BEAM HALO ENHANCEMENT from COLLECTIVE MODES and COLORED NOISE

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(presented to Fermilab Booster Space Charge Study Group, 27 May 2004)

WHY WORRY ABOUT BEAM HALO?

- Proton machines such as spallation-neutron-source drivers:
  - Need □1 nA m^{-1} GeV^{-1} beam loss for hands-on maintenance.
  - For 1 mA, 1 GeV beam, this is just □1 particle in 10^6 per meter.

- Electron machines such as energy-recovery linacs:
  - Need □1 μA beam loss for machine and electronics protection.
  - For 100 mA beam (high-*P* FELs), this is just □1 particle in 10^5.

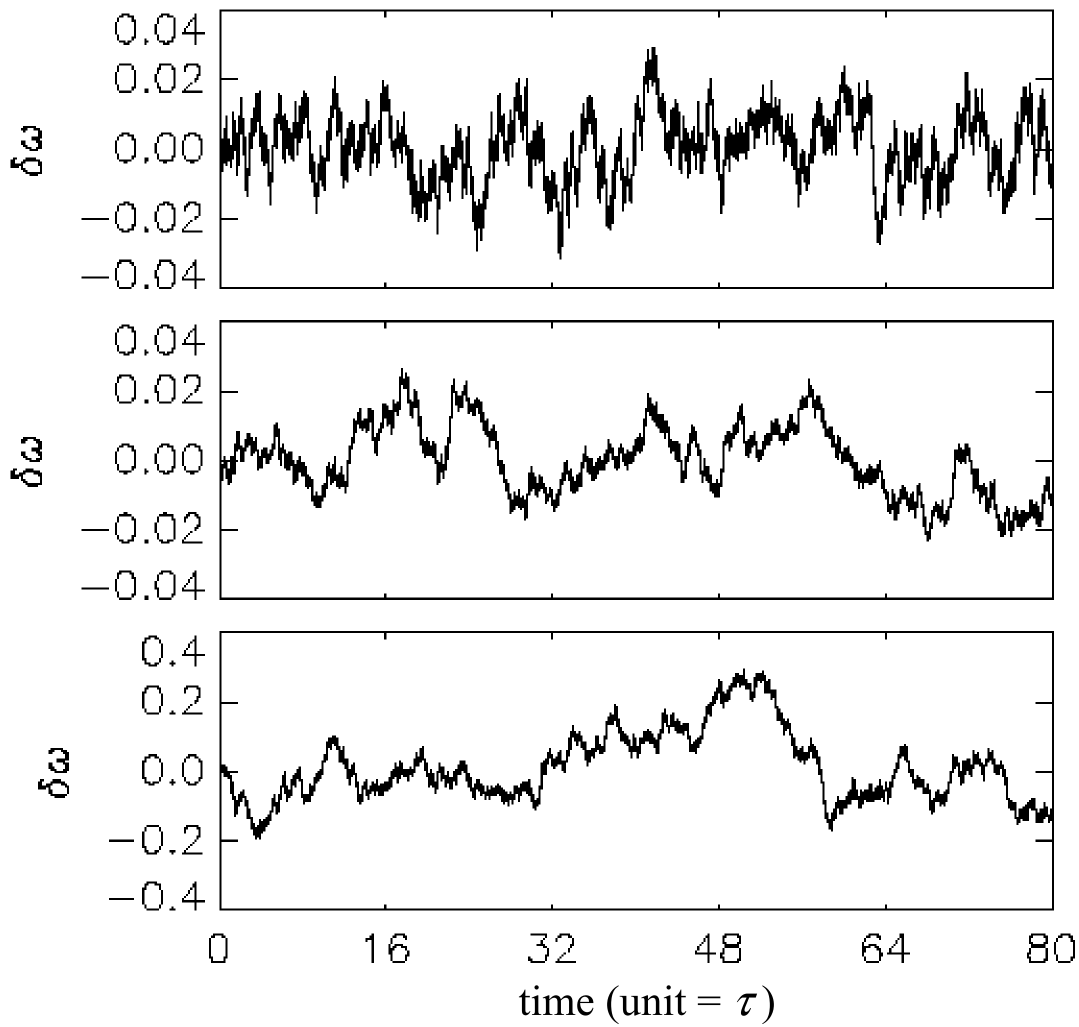
Comprehensive understanding of beam-halo formation is imperative!

- Standard Picture: Parametric resonance
  - Viewed as *the* fundamental mechanism of halo formation.
  - Predicts hard upper bound to halo amplitude.

Question of the Hour: Is parametric resonance *really* everything?

EXAMPLE MANIFESTATIONS OF COLORED NOISE ALONG AN ORBIT

$$\langle \delta\omega(t) \rangle = 0, \quad \langle \delta\omega(t)\delta\omega(t_1) \rangle \propto \exp(-|t - t_1|/t_c),$$



$$\langle |\delta\omega| \rangle = 0.01, \quad t_c = 1.5\tau$$

$$\langle |\delta\omega| \rangle = 0.01, \quad t_c = 12\tau$$

$$\langle |\delta\omega| \rangle = 0.1, \quad t_c = 12\tau$$

(τ = orbital period of a typical halo particle)

TO START: TWO TOY MODELS

[C. L. Bohn and I. V. Sideris, *Phys. Rev. Lett.* **91**, 264801 (2003)]

- Model I – Variant of Gluckstern/Wangler Particle-Core Model:

$$\ddot{x} + x\left[1 - \frac{\Theta(1-|x|)}{[1+(M-1)\cos\omega t]^2} - \frac{\Theta(|x|-1)}{x^2}\right] = 0;$$

$M = R(0)/R$ is the mismatch parameter,

$$\omega = \sqrt{2} \quad (\text{zero tune depression}); \quad \boxed{\omega \rightarrow \omega + \delta\omega(t)}$$

