\psi_y)]/(2\ B\ \rho) = A \cdot cos(\gamma)$
$\Sigma[(i(k) \cdot 0.003717 \cdot \beta_x^{1/2} \cdot \beta_y sin(\psi_x + 2\psi_y)]/(2\ B\ \rho) = A \cdot sin(\gamma)$

here $\quad i(k) \cdot 0.003717 = l \cdot \partial^2 B/\partial x^2$,
$\quad i(44) = -i47, \quad i(55) = -i58$,
and $\quad \nu_x + 2\nu_y = K$ (resonance $\nu_x + 2\nu_y = K$).

Currents for correction of coupling resonance $2\nu_x + \nu_y = L$ may be found from equations:

$\Sigma[(i(k) \cdot 0.003717 \cdot \beta_x \cdot \beta_y^{1/2} cos(2\psi_x + \psi_y)]/(2\ B\ \rho) = A \cdot cos(\gamma)$

$\Sigma[(i(k) \cdot 0.003717 \cdot \beta_x \cdot \beta_y^{1/2} sin(2\psi_x + \psi_y)]/(2\ B\ \rho) = A \cdot sin(\gamma)$

here $\quad i(k) \cdot 0.003717 = l \cdot \partial^2 B/\partial x^2$,
$\quad i(44) = -i47, \quad i(55) = -i58$,
and $\quad 2\nu_x + \nu_y = L$ (resonance $\nu_x + 2\nu_y = L$).

