Fitting the Fully Coupled Orbit Response Matrix to Model

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Fitting the orbit response matrix to model

The orbit response matrix

$$M_{ij} = \frac{\Delta y_i}{\Delta \theta_j}$$

The change of orbit at *i*'th BPM due to a kick at location *j*.

The fully-coupled orbit response matrix

$$\mathbf{M} = \begin{pmatrix} M_{xx} & M_{xz} \\ M_{zx} & M_{zz} \end{pmatrix}$$

Each block is a 48×48 matrix for the Booster

In experiment, each element is measured several times by applying different kicks. The results are fitted to linear curve to obtain the element (the slope) and its error estimate (sigma)

The Model orbit response matrix

The fully-coupled ORM can be computed with an accelerator model (e.g., MAD).

$$(\mathbf{T}-\mathbf{I})\begin{pmatrix} x/\theta_x \\ x'/\theta_x \\ z/\theta_x \\ z'/\theta_x \end{pmatrix} = \begin{pmatrix} 0 \\ -1 \\ 0 \\ 0 \end{pmatrix}$$

Let T be the 4D one-turn map at a kicker. The orbit deviation per unit horizontal kick at the kicker's location can be computed by solving the equation.

$$M_{xx}(i,i) = x/\theta_x, \; M_{zx}(i,i) = z/\theta_x.$$

Other elements can be obtained with the transfer matrix.

In the Booster model, the trim quads, skew quads and sextupoles are set to operation values. But the trim dipoles are left zeros to avoid the spurious sextupole focusing effect of the body quads.

Corrections to the measured ORM

The measured ORM need corrections because of BPM gains, BPM rolls, kicker gains, kicker rolls, orbit shifts due to momentum deviations.

$$B_i^{-1} = \begin{pmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{pmatrix} \begin{pmatrix} 1/b_h & 0 \\ 0 & 1/b_v \end{pmatrix}$$

BPM gains and rolls

$$\begin{pmatrix} \Delta x \\ \Delta z \end{pmatrix}_i^{\mathrm{act}} = B_i^{-1} \begin{pmatrix} \Delta x \\ \Delta z \end{pmatrix}_i^{\mathrm{meas}}$$

Kicker gains and rolls, momentum deviation due to horizontal kickers and rotated vertical kickers. The dispersion function is from the model.

$$\begin{pmatrix} M_{xx} \\ M_{zx} \end{pmatrix}_{ij}^{\mathrm{meas}} = B_i \left\{ \begin{pmatrix} M_{xx} & M_{xz} \\ M_{zx} & M_{zz} \end{pmatrix}_{ij}^{\mathrm{act}} k_h \begin{pmatrix} \cos\theta_h \\ \sin\theta_h \end{pmatrix}_j + \begin{pmatrix} \left(\frac{\Delta p}{p}\right)_h^j D_x^i \\ 0 \end{pmatrix} \right\}$$

$$\begin{pmatrix} M_{xz} \\ M_{zz} \end{pmatrix}_{ij}^{\mathrm{meas}} = B_i \left\{ \begin{pmatrix} M_{xx} & M_{xz} \\ M_{zx} & M_{zz} \end{pmatrix}_{ij}^{\mathrm{act}} k_v \begin{pmatrix} \sin\theta_v \\ \cos\theta_v \end{pmatrix}_j + \begin{pmatrix} \left(\frac{\Delta p}{p}\right)_v^j D_x^i \\ 0 \end{pmatrix} \right\}$$

Inverse the above equation to obtain “corrected” ORM matrix.

χ^2 to be minimized

The objective function of the least-square problem

$$\chi^2 = \sum_{ij} \frac{(M^{\text{corrected}} - M^{\text{model}})_{ij}^2}{\sigma_{ij}^2} + \frac{(\nu_x^{\text{measured}} - \nu_x^{\text{model}})^2}{\sigma_{\nu x}^2} + \frac{(\nu_z^{\text{measured}} - \nu_z^{\text{model}})^2}{\sigma_{\nu z}^2}$$

Form a remainder vector *r* (of dimension 9218 $\times$1) to contain the difference terms in the χ^2 definition, then

$$\chi^2 = \mathbf{r}^T(\alpha)\mathbf{r}(\alpha)$$

α is the column vector that contains all fitting parameters, which are horizontal BPM gains (48), vertical BPM gains (48), horizontal kicker gains (48), vertical BPM gains (48), BPM rolls (48), horizontal kicker rolls (48), vertical kicker rolls (48), momentum deviation due to horizontal kicks (48), momentum deviation due to rotated vertical kicks (48), corrections of body quadrupole gradients (96), magnet rolls (96). There are 624 parameters in total. The last two categories are in the MAD model of the Booster.