- Model II – Thermal-Equilibrium Beam with Pulsation Added:

$$\ddot{\mathbf{x}} = -\nabla\Psi; \quad \Psi = \Psi_0 + \Psi_1;$$

$$\Psi_0 = \frac{1}{2}\Omega^2 r^2 + \Phi(r), \quad \Psi_1 = \mu\Phi(r_1)\sin\omega t,$$

$$r = \sqrt{x^2 + y^2 + z^2}, \quad r_1 = \sqrt{0.8(x^2+y^2)+z^2};$$

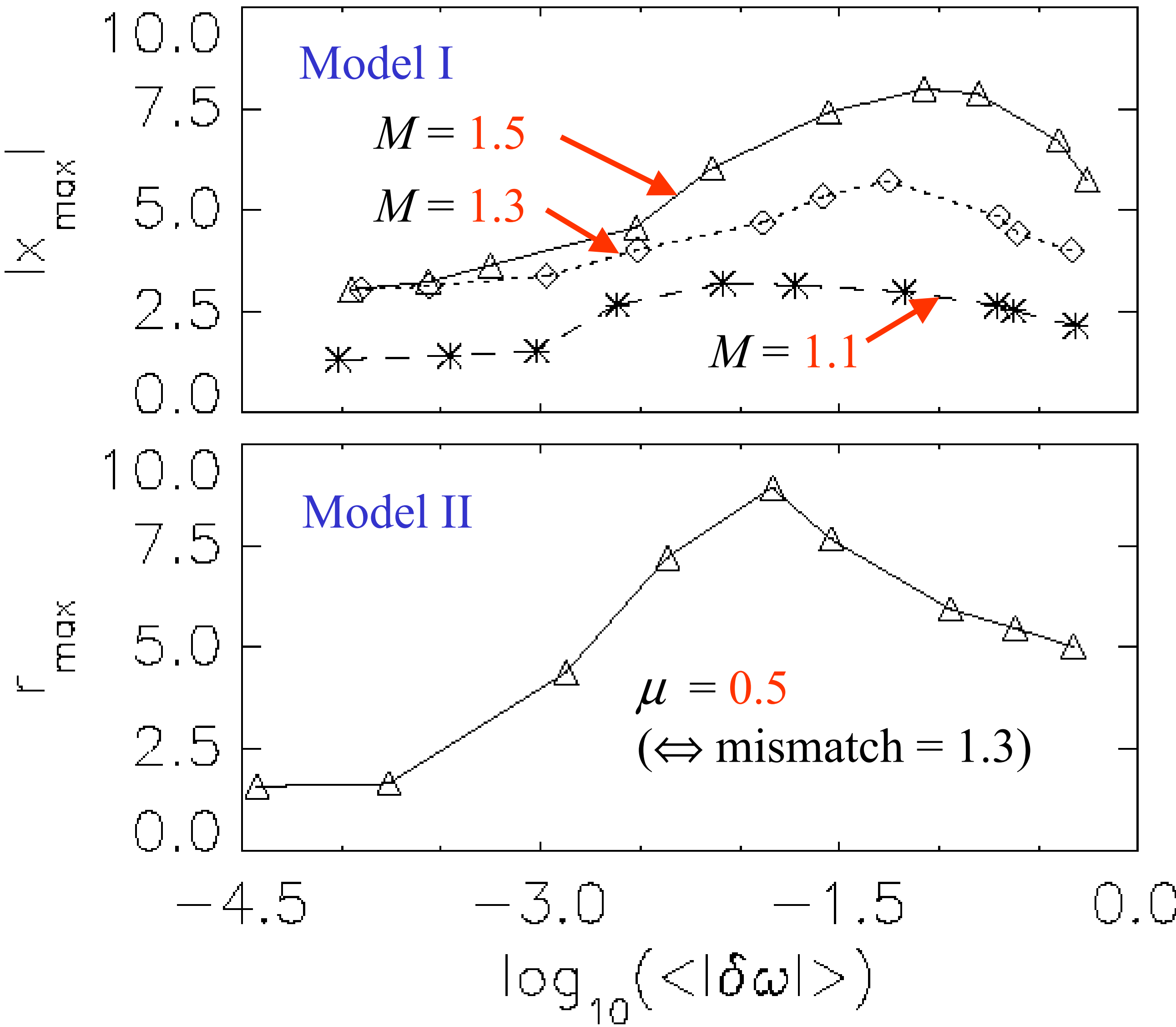
$$\Omega = 1.0001/\sqrt{3} \quad \text{and} \quad \omega = 1.7 \quad (0.36 \text{ tune depression})$$

INVESTIGATIVE STRATEGY

In a real beam each individual particle will have its own distinct initial conditions and thus experience a manifestation of the noise that differs from that seen by each of the other particles. For example, in the axisymmetric Model I, each particle initially occupying a thin annulus centered at radius $x(0)$ will experience noise differing from that seen by each of the other particles initially in that annulus because the particles start at different angular coordinates. The same is true for particles initially occupying a spherical shell centered on radius $r(0)$ in Model II. Accordingly, we adopted a 'survey strategy'. Upon choosing initial conditions $x(0)$ and $r(0)$ for Eqs. (1) and (2), respectively, and for a specific choice of noise parameters, we sequentially computed 10,000 orbits, each experiencing its own random manifestation of the colored noise, and we catalogued the maximum amplitudes of these orbits. We set the initial conditions of the orbit in Model I at $x(0) = 1.20$, $\dot{x}(0) = 0$, and in Model II at $r(0) = 1.23$, $\dot{r}(0) = 0$. In the unperturbed TE sphere of Model II, and for realistic proton beam parameters, there are $\sim 4 \times 10^9$ particles per bunch, i.e., ~ 0.6 nC [17]. There are $\sim 3 \times 10^4$ particles in the range $r = 1.23 \pm (0.5 \times 10^{-4})$, a thin spherical shell centered on $r(0)$ and located well into the Debye tail of the bunch. Accordingly, the chosen sample size is realistic.

LARGEST ORBITAL AMPLITUDES IN MODELS I, II

$$t_c = 12\tau$$



For Comparison:

Zero noise ⇒

$|x_{max}| = 1.20$ for $M = 1.1$

$|x_{max}| = 2.54$ for $M = 1.3$

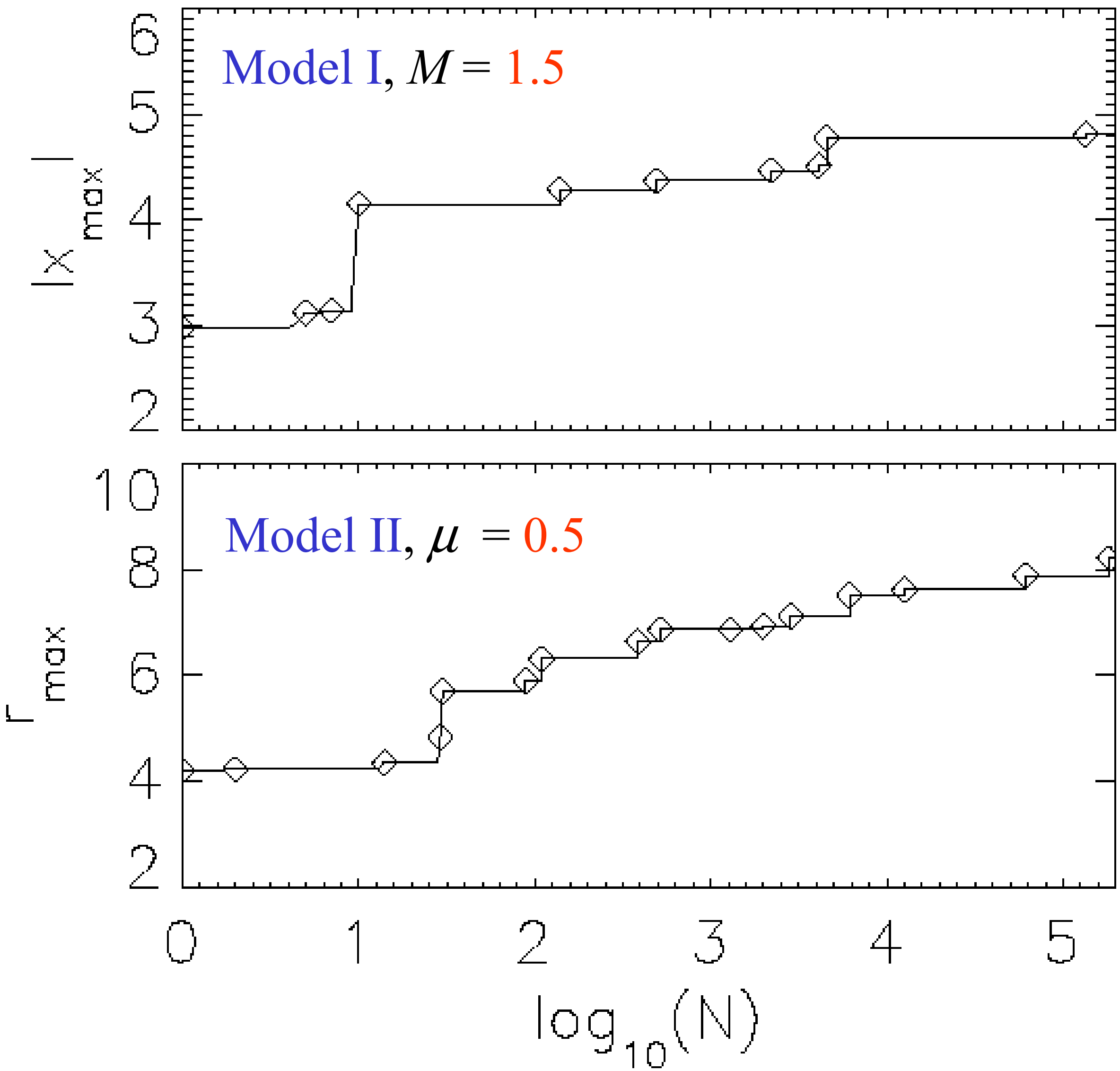
$|x_{max}| = 2.92$ for $M = 1.5$

Zero noise ⇒ $r_{max} = 1.36$

Moral: Noise enhances the orbital amplitudes!

MAXIMUM ORBITAL AMPLITUDE vs. SAMPLE SIZE

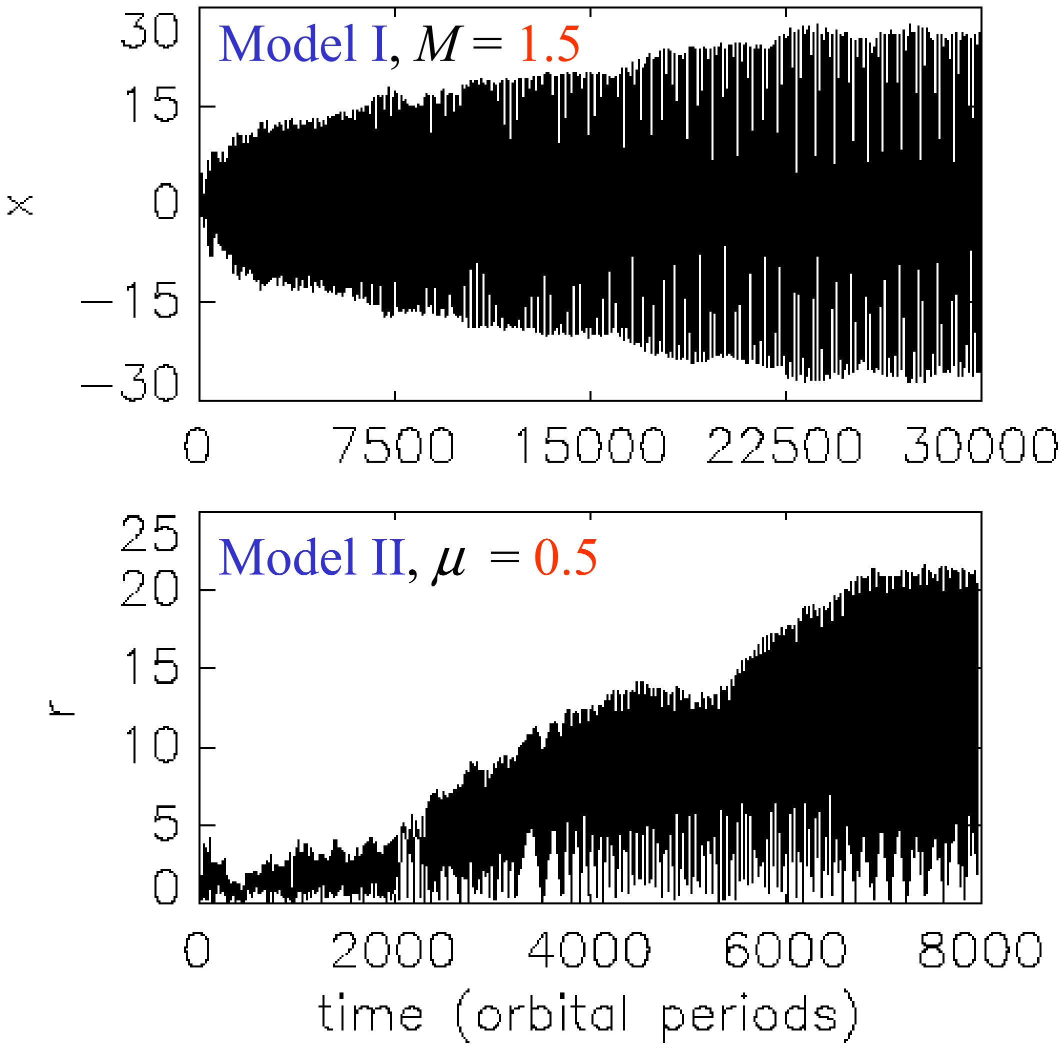
$$\langle|\delta\omega|\rangle = 0.004 \text{ and } t_c = 12\tau$$



Maximum orbital amplitude grows quasi-logarithmically with sample size N.

LONG-TIME EVOLUTION OF LARGE-AMPLITUDE ORBITS

$$\langle|\delta\omega|\rangle = 0.01\,,\ t_c = 12\tau$$



Colored noise REMOVES the hard upper bound to the halo amplitude! This is important in, e.g., storage rings.