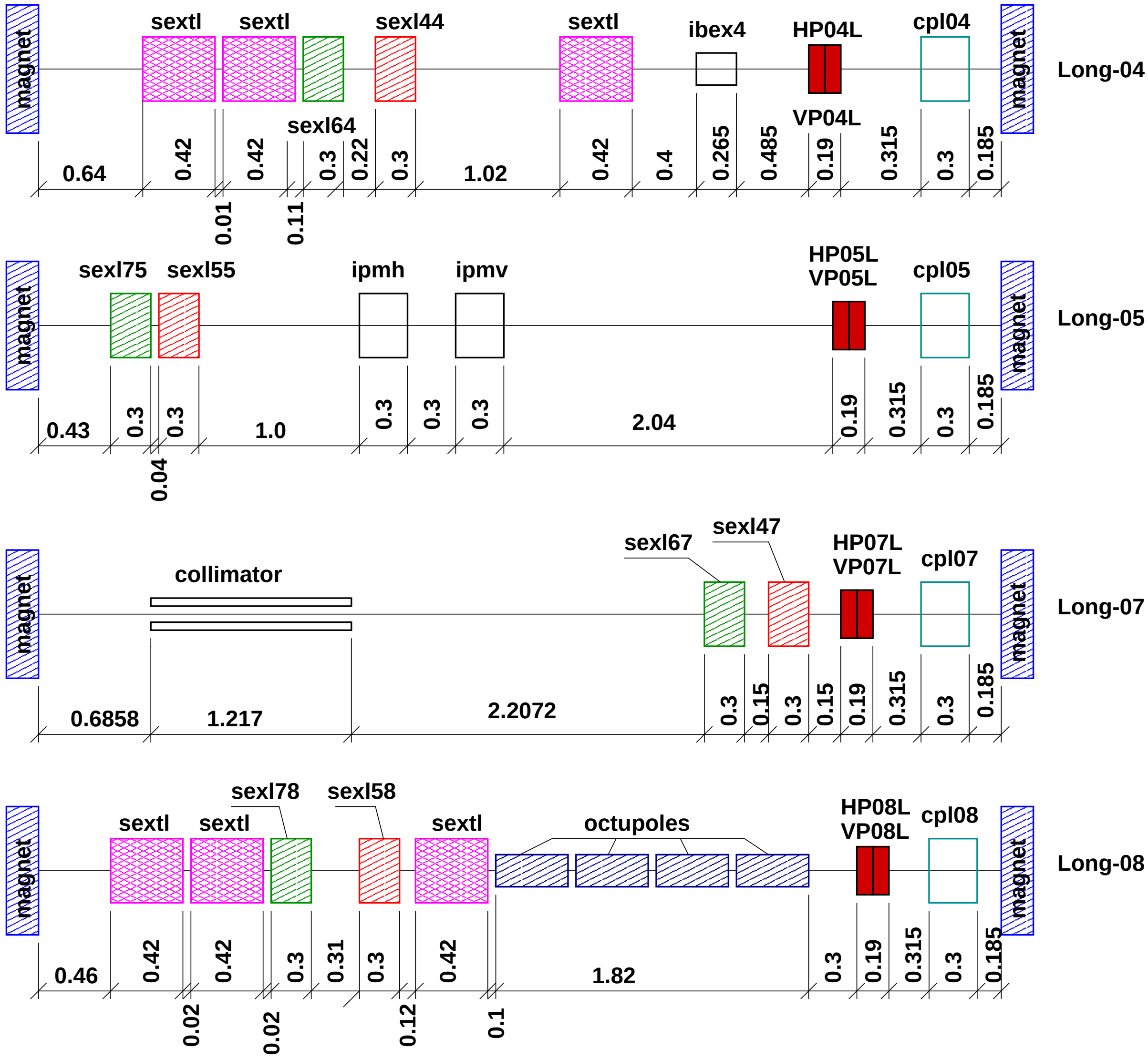

Figure 1: Location of harmonic correction sextupoles in the Booster Long04, 05, 07, 08 straight sections.

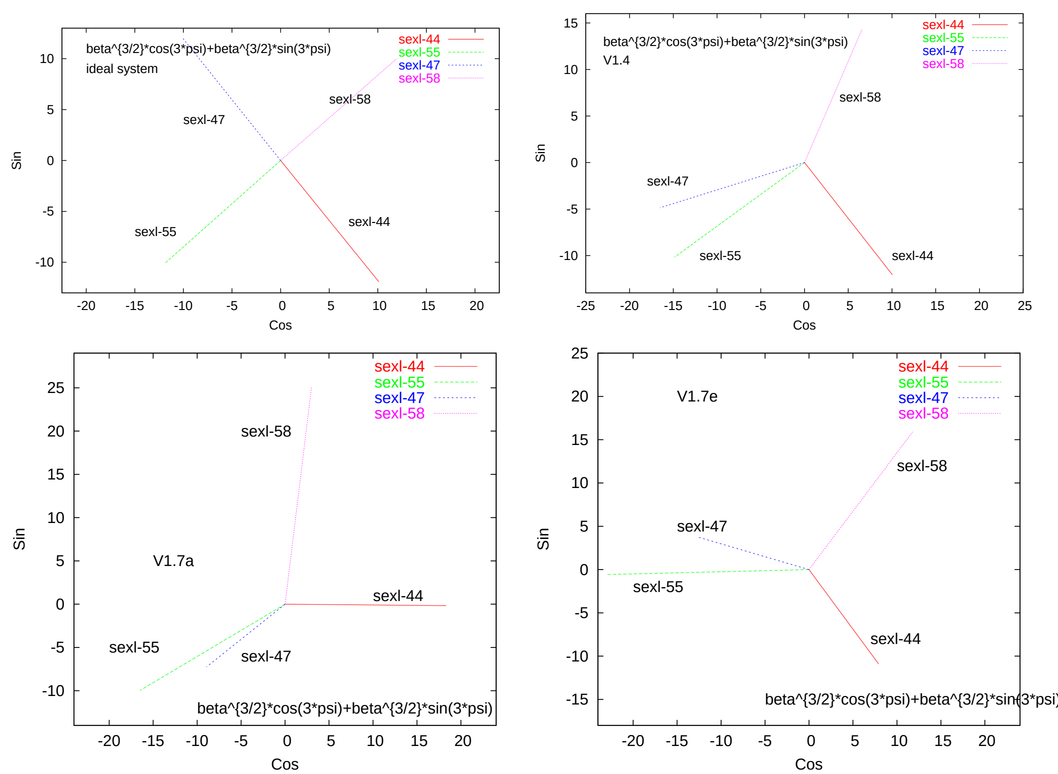

Figure 2: Phase of harmonic correction sextupoles in “ideal” system (top, left), in the Booster without bump magnets (top, right, version V1.4), in the Booster with bump magnets (bottom, left, version V1.7a) and with increased distance between bump magnets (bottom, right, version V1.7e, after shutdown).