Minimization method

The Levenberg-Marquadt method

Compute the Jacobian matrix (of dimension 9218×624)

$$J_{ij} = \frac{\partial r_i}{\partial \alpha_j}$$

Solve the normal equation

$$(\mathbf{J}^T \mathbf{J} + \lambda \mathbf{I}) \Delta \alpha = -\mathbf{J}^T \mathbf{r}$$

λ is a parameter which decrease by a factor of 10 if chi-square is reduced in an iteration, otherwise increase by a factor of 10.

Iterate while adjusting the λ parameter. When it is converged, calculate the covariance matrix.

$$\mathbf{C} = (\mathbf{J}^T \mathbf{J})^{-1}$$

The error sigma of the parameters are the diagonal elements of C.

$$\sigma_i = \sqrt{C_{ii}}$$

ORM Data Taken in November

Nov 05 data, Booster in 8 GeV cycle. TBT data were also taken to measure the tunes.

Nov 15 data, Booster in 6 GeV cycle. Dispersion data were taken by changing ROF curve at injection and extraction. TBT data were also taken to measure the tunes.

Each data set measures the orbit at different times (frames) of the cycle.

frame	Time (ms)	Ek (8GeV)	Ek (6GeV)
1	2.90	0.410	0.408
2	3.81	0.441	0.431

Result - χ^2

nov 05 frm 1

chi20 = 1.8307

type	chi2	chi2-chi20
hbi	3.8028	1.9722
vbi	13.0019	11.1712
hkj	4.4885	2.6578
vkj	3.0238	1.1931
bpmroll	2.6345	0.80387
vdpp	2.7931	0.96244
hkickroll	2.4052	0.57454
vkickroll	5.1643	3.3336
hdpp	22.472	20.6412
quads	17.21	15.3793
rolls	18.7736	16.9429

nov 15 frm 1

chi20 = 1.9381

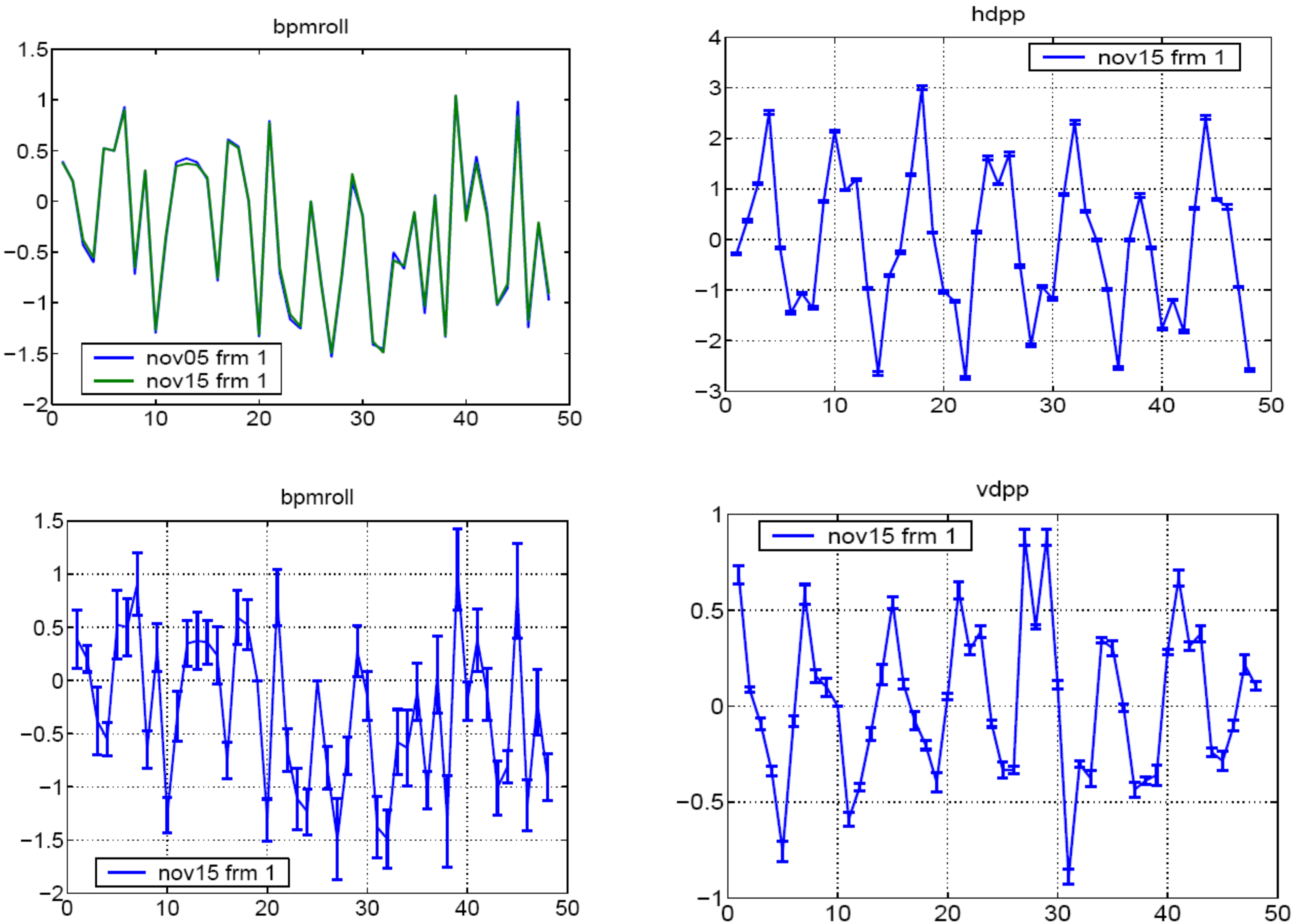
type	chi2	chi2-chi20
hbi	4.5491	2.611
vbi	13.5001	11.562
hkj	4.9243	2.9862
vkj	2.8923	0.95425
bpmroll	2.7017	0.76359
vdpp	2.9521	1.014
hkickroll	2.4947	0.5566
vkickroll	5.3751	3.437
hdpp	22.6355	20.6974
quads	7.1204	5.1823
rolls	18.8456	16.9075