WARM-FLUID MODEL OF A CYLINDRICAL DC BEAM

S. Lund and R. C. Davidson, *Phys. Plasmas* **5**, 3028 (1998);
S. Strasburg and R. C. Davidson, *Phys. Rev. E* **61**, 5753 (2000).

Equation of motion for radial orbits :

$$\ddot{x} + [\eta^2 - \sqrt{\Gamma}(1-\eta^2)\cos\omega t]x = 0 \quad \text{for } \; x < 1.0;$$

$$\ddot{x} + x - \frac{1-\eta^2}{x} = 0 \qquad\qquad \text{for } x \geq 1.0;$$

Model III

η is the tune depression, Γ is the ratio of the electrostatic energy in the collective mode to the electrostatic energy in the equilibrium beam, and, for the lowest - order pulsational mode, $\omega = \sqrt{2(1+\eta^2)}$.

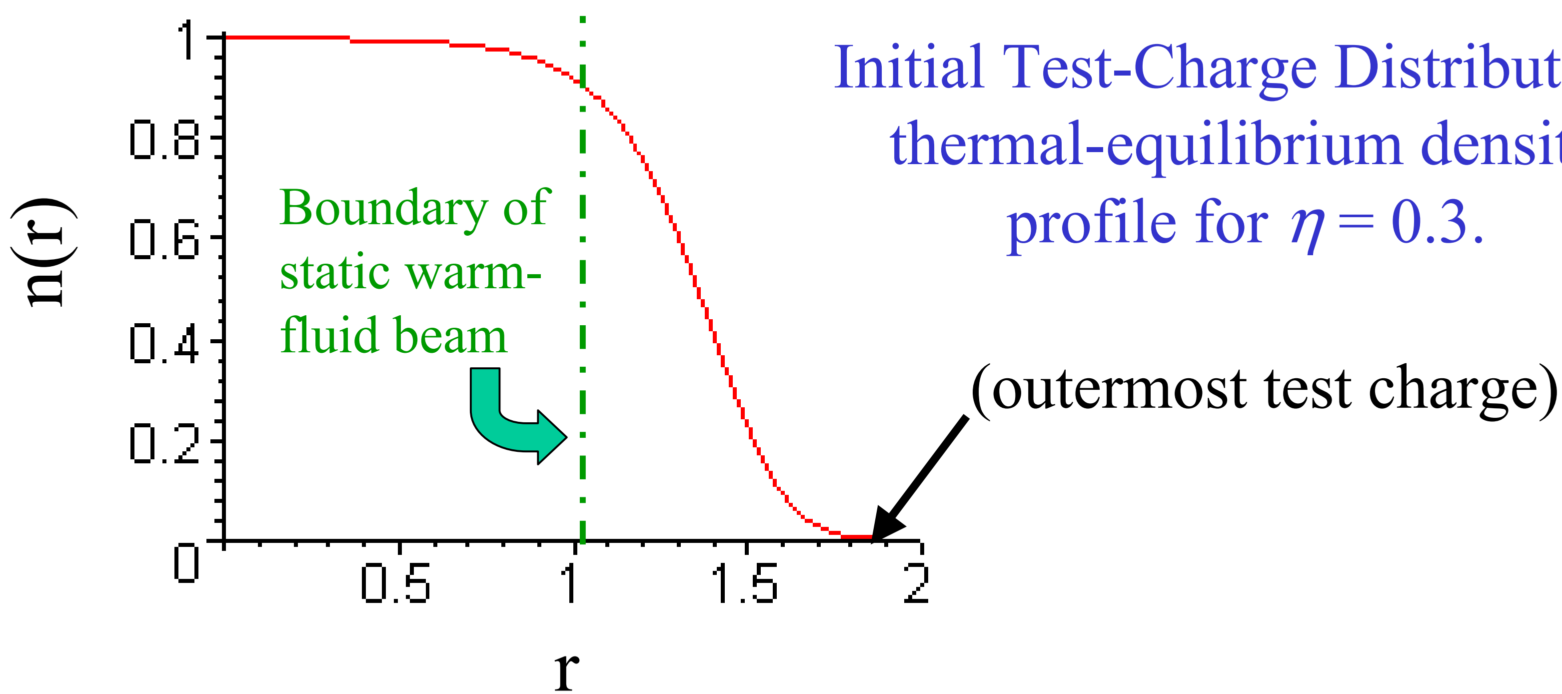
To include colored noise, we set $\omega \rightarrow \omega + \delta\omega$ and $\eta^2 \rightarrow \eta^2 + \omega\delta\omega$.

Consequently, noise now appears in *both* the net focusing and collective oscillation frequencies.

MODEL III: INVESTIGATIVE STRATEGY

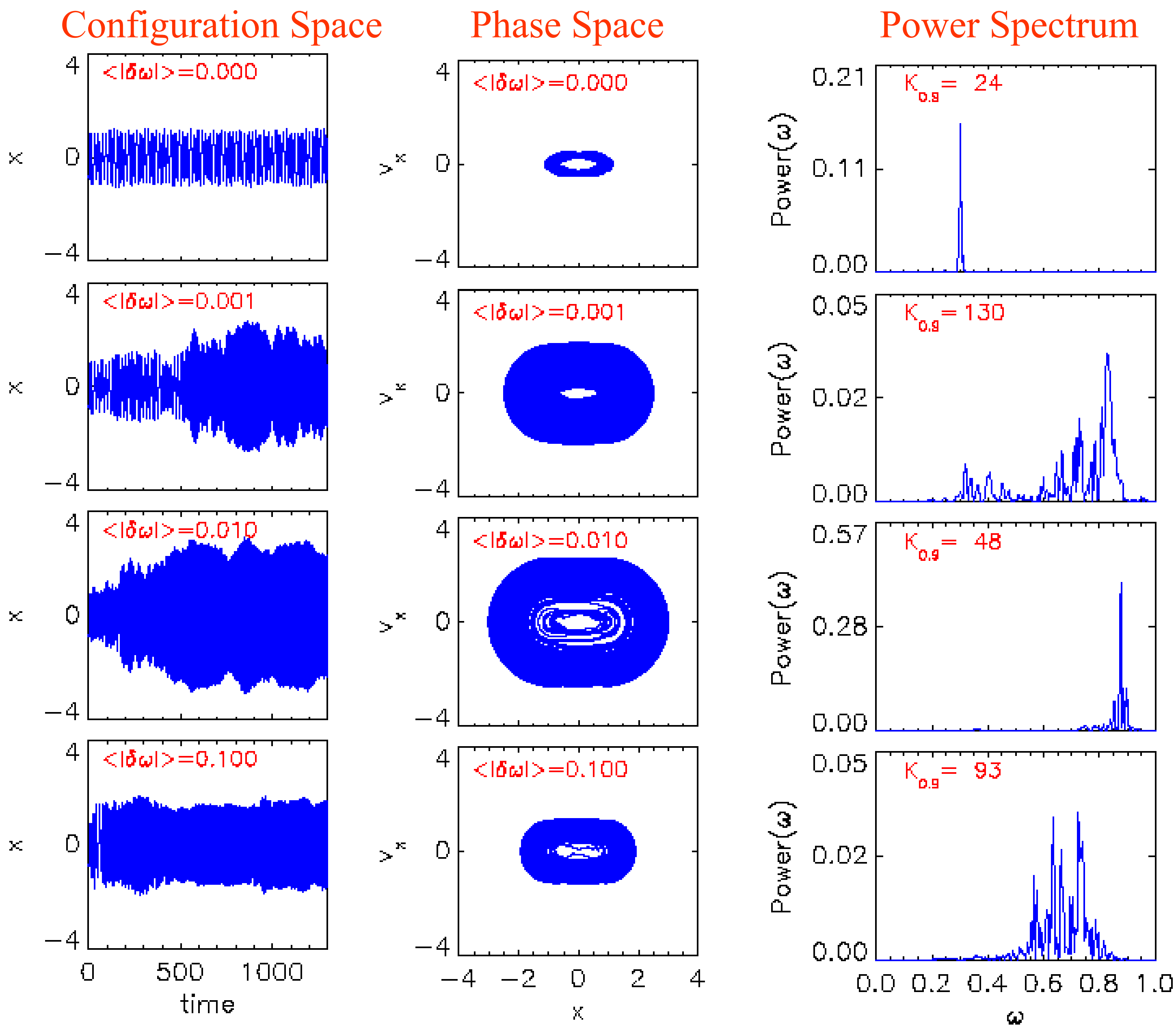
[I. V. Sideris and C. L. Bohn, *Phys. Rev. ST Accel. Beams* (submitted)]

