Compensation of Dogleg Effect
and
Local Beta Bump

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1 Outline

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2 Harmonic compensation

2.1 Perturbation to beta function and dispersion by doglegs

The integrated quadrupole strength of edge focusing effect

$$k\Delta l = \frac{\tan\delta}{\rho} = \frac{\delta\tan\delta}{L} \tag{2.1}$$

The p'th harmonic half-integer stopband integral

$$J_p = \frac{1}{2\pi}\oint \beta k(s) e^{-jp\phi}\,\mathrm{d}s = \frac{1}{2\pi}\sum_i \beta_i [k\Delta l]_i e^{-jp\phi_i} \tag{2.2}$$

The perturbation to beta function

$$\frac{\Delta\beta(s)}{\beta(s)} = -\frac{\nu_0}{2}\sum_{p=-\infty}^{\infty}\frac{J_p e^{jp\phi}}{\nu_0^2 - (p/2)^2} \tag{2.3}$$

For off-momentum particles, there is also a dipole effect because of dispersion. The equivalent dipole strength is

$$\frac{\Delta B}{B\rho} = -[k\Delta l] D \frac{\Delta p}{p_0} \tag{2.4}$$

The minus sign indicates a weaker bending strength for positive momentum deviation. For a unit value of $\frac{\Delta p}{p_0}$, the integer stopband integral

$$\begin{aligned} f_n &= \frac{1}{2\pi\nu}\oint \sqrt{\beta}\frac{\Delta B}{B\rho} e^{-jn\phi}\mathrm{d}s \\ &= -\frac{1}{2\pi\nu}\sum_i \sqrt{\beta_i}[k\Delta l]_i D_i e^{-jn\phi_i} \end{aligned} \tag{2.5}$$

The change to dispersion function is then

$$\Delta D = \frac{\Delta x_{\mathrm{co}}}{\Delta p/p_0} = \sqrt{\beta(s)}\sum_{n=-\infty}^{\infty}\frac{\nu^2 f_n}{\nu^2 - n^2} e^{jn\phi} \tag{2.6}$$

Table (1) lists related parameters of doglegs.

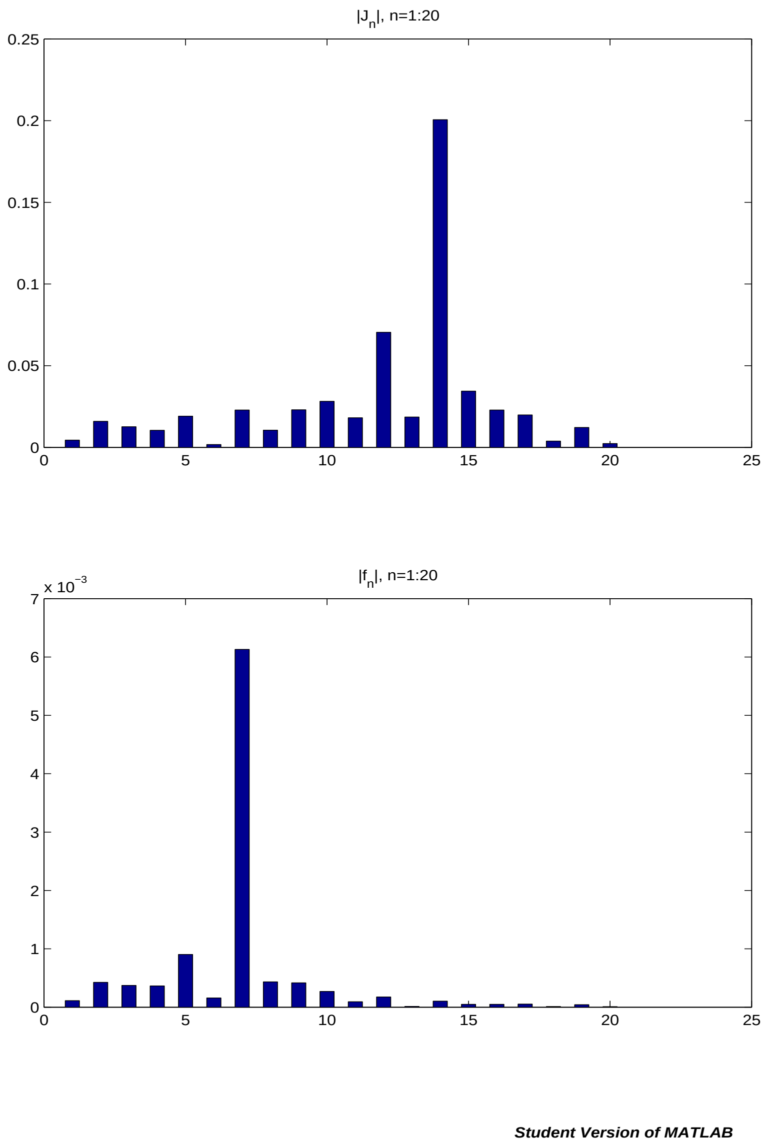

Figure 1: The weighted harmonics for current dogleg layout. Top, $|J_p|/(\nu^2 - p^2/4)$. Bottom $|f_n|/(\nu^2 - n^2)$.