data set	init	final
nov05, frm 1	62.5	1.8
nov15, frm 1	133.7	1.9

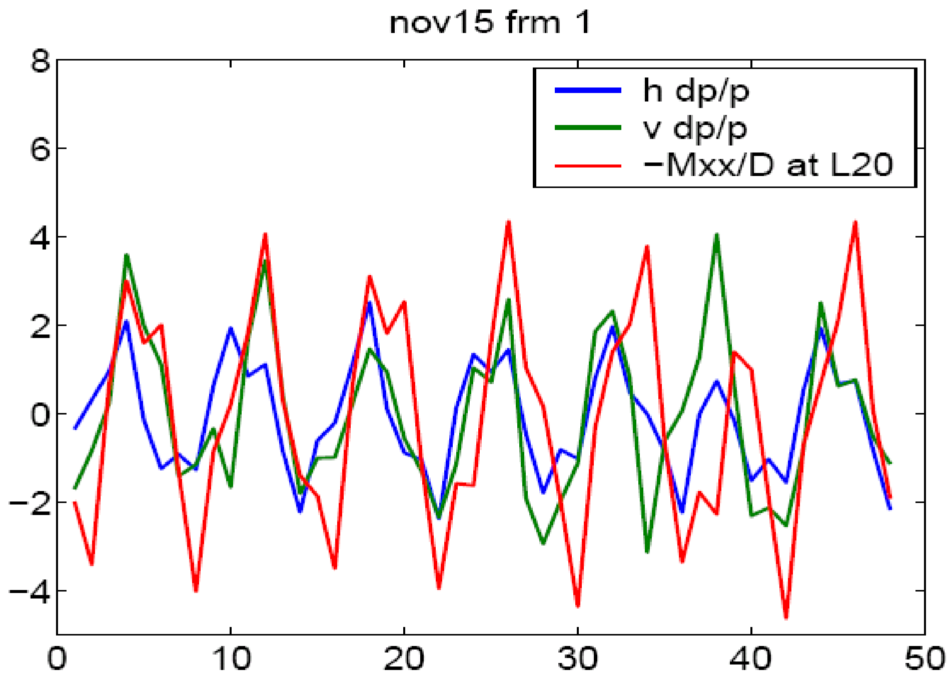
Table 1: Initial and final χ^2.

χ^2 is normalized by the number of data points. So it should be 1.0 after fitting if what remains are due to random errors.