Populate the oscillating warm-fluid beam with 10^6 test charges distributed according to the thermal-equilibrium density profile. Radial orbits have zero initial velocity. Assign each orbit its own noise. Integrate and track.

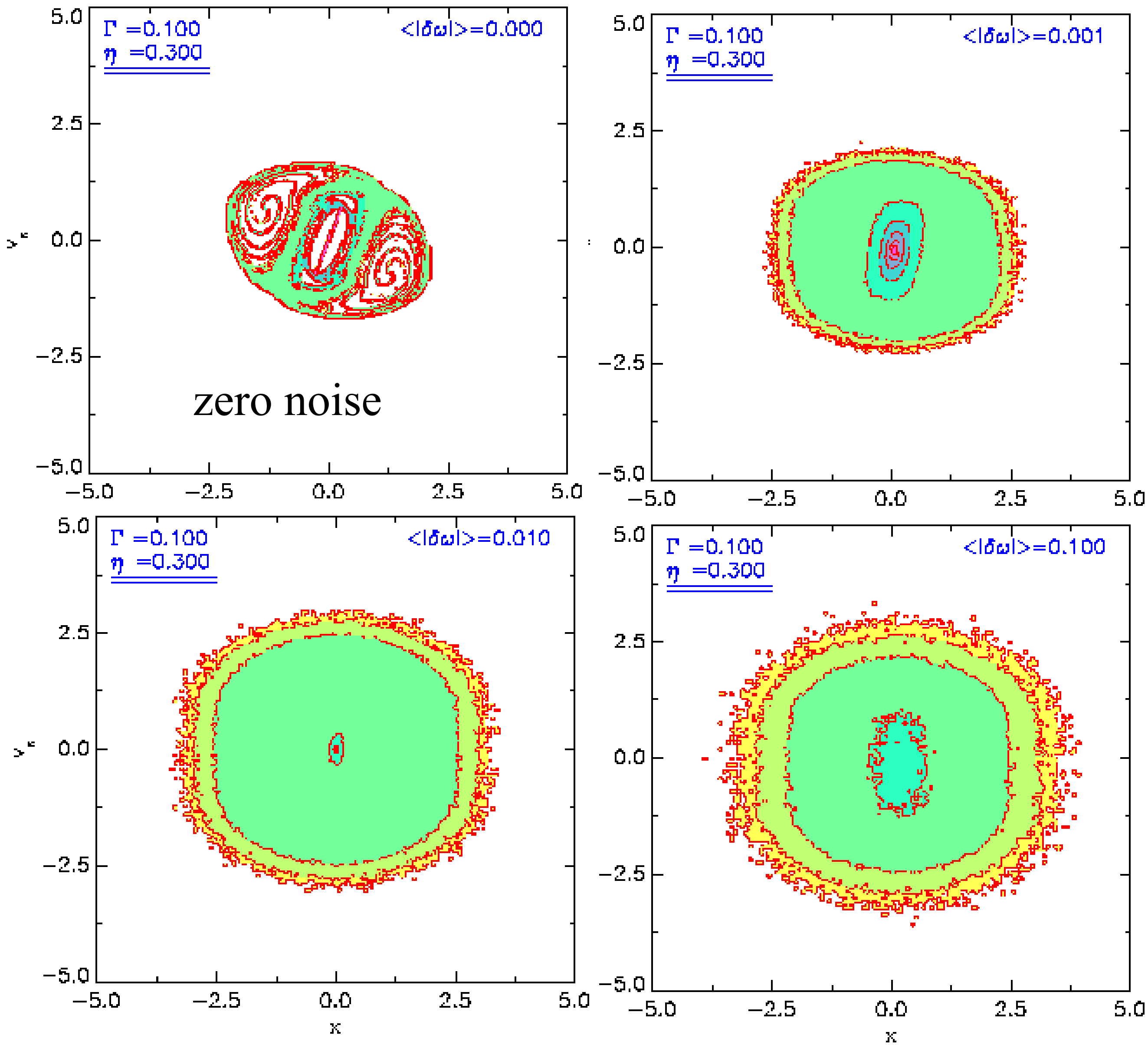


Initial Test-Charge Distribution: thermal-equilibrium density profile for $\eta = 0.3$.