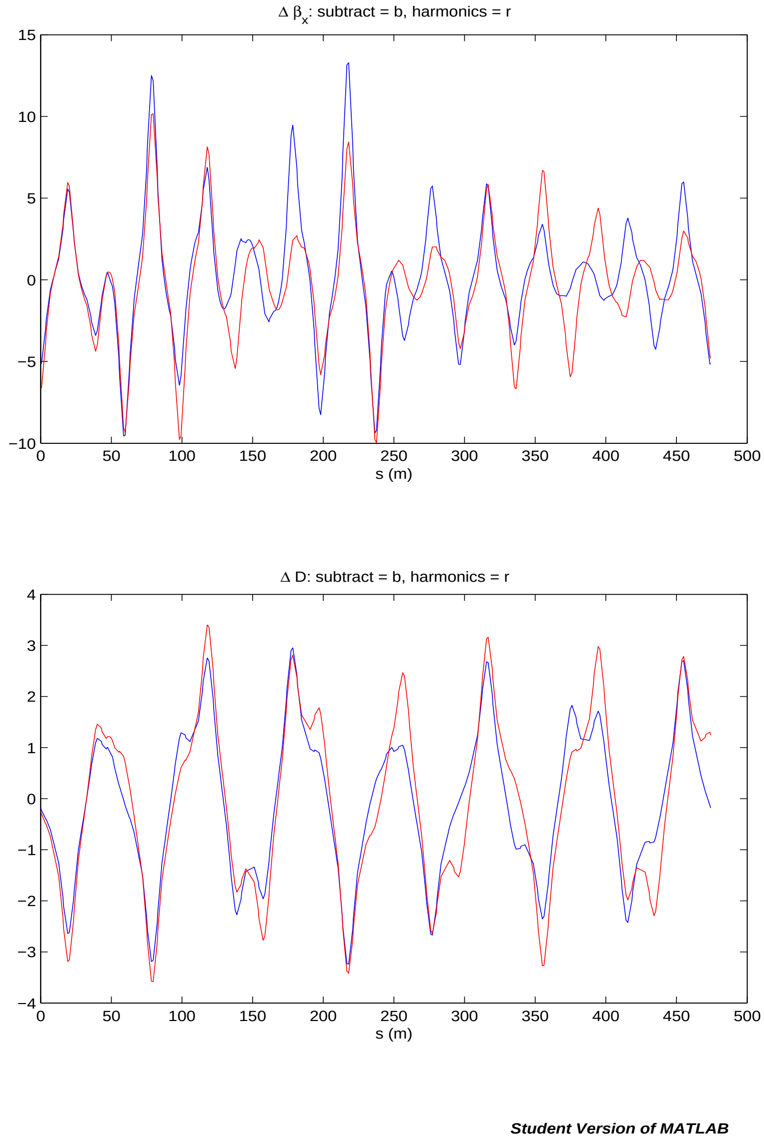

Figure 2: Changes to beta amplitude β_x and dispersion D_x due to current dogleg layout. Blue curve is by subtracting MAD output with and without doglegs. Red one is obtained using equations (2.3) and (2.6). Top, β_x. Bottom, D_x.

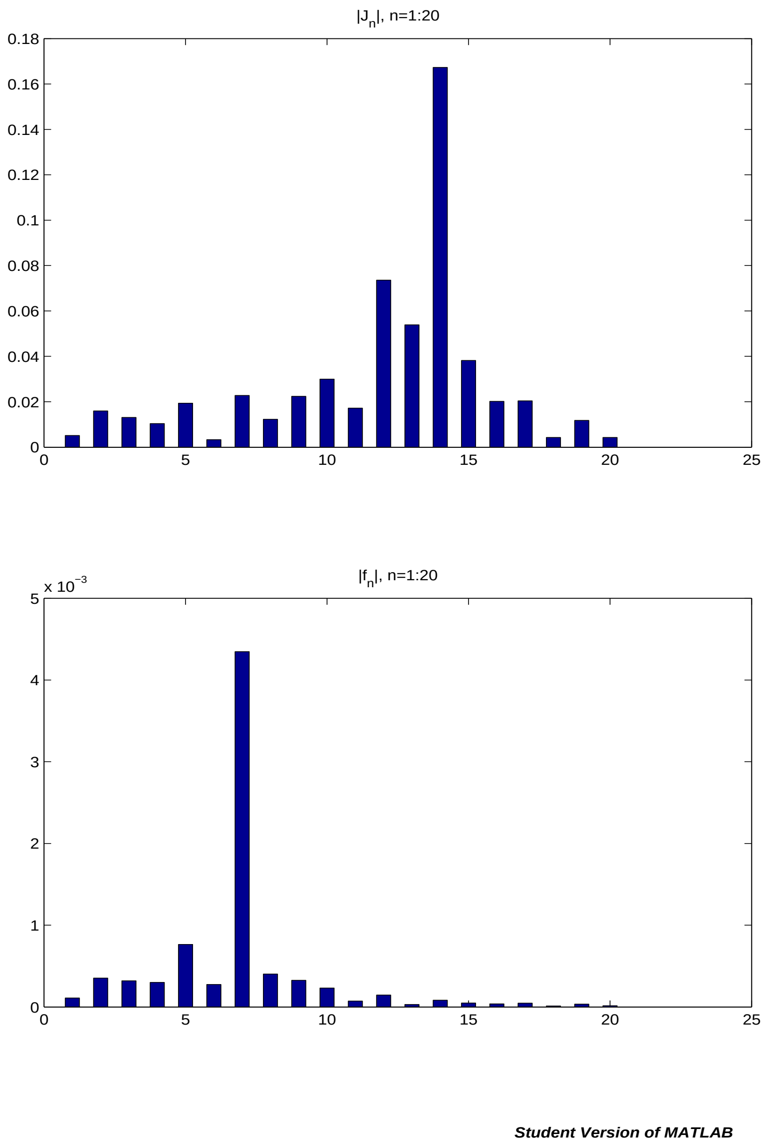

Figure 3: The weighted harmonics for repositioned dogleg layout. Top, $|J_p|/(\nu^2 - p^2/4)$. Bottom $|f_n|/(\nu^2 - n^2)$.