Results – bpm rolls (degree) and dp/p



Result, dp/p

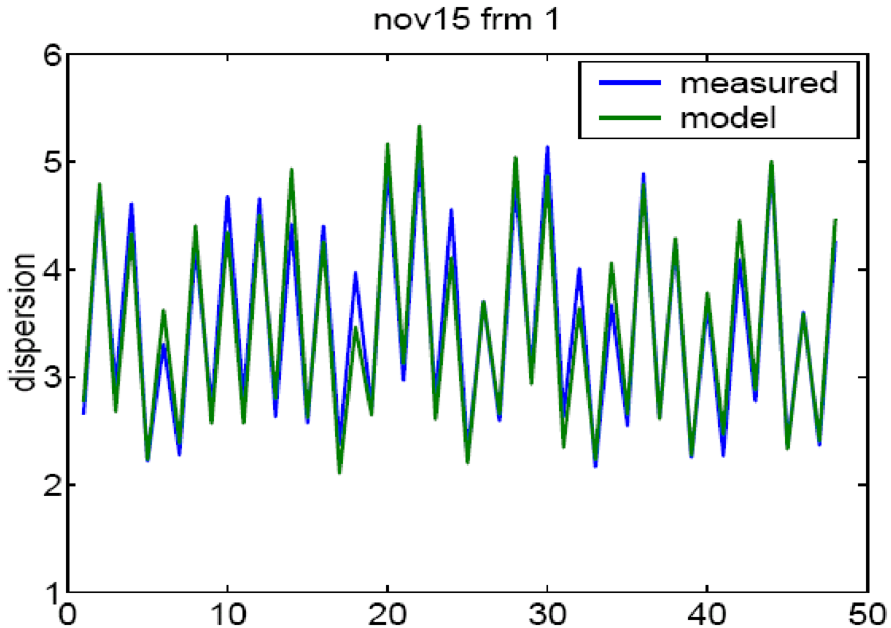


A kick will cause orbit deviation at L20, where the RPOS signal is picked up. Momentum deviation is induced to compensate such orbit deviation at L20. Therefore we expect

$$\left(\frac{\Delta p}{p}\right)_h D_i + M_{xx}(i,\mathrm{L20})\theta = 0$$

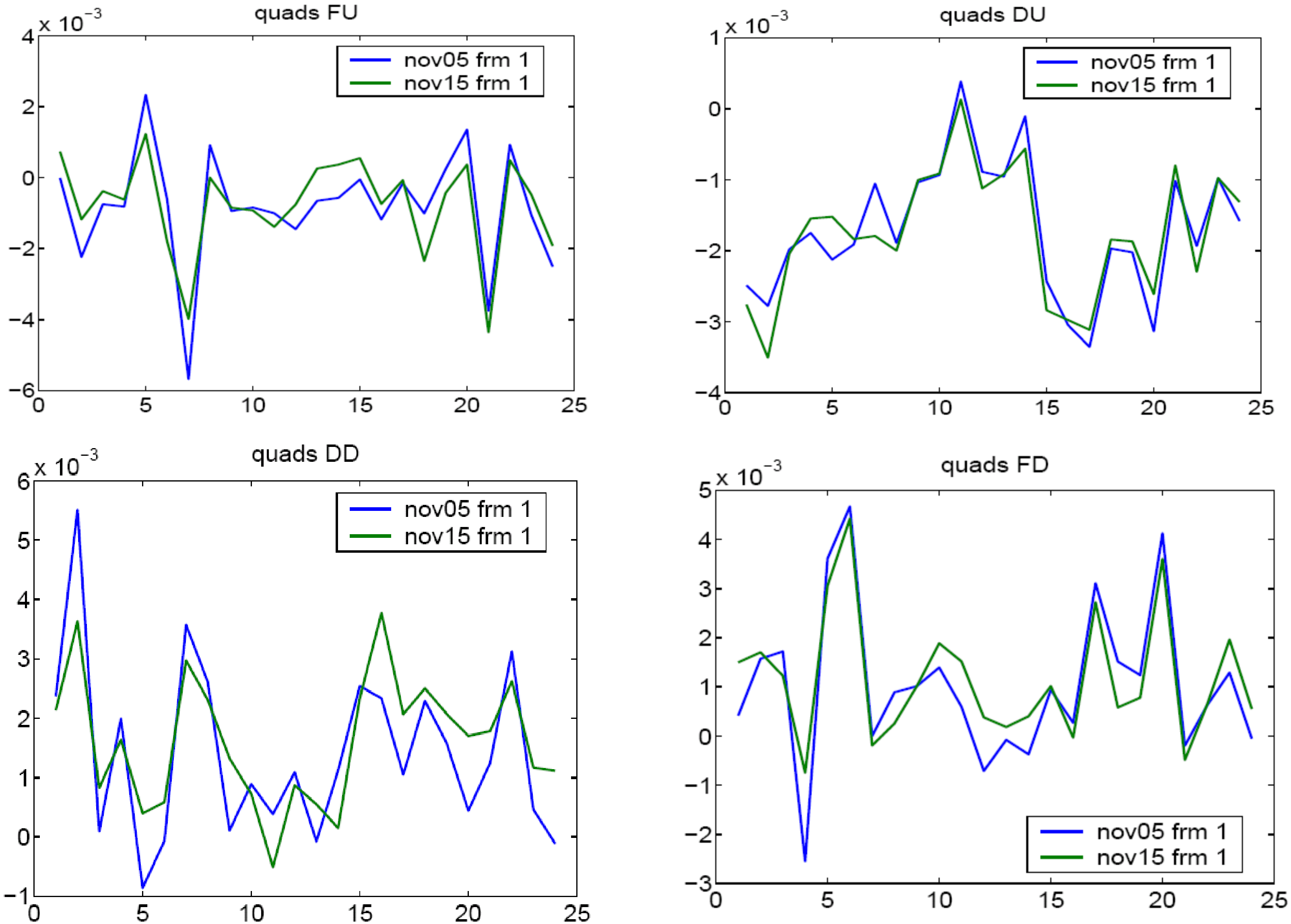
Momentum deviation (1E-3) per actual horizontal kick (mrad) by the horizontal (blue) and vertical (green) kickers. Note the vertical kicker are rotated.