MODEL III: ORBITAL CHAOTICITY [Γ=0.10, x(0)=-0.733407]



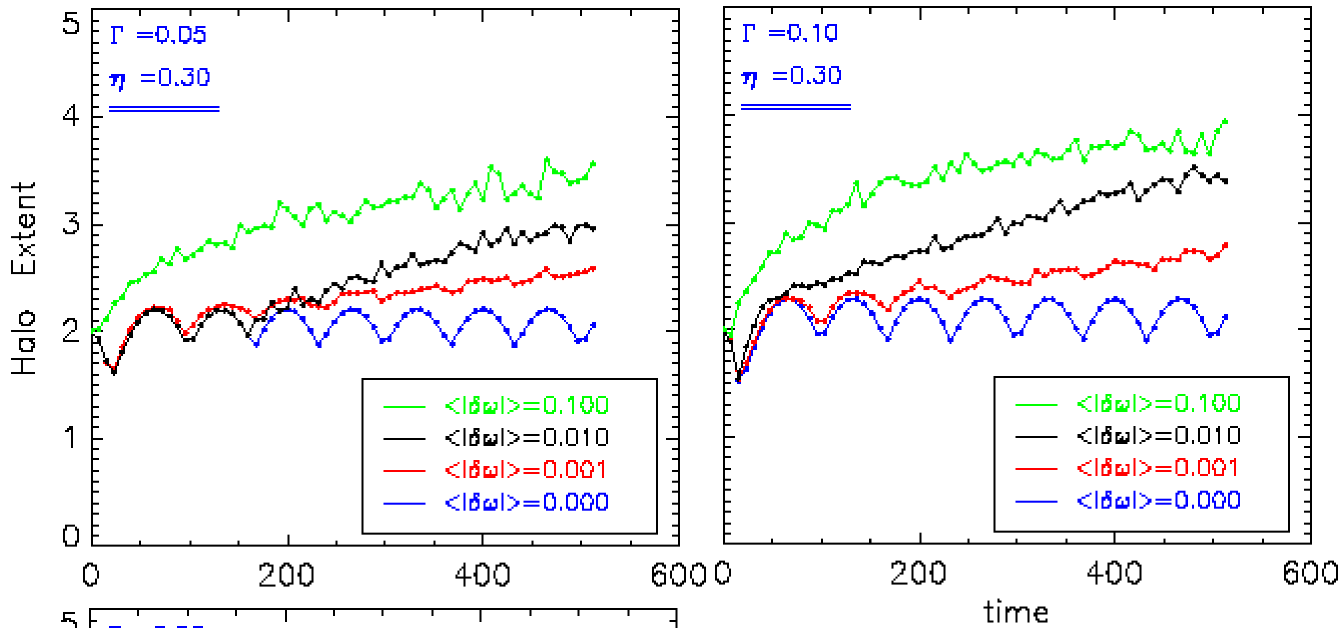
(x,v_x) PHASE SPACE, WARM-FLUID MODEL: $\eta = 0.3$, $\Gamma = 0.1$

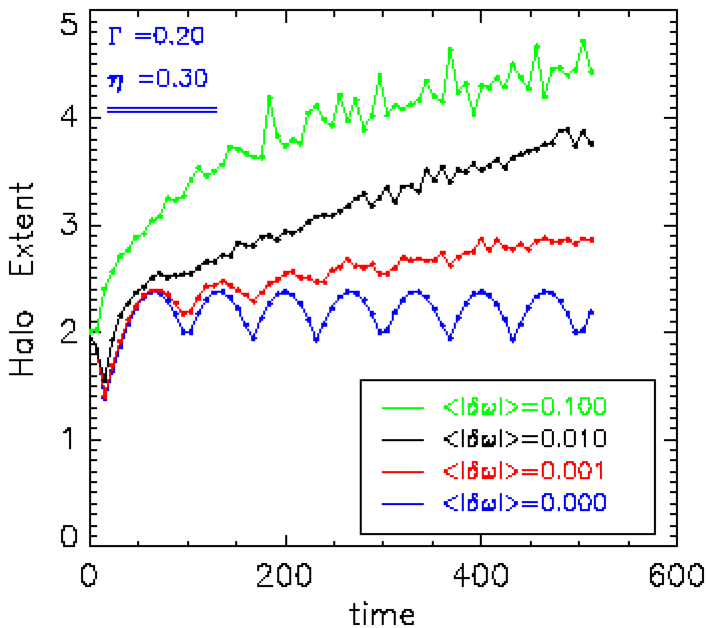


$t_c \cong 6\tau;\ t = 40\tau$

Noise *expands* the phase space and "*smears*" particles through phase space.

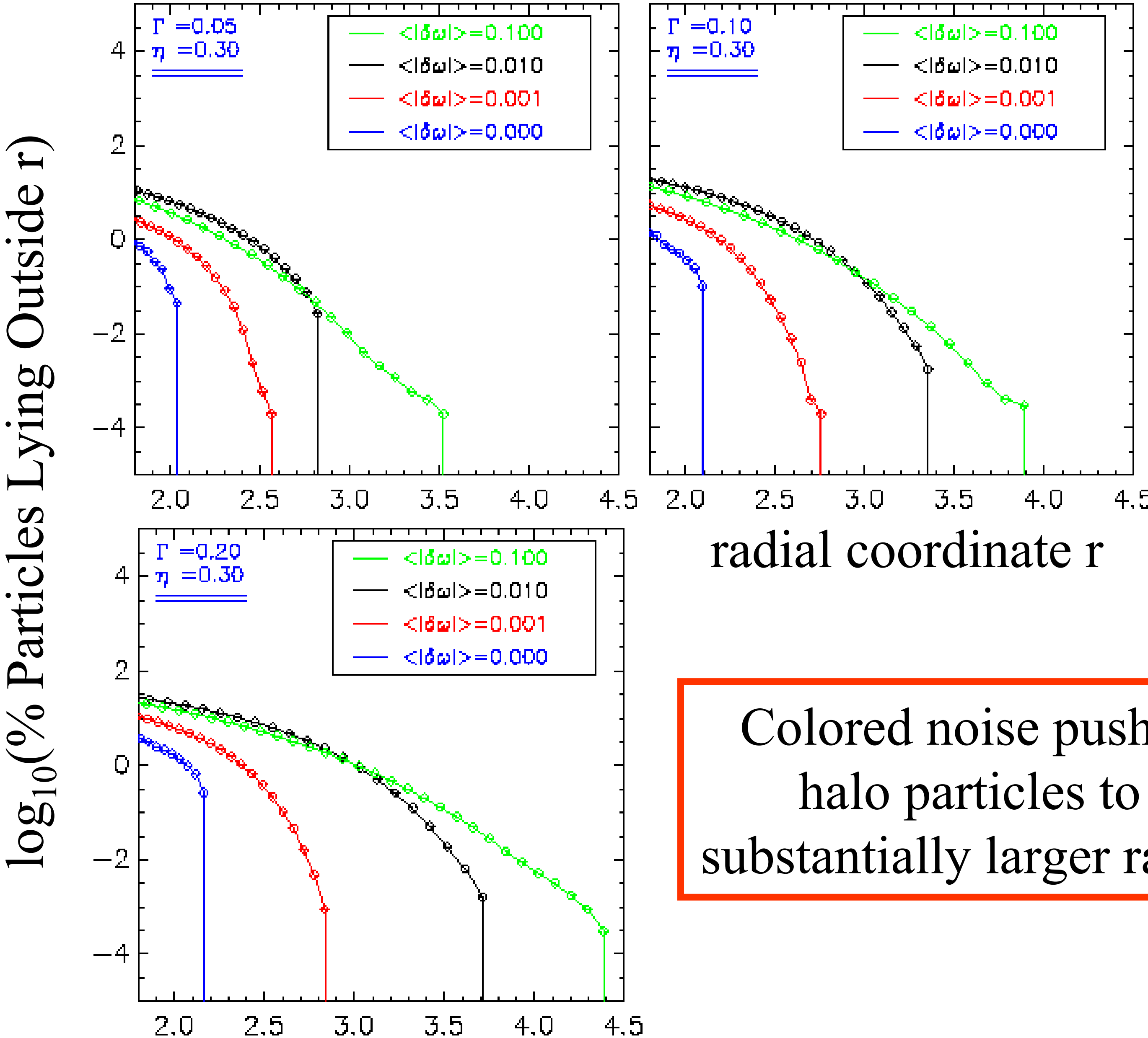
MODEL III: HALO EXTENT vs. OSCILLATION AMPLITUDE