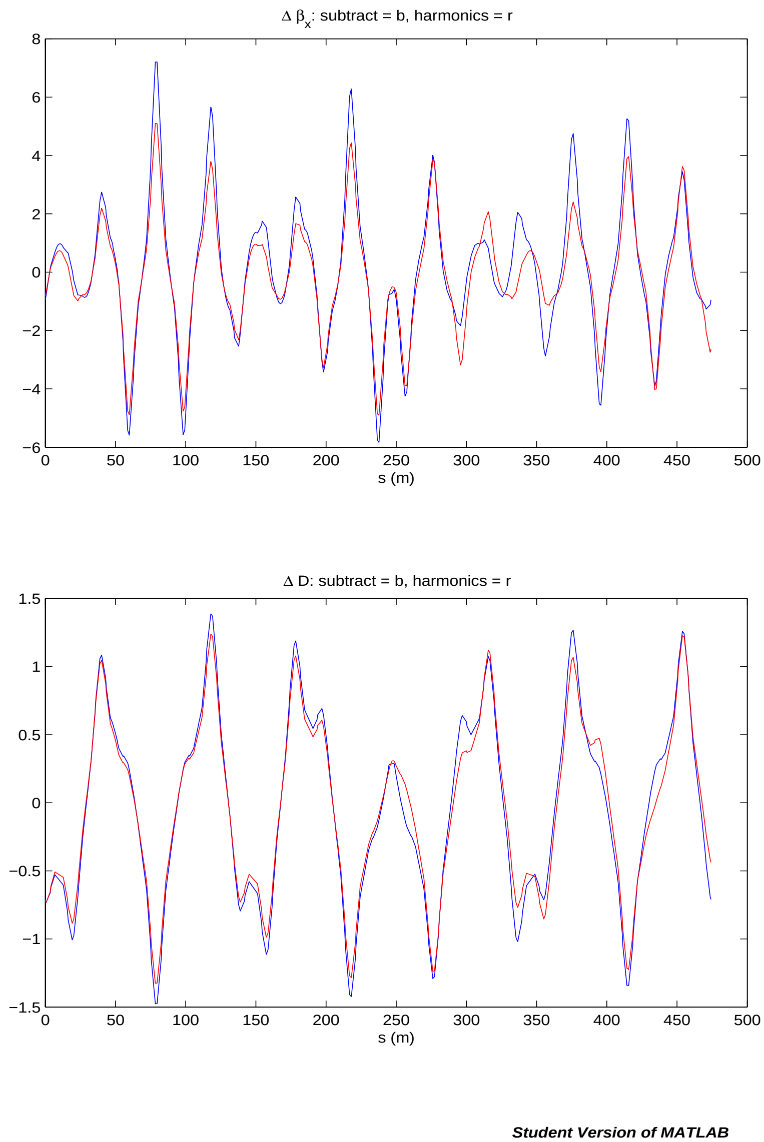

Figure 4: Changes to beta amplitude β_x and dispersion D_x due to repositioned dogleg layout. Blue curve is by subtracting MAD output with and without doglegs. Red one is obtained using equations (2.3) and (2.6).

dogleg #	δ	L (m)	$k\Delta l$ (m^{-1})
03 old	0.062510	0.24722	0.015826
03 new	0.023440	0.24722	0.002223
13	0.059876	0.24722	0.014519

Table 1: dogleg parameters. '03 old' and '13' are current layout. '03 new' is dogleg 03 after repositioning.

2.2 Implementing the compensation

The harmonic compensation is to minimize the total harmonics of doglegs and QS quadrupoles.

Use QS quads that are most efficient. Define efficiency

$$A_p^i = \frac{|J_p^i|}{|J_p|}\cos(\Delta\phi_a^i) = \mathrm{Re}\left(\frac{J_p^i}{J_p}\right) \tag{2.7}$$

$$B_n^i = \frac{|f_n^i|}{|f_n|}\cos(\Delta\phi_b^i) = \mathrm{Re}\left(\frac{f_n^i}{f_n}\right) \tag{2.8}$$

where $\Delta\phi_a^i$ ang $\Delta\phi_b^i$ are angles between J_p^i and J_p, f_n^i and f_n respectively.

Computerized compensation procedure

Step 1 sort the magnitude of the weighted overall harmonics of J_p and f_n to descending order and locate the most important harmonics, the 7th of f_n and the 14th of J_p, for example.

Step 2 sort the 24 A_{14}^i to descending order

Step 3 for each of the first 6 sorted A_{14}^i, if it has the same sign as B_7^i, change the current of the i'th QS by adding $0.1/A_{14}^i$. (i.e. cancel 10% of J_{14} with this quadrupole)

Step 4 sort the 24 B_7^i to descending order

Step 5 for each of the first 6 sorted B_7^i, if it has the same sign as A_{14}^i, change the current of the i'th QS by adding $0.1/B_7^i$. (i.e. cancel 10% of f_7 with this quadrupole)

Step 6 go to step 1

2.3 Results

case 0 Ideal lattice, no doglegs

case 1 Current lattice, with two doglegs, no compensation

case 1^* Current lattice, with two doglegs, with compensation

case 2 New lattice, dogleg 03 repositioned, no compensation

case 2^* New lattice, dogleg 03 repositioned, with compensation

case	max(β_x)	max(β_y)	max(D_x)	R_x (mm)
0	33.68	20.46	3.19	32.04
1	46.95	24.16	6.14	43.86
1^*	40.61	23.16	4.19	34.53
2	40.88	23.00	4.58	38.02
2^*	37.66	22.60	4.01	34.23

Table 2: compare maxima of β_x, β_y, D_x (in meters) and R_x.

$$R_x = \sqrt{\frac{\beta_x \epsilon_x}{\pi}} + D_x \frac{\Delta p}{p_0} \tag{2.9}$$

Typical values $\epsilon_x = 15\pi$ m $\cdot$ mrad and $\frac{\Delta p}{p_0} = \pm 0.3\%$ are used.