Measured and model dispersion

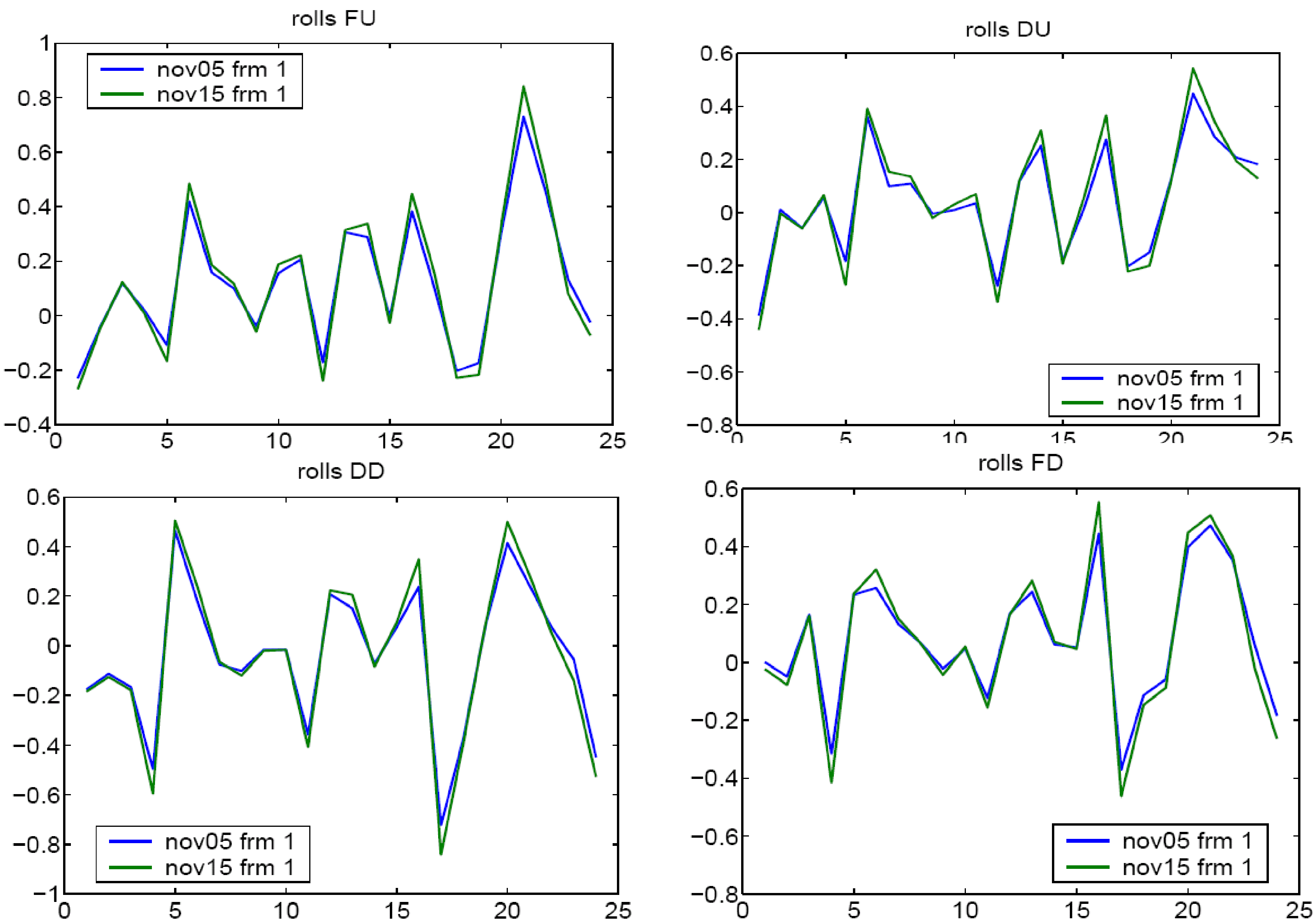


Dispersion were measured by changing ROF curve near injection (t=3.0ms). Horizontal gains are divided. A ratio of 0.81 has also been divided from the measured dispersion to scale it up. It is necessary because console Page B40 assumes the ideal model to calculate dp/p.