The halo amplitude grows with both the noise strength and the oscillation amplitude (mismatch).

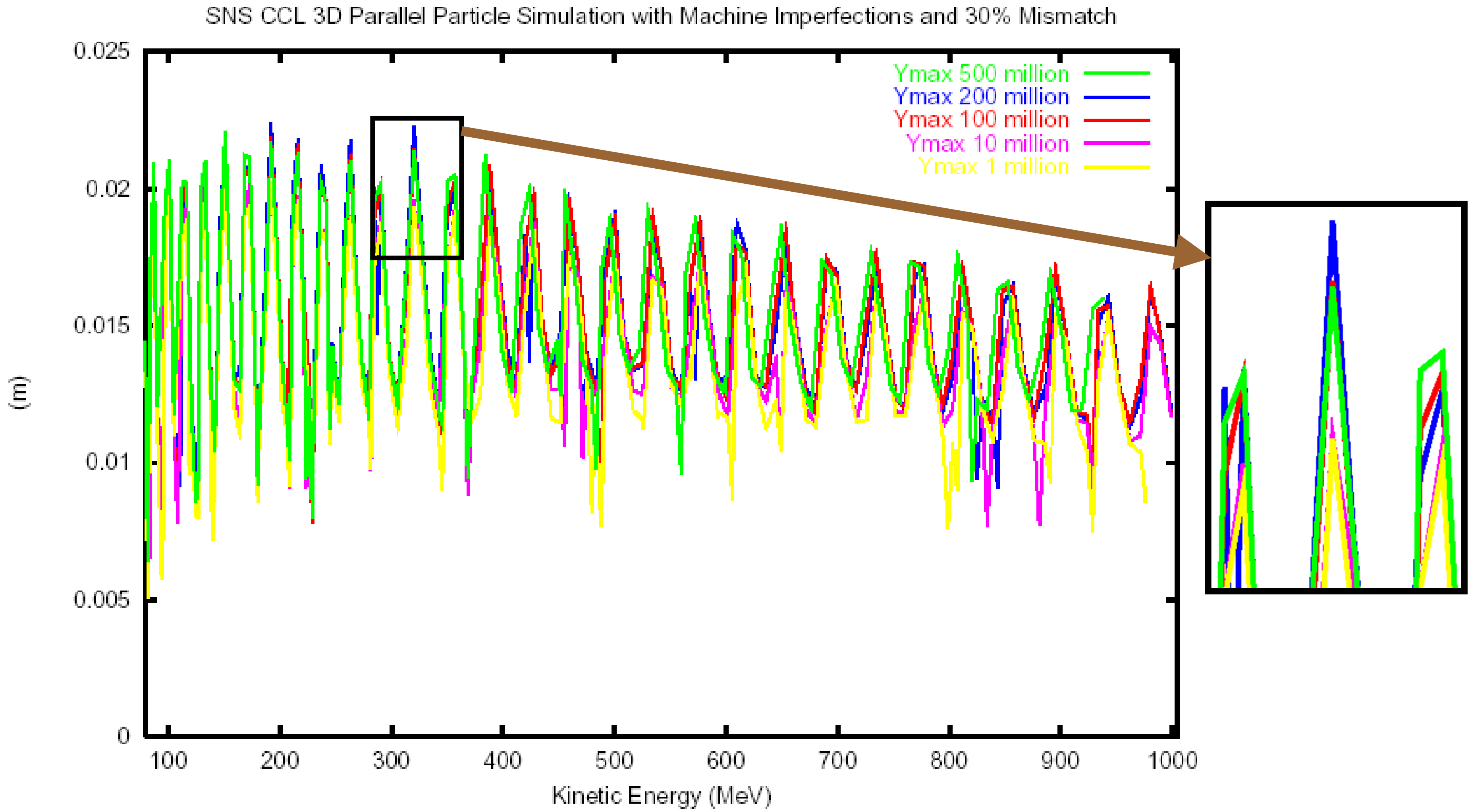
MODEL III: HALO DENSITY PROFILE



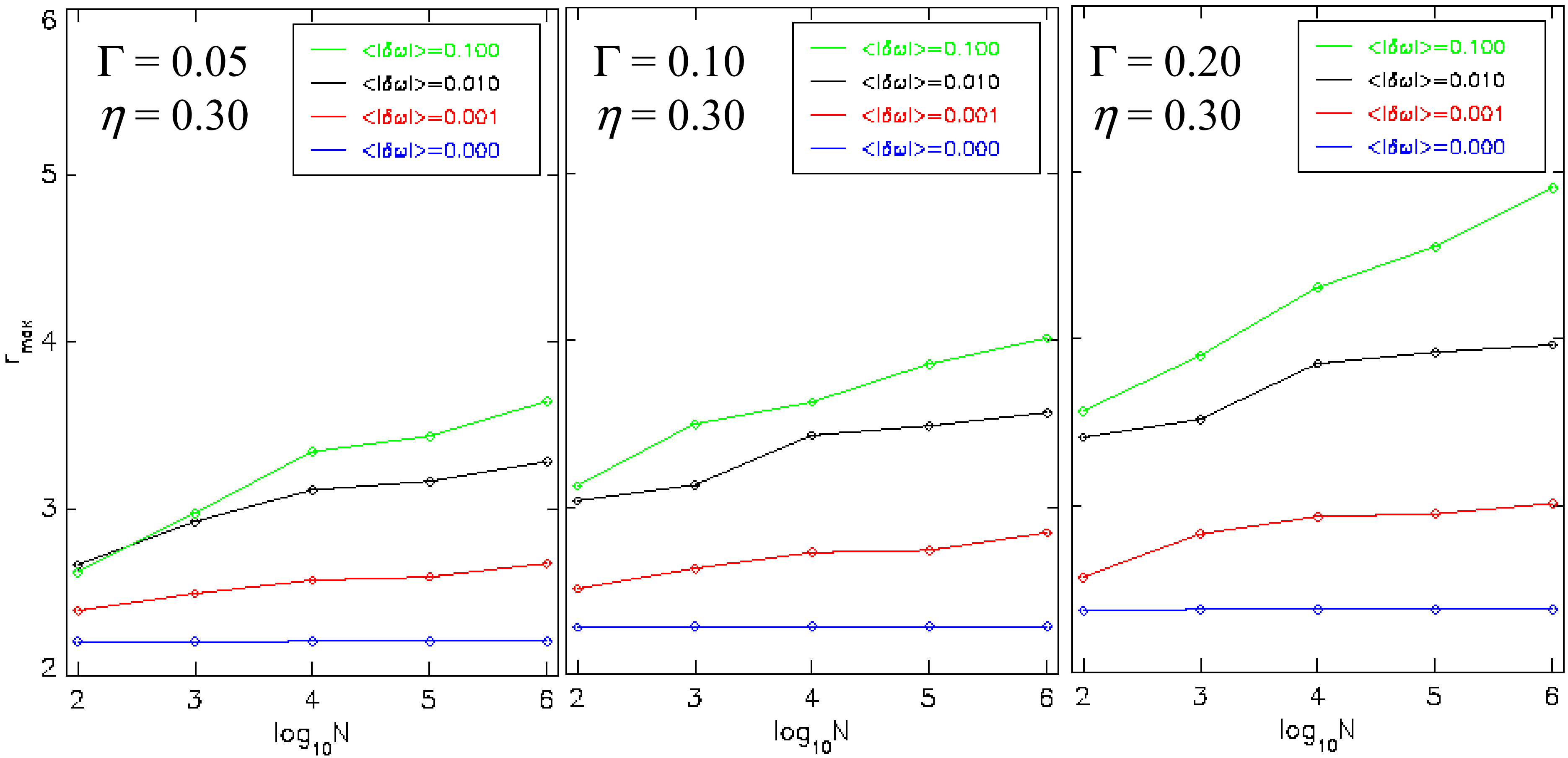
Colored noise pushes halo particles to substantially larger radii!