The quadrupole current values for case 1^* and case 2^* are well below the limit 2A. The tune shifts are small. $Q_y = 6.78111$ and $Q_x = 6.71748$ for case 1^*. $Q_y = 6.77953$ and $Q_x = 6.74996$ for case 2^*.

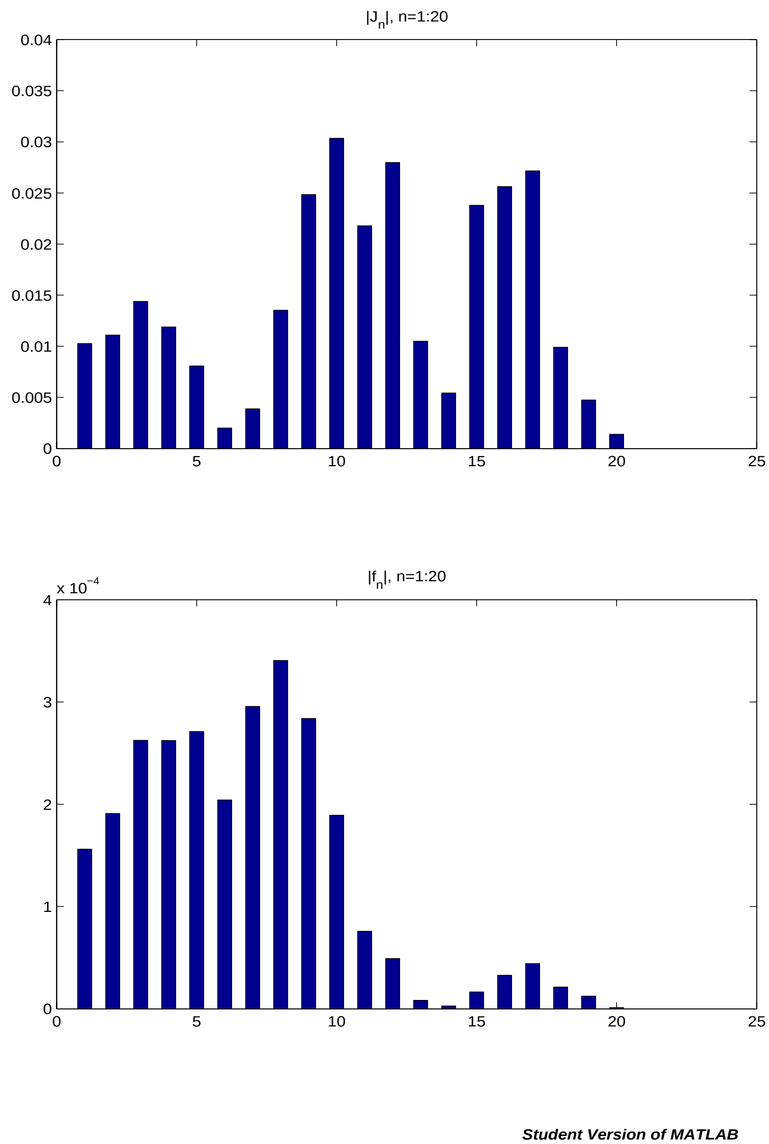

Figure 5: The weighted harmonics after compsensation, $|J_p|/(\nu^2 - p^2/4)$ and $|f_n|/(\nu^2 - n^2)$ for new layout