Quadrupole corrections (m^{-2})



Magnet rolls (in degrees)

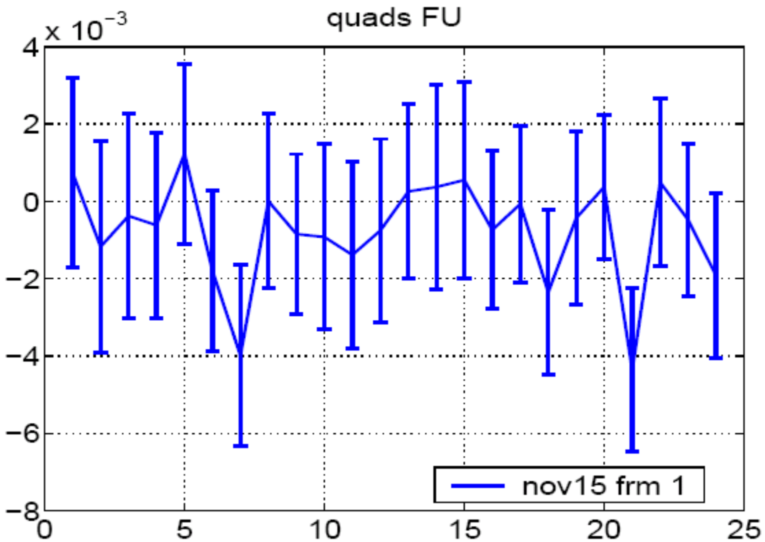


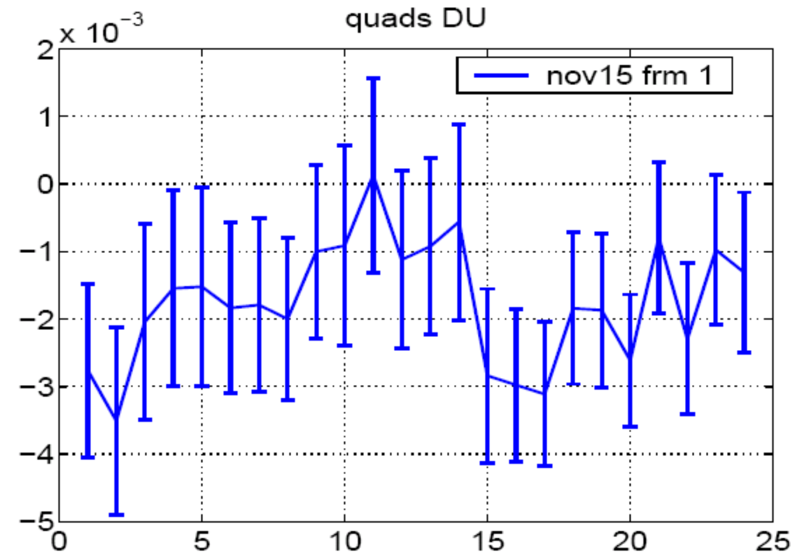
Observations

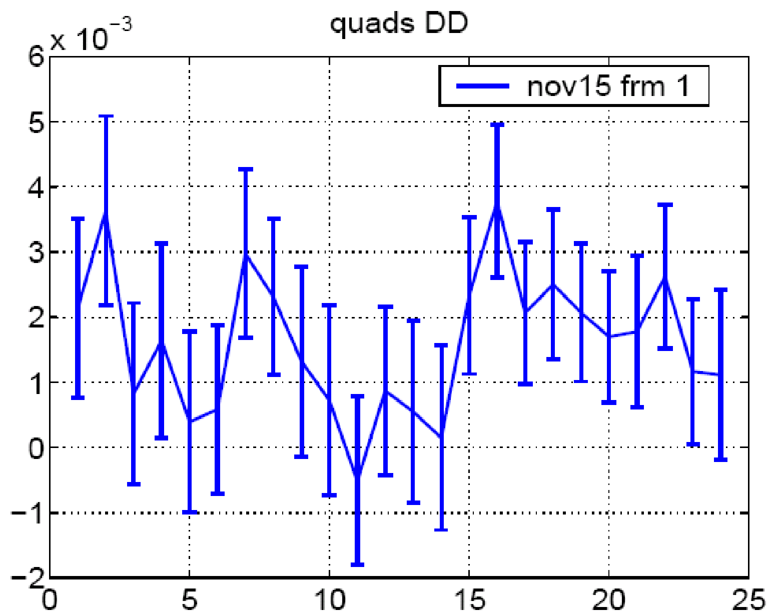
A confusing observation (Sho Ohnuma):
The quadrupole gradient corrections tend to be positive for DD magnets and negative for DU magnets. How to explain this “asymmetry”?