angle	i44	i55	i47	i58	A_{cos}	A_{sin}	$\Delta\xi_x$	$\Delta\xi_y$
degree	A	A	A	A				
real system, V1.7a								
0.000	34.881	7.036	-34.881	-7.036	0.950000	0.000002	-0.002866	0.024069
10.000	30.985	2.222	-30.985	-2.222	0.935567	0.164966	-0.002985	0.022417
20.000	26.147	-2.660	-26.147	2.660	0.892708	0.324919	-0.003012	0.020085
30.000	20.515	-7.461	-20.515	7.461	0.822724	0.475000	-0.002949	0.017142
40.000	14.259	-12.035	-14.259	12.035	0.727742	0.610648	-0.002796	0.013679
50.000	7.570	-16.244	-7.570	16.244	0.610648	0.727742	-0.002557	0.009800
60.000	0.651	-19.959	-0.651	19.959	0.475000	0.822724	-0.002241	0.005623
70.000	-6.287	-23.067	6.287	23.067	0.324919	0.892708	-0.001857	0.001275
80.000	-13.035	-25.475	13.035	25.475	0.164965	0.935567	-0.001417	-0.003111
90.000	-19.386	-27.109	19.386	27.109	-0.000001	0.950000	-0.000933	-0.007403
100.000	-25.149	-27.919	25.149	27.919	-0.164966	0.935567	-0.000422	-0.011470
110.000	-30.147	-27.881	30.147	27.881	-0.324920	0.892708	0.000103	-0.015189
120.000	-34.230	-26.995	34.230	26.995	-0.475001	0.822724	0.000625	-0.018445
130.000	-37.272	-25.289	37.272	25.289	-0.610649	0.727742	0.001127	-0.021142
140.000	-39.182	-22.815	39.182	22.815	-0.727743	0.610648	0.001596	-0.023196
150.000	-39.901	-19.648	39.901	19.648	-0.822725	0.474999	0.002015	-0.024545
160.000	-39.408	-15.884	39.408	15.884	-0.892708	0.324918	0.002374	-0.025149
170.000	-37.717	-11.637	37.717	11.637	-0.935568	0.164965	0.002660	-0.024988
180.000	-34.881	-7.036	34.881	7.036	-0.950000	-0.000001	0.002866	-0.024069
190.000	-30.985	-2.222	30.985	2.222	-0.935567	-0.164967	0.002985	-0.022417
200.000	-26.147	2.660	26.147	-2.660	-0.892708	-0.324920	0.003012	-0.020085
210.000	-20.514	7.461	20.514	-7.461	-0.822723	-0.475001	0.002949	-0.017142
220.000	-14.259	12.035	14.259	-12.035	-0.727741	-0.610649	0.002796	-0.013679
230.000	-7.570	16.244	7.570	-16.244	-0.610647	-0.727743	0.002557	-0.009800
240.000	-0.651	19.959	0.651	-19.959	-0.474999	-0.822725	0.002241	-0.005623
250.000	6.287	23.068	-6.287	-23.068	-0.324918	-0.892708	0.001857	-0.001275
260.000	13.035	25.475	-13.035	-25.475	-0.164964	-0.935568	0.001417	0.003111
270.000	19.387	27.109	-19.387	-27.109	0.000002	-0.950000	0.000933	0.007403
280.000	25.149	27.919	-25.149	-27.919	0.164968	-0.935567	0.000422	0.011470
290.000	30.147	27.881	-30.147	-27.881	0.324921	-0.892707	-0.000103	0.015189
300.000	34.230	26.995	-34.230	-26.995	0.475002	-0.822723	-0.000625	0.018446
310.000	37.272	25.289	-37.272	-25.289	0.610650	-0.727741	-0.001127	0.021142
320.000	39.182	22.815	-39.182	-22.815	0.727744	-0.610647	-0.001596	0.023196
330.000	39.901	19.648	-39.901	-19.648	0.822725	-0.474998	-0.002015	0.024545
340.000	39.408	15.884	-39.408	-15.884	0.892709	-0.324917	-0.002374	0.025149
350.000	37.717	11.637	-37.717	-11.637	0.935568	-0.164963	-0.002660	0.024988

Table 2: Current in the harmonic correction sextupoles required for excitation of harmonic amplitude of $A = sqrt(A_{cos}^2 + A_{sin}^2) = 0.002$ in the system with bump magnets (version V1.7a).