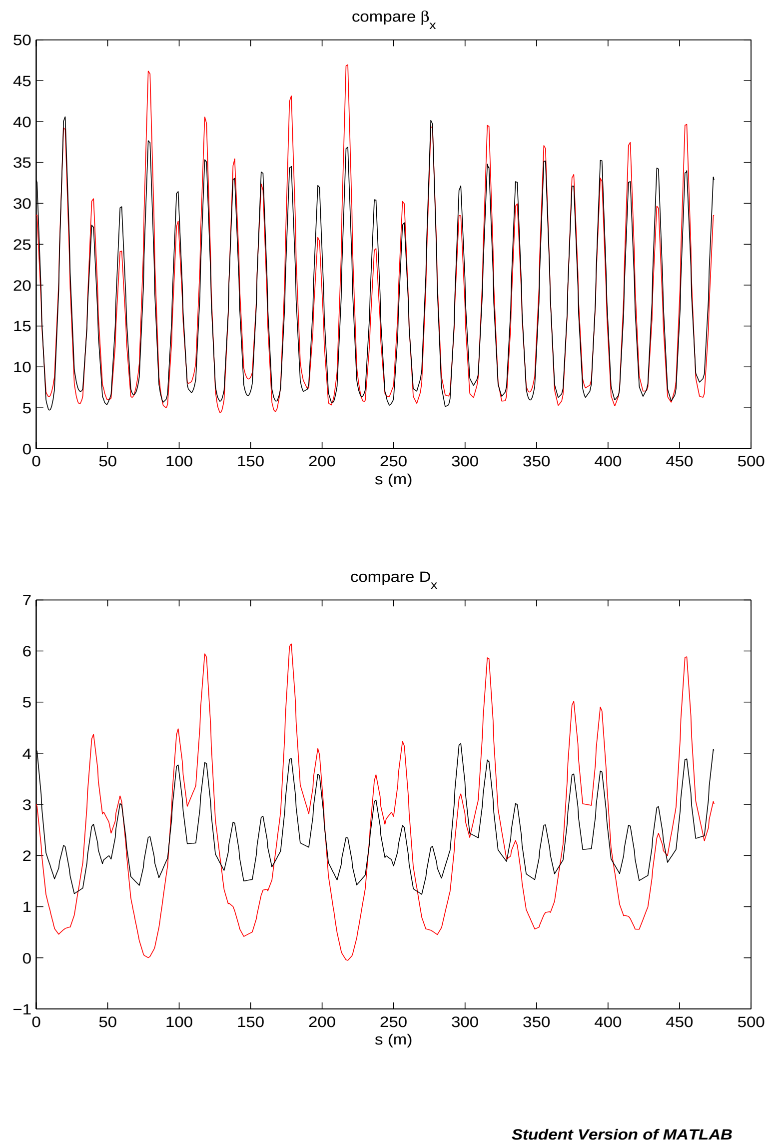

Figure 6: Comparison of horizontal beta function and dispersion for the current dogleg layout. Red curve is original and black one is compensated. Top, β_x. Bottom, D_x.

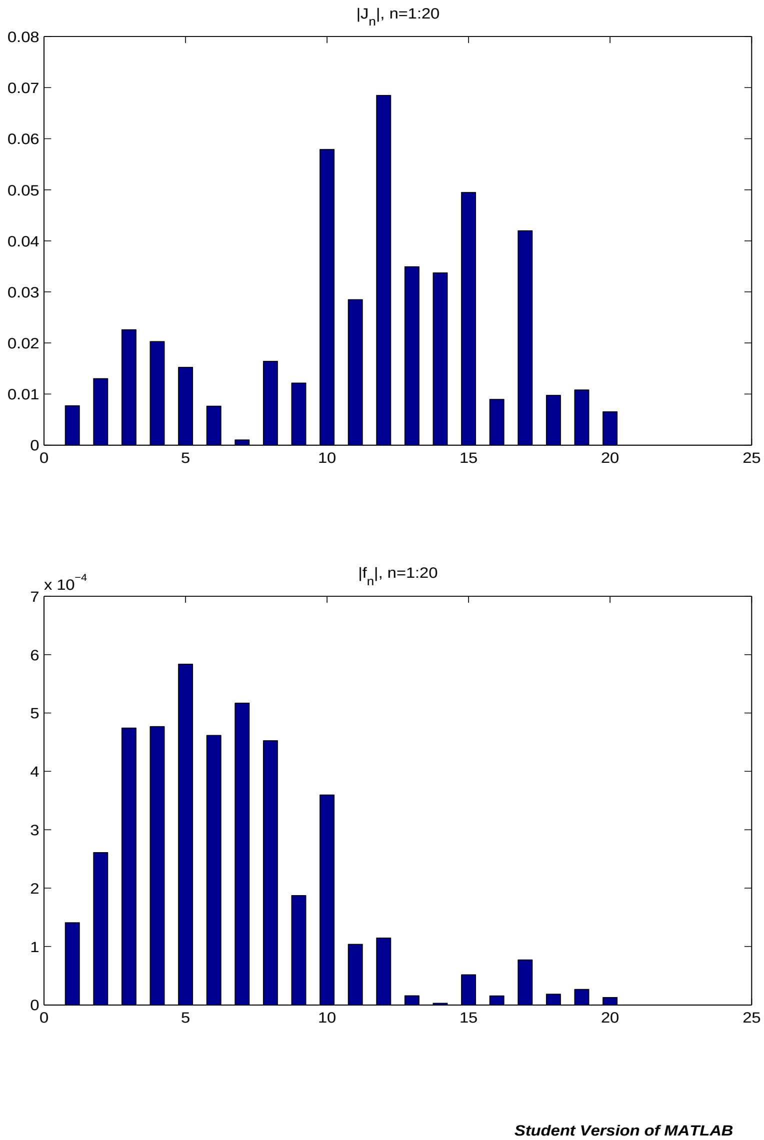

Figure 7: The weighted harmonics after compsensation, $|J_p|/(\nu^2 - p^2/4)$ and $|f_n|/(\nu^2 - n^2)$ for current layout