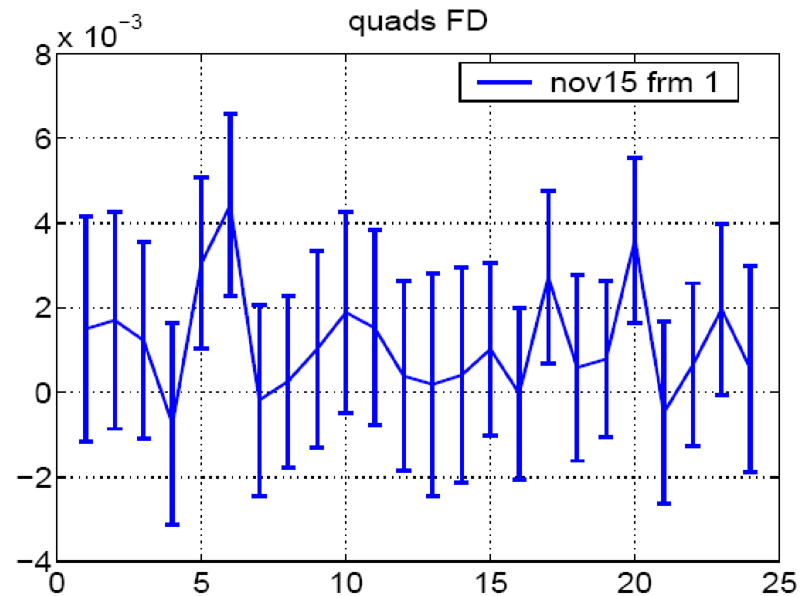
A tentative explanation:
The magnets are assumed to be nearly identical. But the gradient inside the magnets are not uniform everywhere. Let assume one side is stronger than the other. If the beam tend to pass one particular side of the DU magnets and the other side of DD magnets, then the effective gradients it sees will appear as observed. But,
Does the beam orbit have such a pattern in the [FU,DU,DD,FD] combination of a sub-period?