angle	i44	i55	i47	i58	A_{cos}	A_{sin}	$\Delta\xi_x$	$\Delta\xi_y$
degree	A	A	A	A				
real system, V1.7e								
0.000	15.839	-14.093	-15.839	14.093	0.950000	0.000002	-0.004573	-0.008615
10.000	9.812	-17.308	-9.812	17.308	0.935567	0.164966	-0.003891	-0.009714
20.000	3.486	-19.998	-3.486	19.998	0.892708	0.324919	-0.003092	-0.010517
30.000	-2.945	-22.079	2.945	22.079	0.822724	0.475000	-0.002199	-0.011000
40.000	-9.287	-23.490	9.287	23.490	0.727742	0.610648	-0.001238	-0.011149
50.000	-15.346	-24.187	15.346	24.187	0.610648	0.727742	-0.000241	-0.010960
60.000	-20.940	-24.149	20.940	24.149	0.475000	0.822724	0.000764	-0.010438
70.000	-25.897	-23.377	25.897	23.377	0.324919	0.892708	0.001746	-0.009598
80.000	-30.067	-21.895	30.067	21.895	0.164965	0.935567	0.002675	-0.008467
90.000	-33.324	-19.748	33.324	19.748	-0.000001	0.950000	0.003523	-0.007078
100.000	-35.568	-17.000	35.568	17.000	-0.164966	0.935567	0.004263	-0.005475
110.000	-36.732	-13.737	36.732	13.737	-0.324920	0.892708	0.004874	-0.003705
120.000	-36.779	-10.055	36.779	10.055	-0.475001	0.822724	0.005337	-0.001822
130.000	-35.709	-6.069	35.709	6.069	-0.610649	0.727742	0.005638	0.000116
140.000	-33.554	-1.897	33.554	1.897	-0.727743	0.610648	0.005767	0.002050
150.000	-30.379	2.331	30.379	-2.331	-0.822725	0.474999	0.005721	0.003922
160.000	-26.281	6.489	26.281	-6.489	-0.892708	0.324918	0.005502	0.005675
170.000	-21.385	10.450	21.385	-10.450	-0.935568	0.164965	0.005115	0.007255
180.000	-15.839	14.093	15.839	-14.093	-0.950000	-0.000001	0.004573	0.008615
190.000	-9.812	17.308	9.812	-17.308	-0.935567	-0.164967	0.003891	0.009714
200.000	-3.486	19.998	3.486	-19.998	-0.892708	-0.324920	0.003092	0.010517
210.000	2.945	22.079	-2.945	-22.079	-0.822723	-0.475001	0.002199	0.011000
220.000	9.287	23.490	-9.287	-23.490	-0.727741	-0.610649	0.001238	0.011149
230.000	15.346	24.187	-15.346	-24.187	-0.610647	-0.727743	0.000241	0.010960
240.000	20.940	24.149	-20.940	-24.149	-0.474999	-0.822725	-0.000764	0.010438
250.000	25.897	23.377	-25.897	-23.377	-0.324918	-0.892708	-0.001746	0.009598
260.000	30.067	21.895	-30.067	-21.895	-0.164964	-0.935568	-0.002675	0.008467
270.000	33.324	19.748	-33.324	-19.748	0.000002	-0.950000	-0.003523	0.007078
280.000	35.568	17.000	-35.568	-17.000	0.164968	-0.935567	-0.004263	0.005475
290.000	36.732	13.736	-36.732	-13.736	0.324921	-0.892707	-0.004874	0.003705
300.000	36.779	10.055	-36.779	-10.055	0.475002	-0.822723	-0.005337	0.001822
310.000	35.709	6.069	-35.709	-6.069	0.610650	-0.727741	-0.005638	-0.000116
320.000	33.554	1.897	-33.554	-1.897	0.727744	-0.610647	-0.005767	-0.002050
330.000	30.379	-2.331	-30.379	2.331	0.822725	-0.474998	-0.005721	-0.003922
340.000	26.281	-6.489	-26.281	6.489	0.892709	-0.324917	-0.005502	-0.005675
350.000	21.385	-10.450	-21.385	10.450	0.935568	-0.164963	-0.005115	-0.007255

Table 3: Current in the harmonic correction sextupoles required for excitation of harmonic amplitude of $A = sqrt(A_{cos}^2 + A_{sin}^2) = 0.95$ in the system with increased distance between bump magnets (version V1.7e, after shutdown).