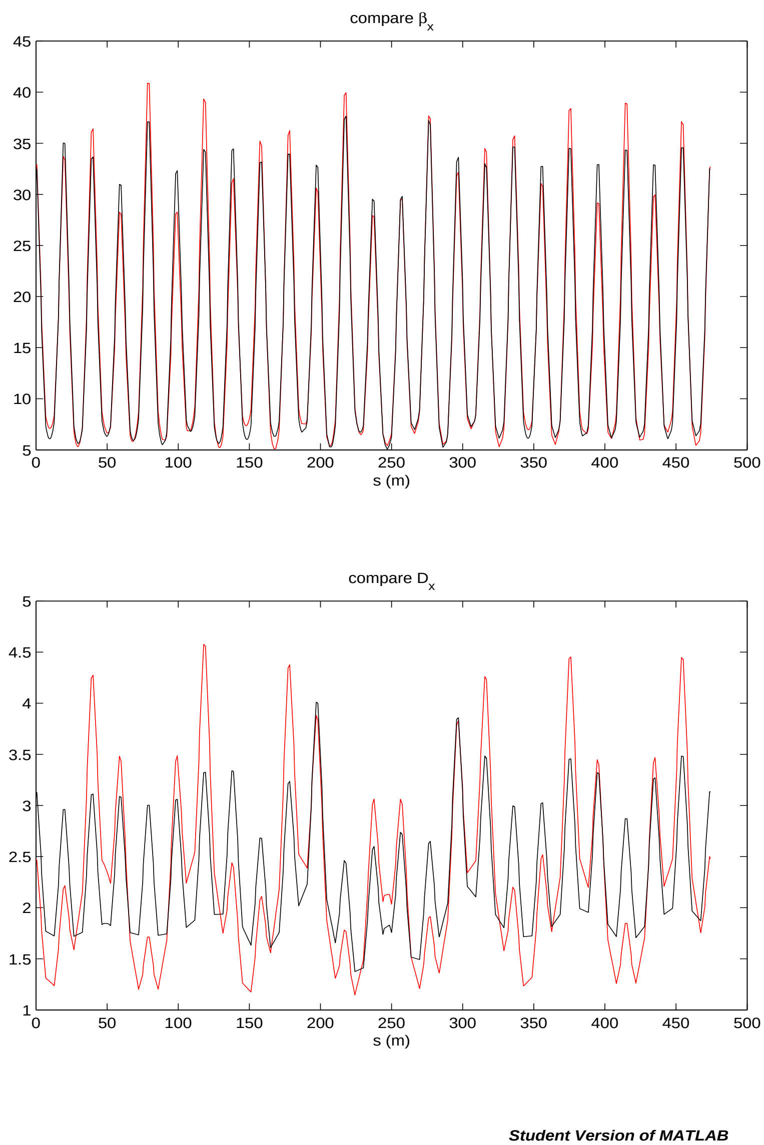

Figure 8: Comparison of horizontal beta function and dispersion for the new dogleg layout with repositioned dogleg 03. Red curve is original and black one is compensated. Top, β_x. Bottom, D_x.

3 Local beta bump

3.1 Theory

For the normalized betatron phase-space coordinates Y, $\mathcal{P}$, which are related to the usual coordinates y, y' by

$$\begin{pmatrix} Y \\ \mathcal{P} \end{pmatrix} = \begin{pmatrix} 1/\sqrt{\beta} & 0 \\ \alpha/\sqrt{\beta} & \sqrt{\beta} \end{pmatrix} \begin{pmatrix} y \\ y' \end{pmatrix}$$

the transfer matrix for a section

$$M(s_2|s_1) = \begin{pmatrix} \cos\psi & \sin\psi \\ -\sin\psi & \cos\psi \end{pmatrix} \tag{3.1}$$

For a thin quadrupole

$$M = \begin{pmatrix} 1 & 0 \\ \beta[k\Delta l] & 1 \end{pmatrix} \tag{3.2}$$

where $k = \pm\frac{B'}{B\rho}$ with '$-$' for horizontal plane and '$+$' for vertical plane.

Beta function transfer matrix

$$\begin{pmatrix} \beta_2 \\ \alpha_2 \\ \gamma_2 \end{pmatrix} = \begin{pmatrix} M_{11}^2 & -2M_{11}M_{12} & M_{12}^2 \\ -M_{11}M_{21} & M_{11}M_{22} + M_{12}M_{21} & -M_{12}M_{22} \\ M_{21}^2 & -2M_{21}M_{22} & M_{22}^2 \end{pmatrix} \begin{pmatrix} \beta_1 \\ \alpha_1 \\ \gamma_1 \end{pmatrix} \tag{3.3}$$

So for a section, with $c = \cos\psi$ and $s = \sin\psi$

$$\begin{pmatrix} \beta_2^N \\ \alpha_2^N \\ \gamma_2^N \end{pmatrix} = \begin{pmatrix} c^2 & -2cs & s^2 \\ cs & c^2 - s^2 & -cs \\ s^2 & 2cs & c^2 \end{pmatrix} \begin{pmatrix} \beta_1^N \\ \alpha_1^N \\ \gamma_1^N \end{pmatrix} \tag{3.4}$$

Across a thin quadrupole

$$\begin{pmatrix} \beta^{N+} \\ \alpha^{N+} \\ \gamma^{N+} \end{pmatrix} = \begin{pmatrix} 1 & 0 & 0 \\ -\beta[k\Delta l] & 1 & 0 \\ (\beta[k\Delta l])^2 & -2\beta[k\Delta l] & 1 \end{pmatrix} \begin{pmatrix} \beta^{N-} \\ \alpha^{N-} \\ \gamma^{N-} \end{pmatrix} \tag{3.5}$$

$+$ indicates the downstream side and $-$ indicates the upstream side.

For the unperturbed lattice, $(\beta^N, \alpha^N, \gamma^N) = (1, 0, 1)$ everywhere.

labeling parameters at the three quadrupoles with subscript 1,2,3 with 1 for the one at upstream, 3 for the one at downstream and 2 in between.

Three conditions to meet

1 $\beta_2^N = r$

2 $\beta_3^N = 1$

3 $\alpha_3^{N+} = 0$

Define $x_i = \beta_1[k\Delta l]_i$, $c_1 = \cos\psi_{12}$,$s_1 = \sin\psi_{12}$. Also we let $c_2 = \cos\psi_{23}$ and $s_2 = \sin\psi_{23}$.