Error bars – quads m^{-2}

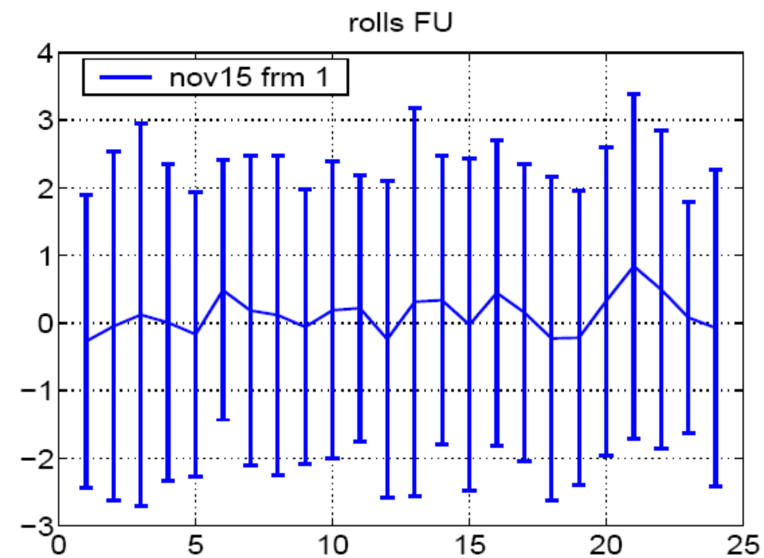


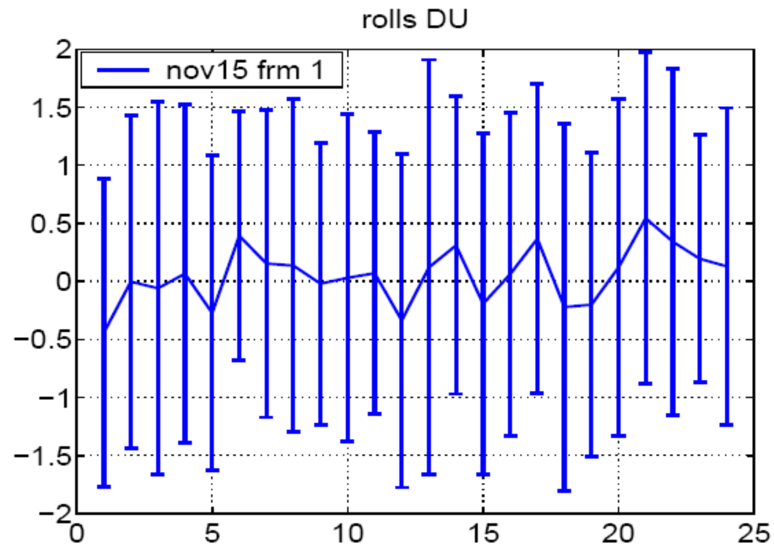


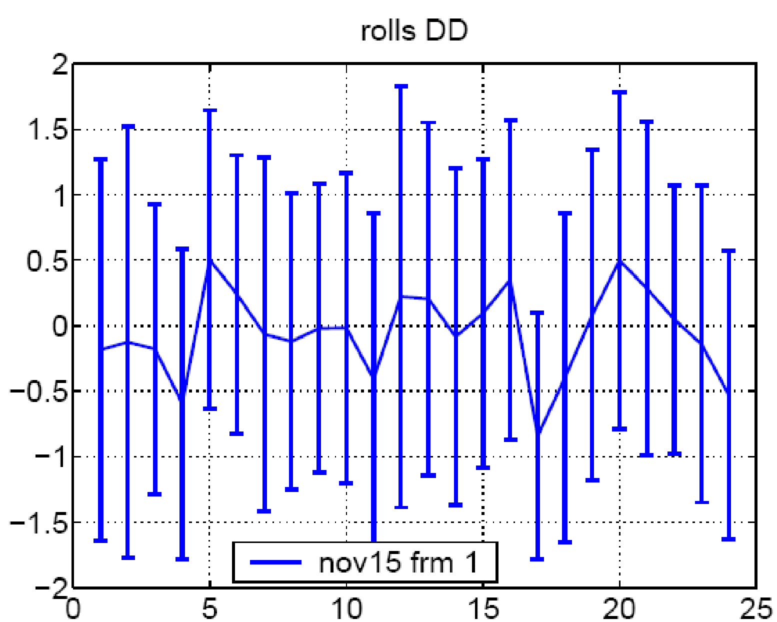


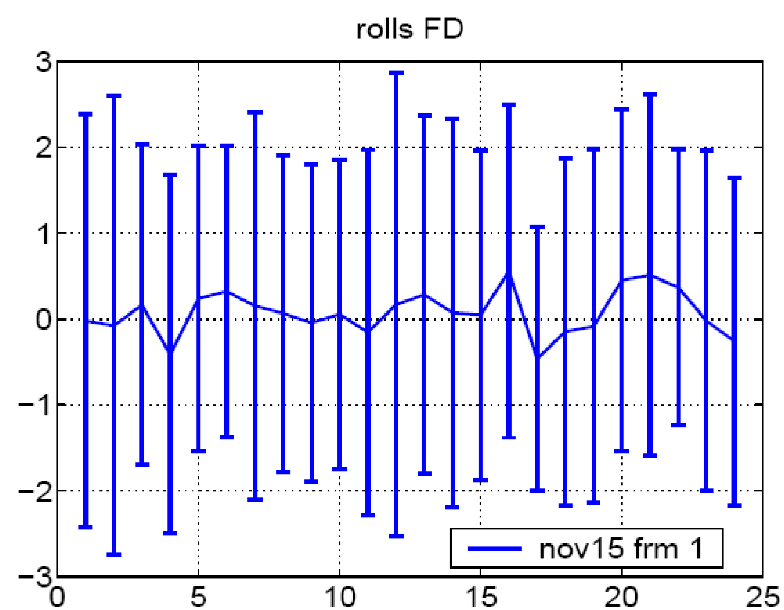


Error bars – quad rolls (degree)









Believe It or Not

Don't believe it because:

1. The values of the gradient errors and magnet rolls are out of the believable range. Too large!
2. The asymmetry in the result of gradient errors of DU/DD magnets.
3. The surveyed magnets tilts are below 5 mrads.

Believe it because:

1. The ORM is a reliable representation of the lattice, e.g., the misalignments of the BPMs are irrelevant.
2. The χ^2 are reduced to a very low level.
3. All variations of the fitting scheme that have been tried always converge to nearly the same result.
4. The measured dispersion agrees to the fitted model.

Summary

- The fully coupled ORM fitting problem has been carried out for the Booster.
- χ^2 are brought down to near random noise level.
- Results of different data sets are consistent.
- Error bars for quadrupole rolls are too large, but are moderate for quadrupole gradients.
- BPM gains, rolls, (dp/p), etc are also obtained.