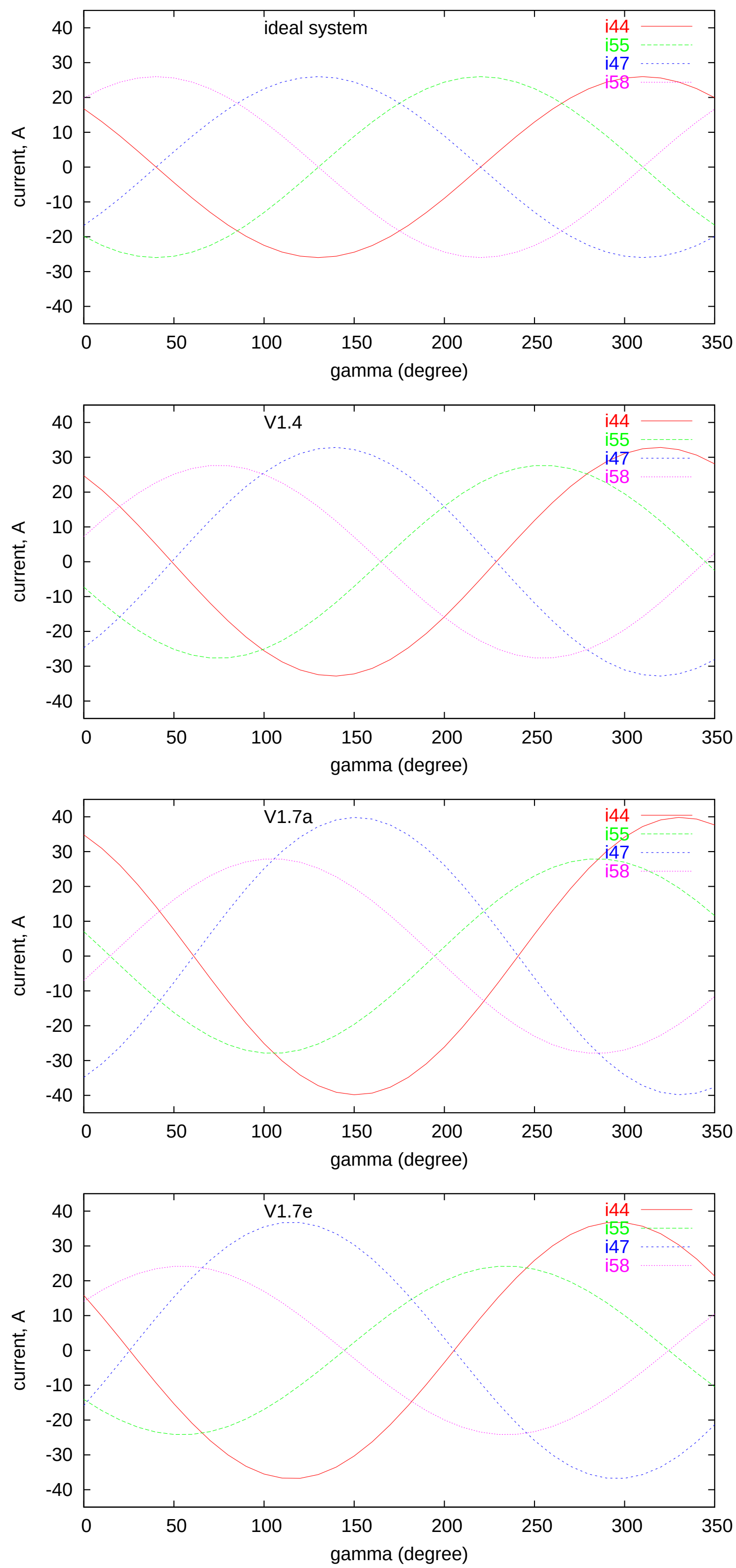

Figure 3: Current in the harmonic correction sextupoles required for excitation of harmonic amplitude of $A = sqrt(A_{cos}^2 + A_{sin}^2) = 0.95$.