A thin quadrupole doesn't change β^N at its location. But it changes α^N to $\alpha^N - \beta[k\Delta l]\beta^N$. Thus at the downstream side of quadrupole 1

$$\begin{pmatrix} \beta_1^N \\ \alpha_1^{N+} \\ \gamma_1^{N+} \end{pmatrix} = \begin{pmatrix} 1 \\ -x_1 \\ 1+x_1^2 \end{pmatrix} \tag{3.6}$$

Combining with equation (3.4), the condition $\beta_2^N = r$ becomes

$$s_1^2 x_1^2 + 2c_1 s_1 x_1 + (1-r) = 0 \tag{3.7}$$

with a solution

$$x_1 = \frac{1}{s_1}(-c_1 \pm \sqrt{r - s_1^2}) \tag{3.8}$$

Quadrupole 2 turns $(\beta_2^N, \alpha_2^{N-}, \gamma_2^{N-})$ to $(\beta_2^N, X_2, 1+X_2^2)$ with $X_2 = \alpha_2^{N+} = \alpha_2^{N-} - x_2\beta_2^N$. The latter is then transfered to quadrupole 3 to meet condition $\beta_3^N = 1$, which yields

$$\frac{s_2^2}{r}X_2^2 - 2c_2 s_2 X_2 + (\frac{s_2^2}{r} + c_2^2 r - 1) = 0 \tag{3.9}$$

with solution

$$X_2 = \frac{r}{s_2}(c_2 \pm \frac{1}{r}\sqrt{r - s_2^2}) \tag{3.10}$$

$$x_2 = \frac{\alpha_2^{N-} - X_2}{\beta_2^N} \tag{3.11}$$

α_3^{N-} with equation (3.4)

$$\alpha_3^{N-} = c_2 s_2 (r - 1 - X_2^2) + (c_2^2 - s_2^2) X_2 \tag{3.12}$$

Condition $\beta_3^N = 1$ and condition $\alpha_3^{N+} = \alpha_3^{N-} - x_3\beta_3^N = 0$ together lead to

$$x_3 = \alpha_3^{N-} \tag{3.13}$$

3.2 Application to the Booster

For the Booster, to suppress β_x at section n, need to use QS quads at section n-2, n and n+2.

For the current dogleg layout, all quadrupole correctors off, the maximum of β_x is 46.95m at S11. To suppress 10% of it,

- iqs09=0.6490A
- iqs11=-0.7202A
- iqs13=0.5674A

β_x at S11 becomes 42.47m, which is 90.5% of the unperturbed value.

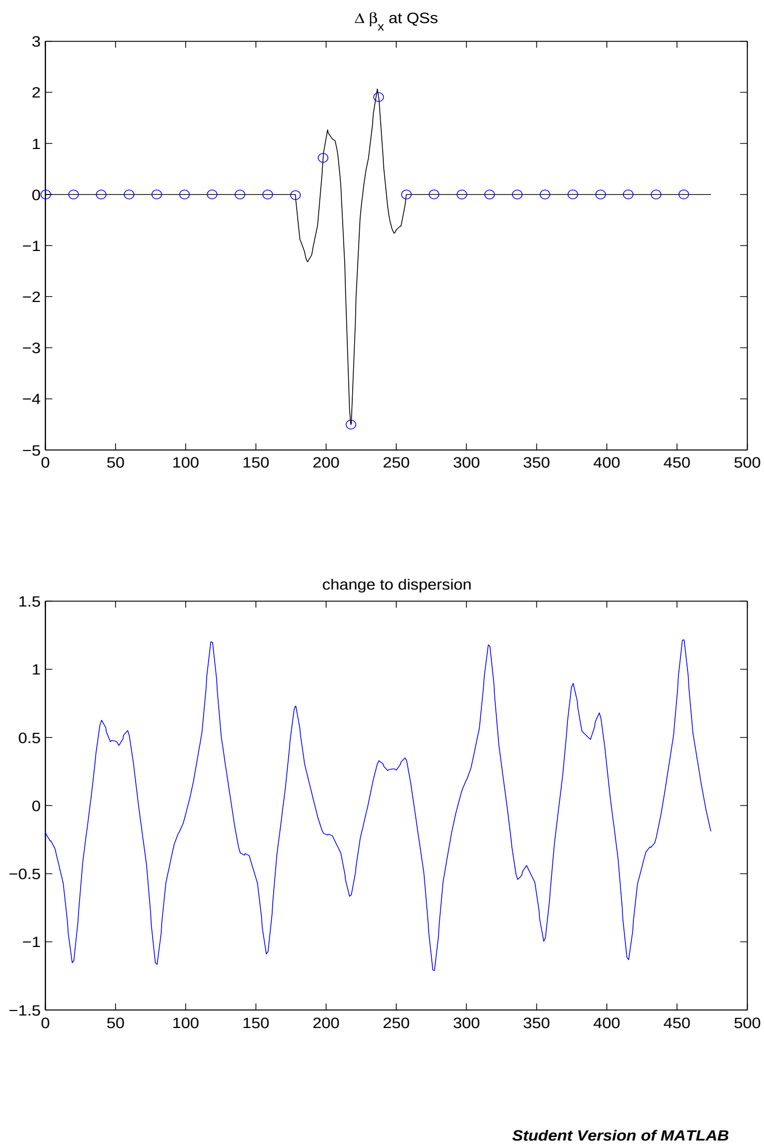

Figure 9: Changes to β_x and dispersion due to QS09, QS11, QS13 set up for local beta bump. Top, β_x. Bottom, D_x.

3.3 Locality error analysis

Our knowledge about Booster's beta function and phase advance is not perfect, will the perturbation still be local?

Monte Carlo method is used to find out the error.

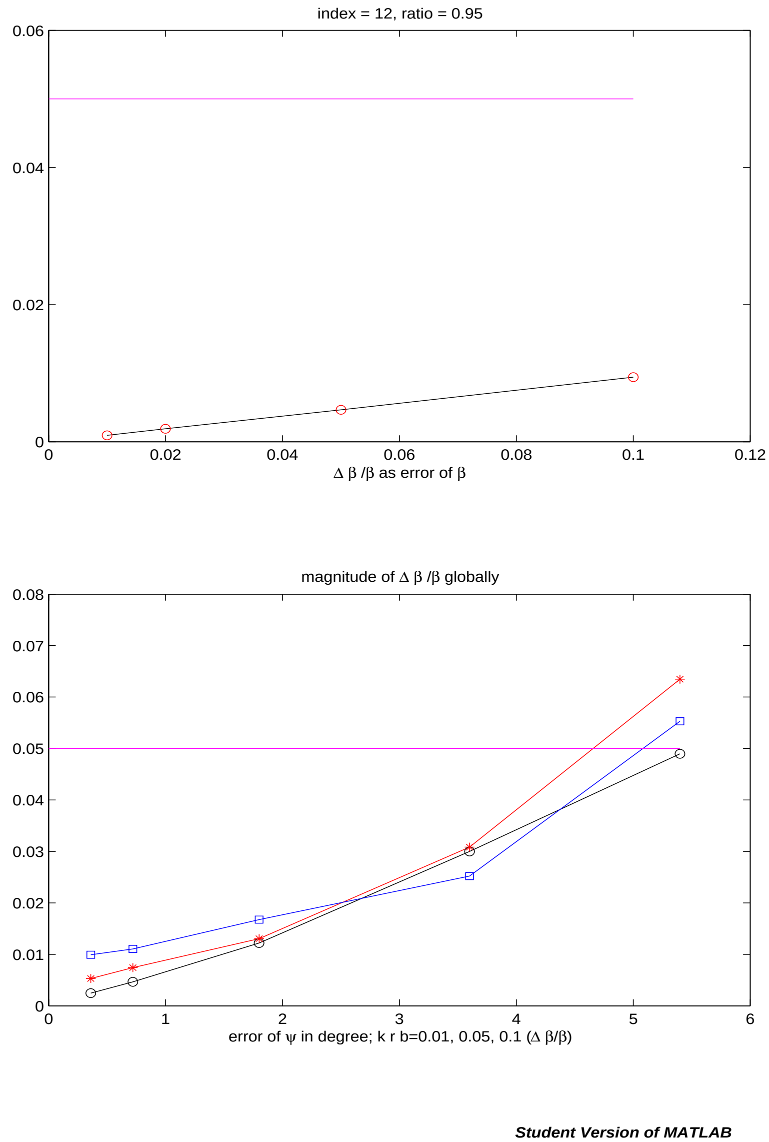

Figure 10: Magnitude of global $\frac{\Delta\beta}{\beta}$ indicating the locality error. To suppress 5% of β_x at S11 of the current layout. Top, consider β error only. Bottom, both phase and β considered, with $\frac{\Delta\beta}{\beta} = 0.01, 0.05, 0.1$ for black, red and blue curves, respectively

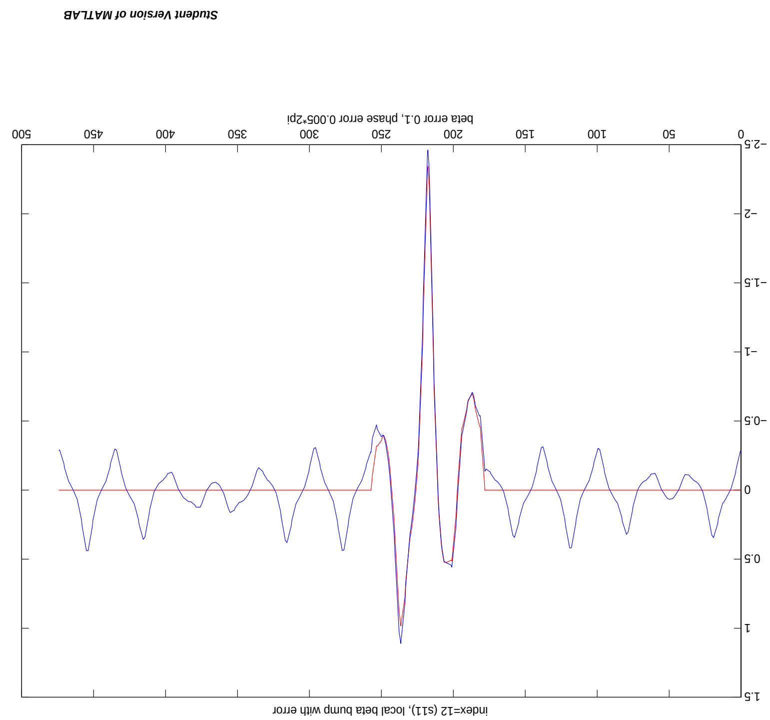

Figure 11: Changes to β with or without error in the case of suppressing 5% of β at S11 of the current layout. Phase error level 2 degree and $\frac{\Delta\beta}{\beta} = 0.1$