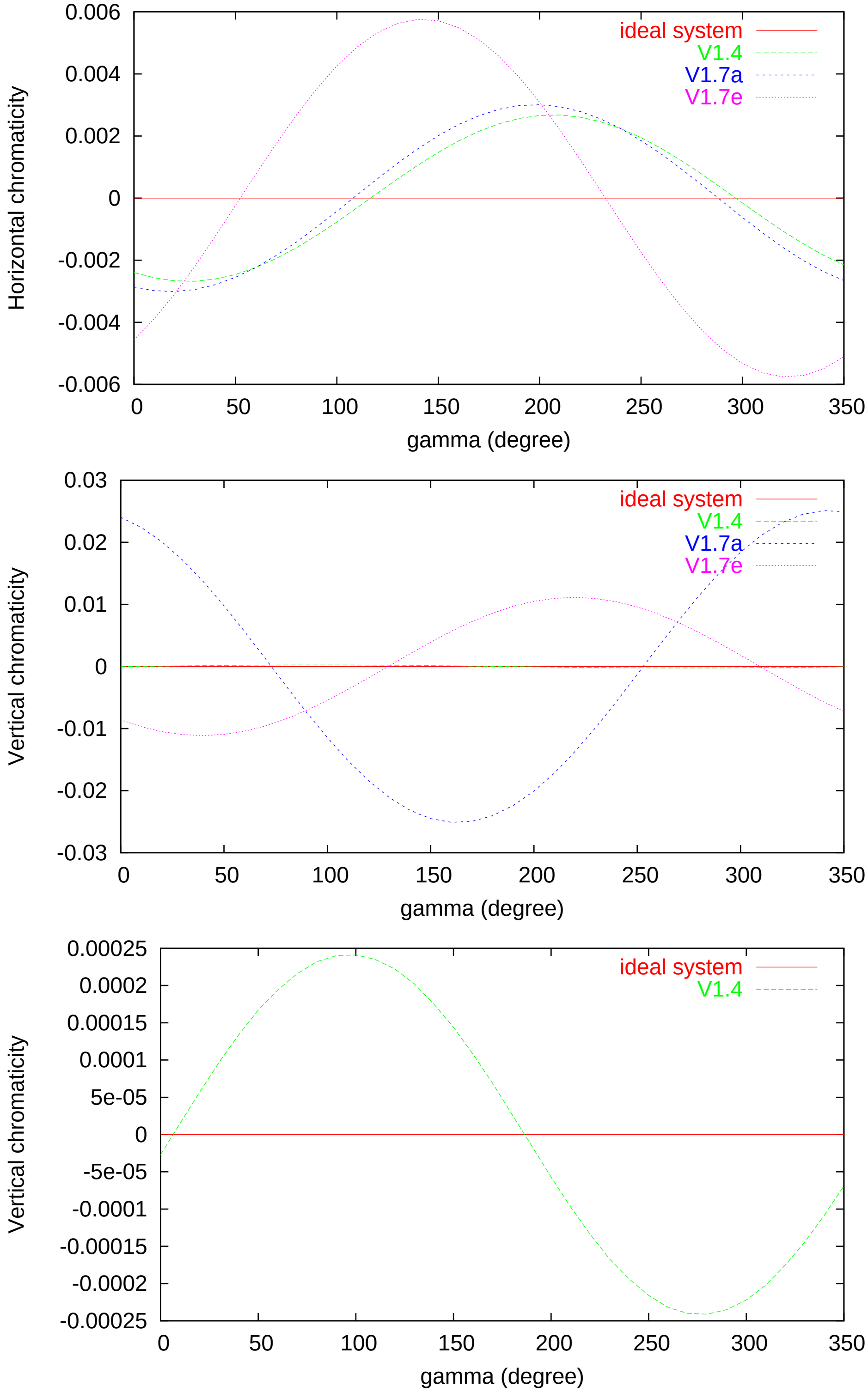

Figure 4: Horizontal (top) and vertical (middle and bottom) chromaticity excited by the harmonics correction sextupoles.

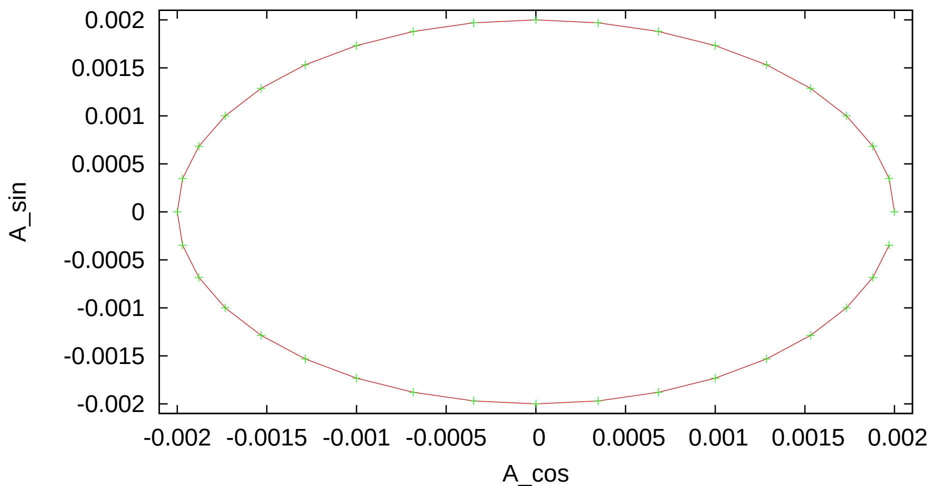

Figure 5: Phase space of sextupole harmonic amplitude A.