HALO AMPLITUDE vs. MACROPARTICLE NUMBER: SIMULATION OF (ALL-COPPER) SNS LINAC

[J. Qiang, et al., *Nucl. Instrum. and Methods* **A457**, 1 (1999)]

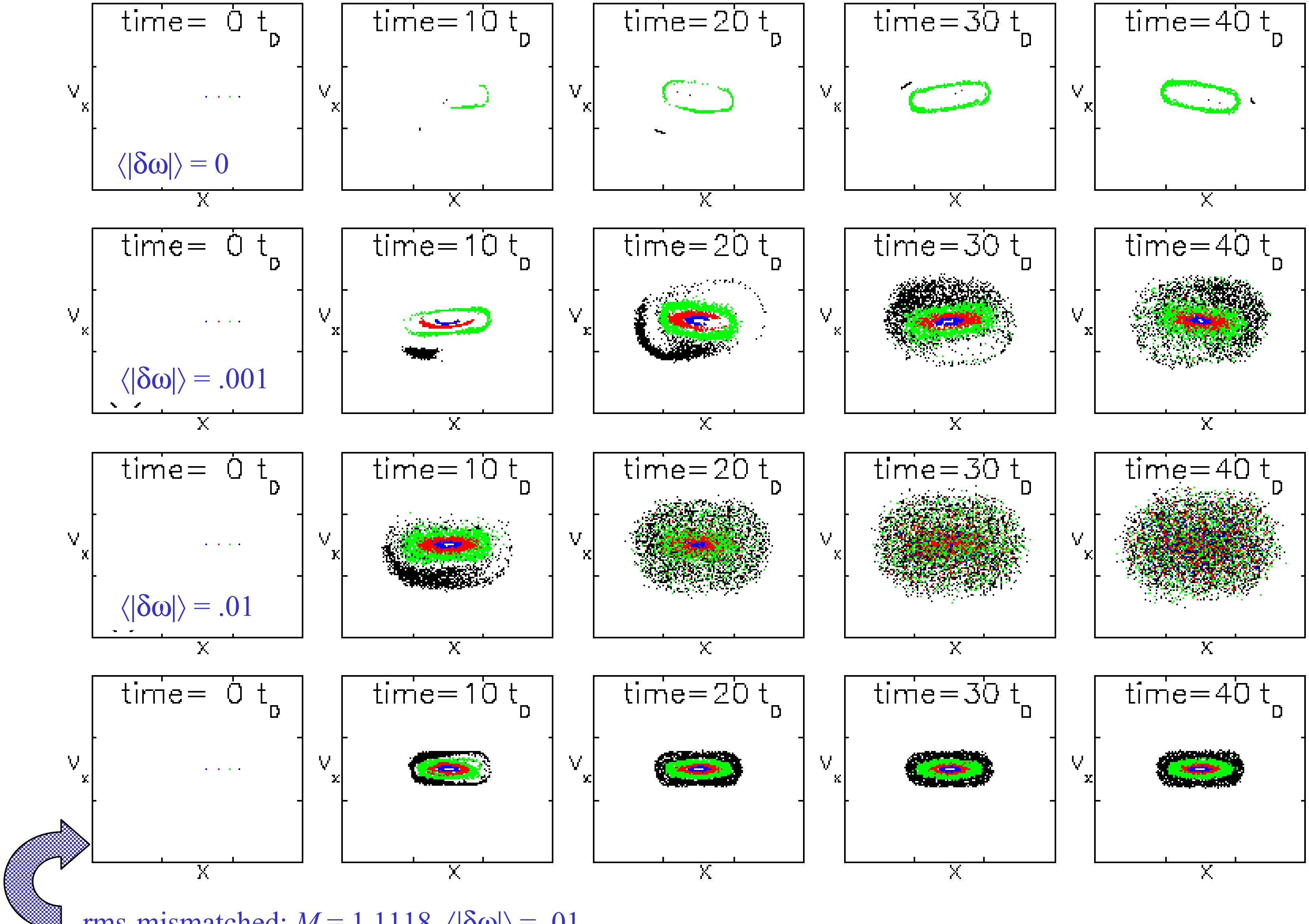


MODEL III: HALO AMPLITUDE vs. NUMBER OF PARTICLES



Regardless of noise strength or mismatch, the halo amplitude scales as $\log_{10}$N.

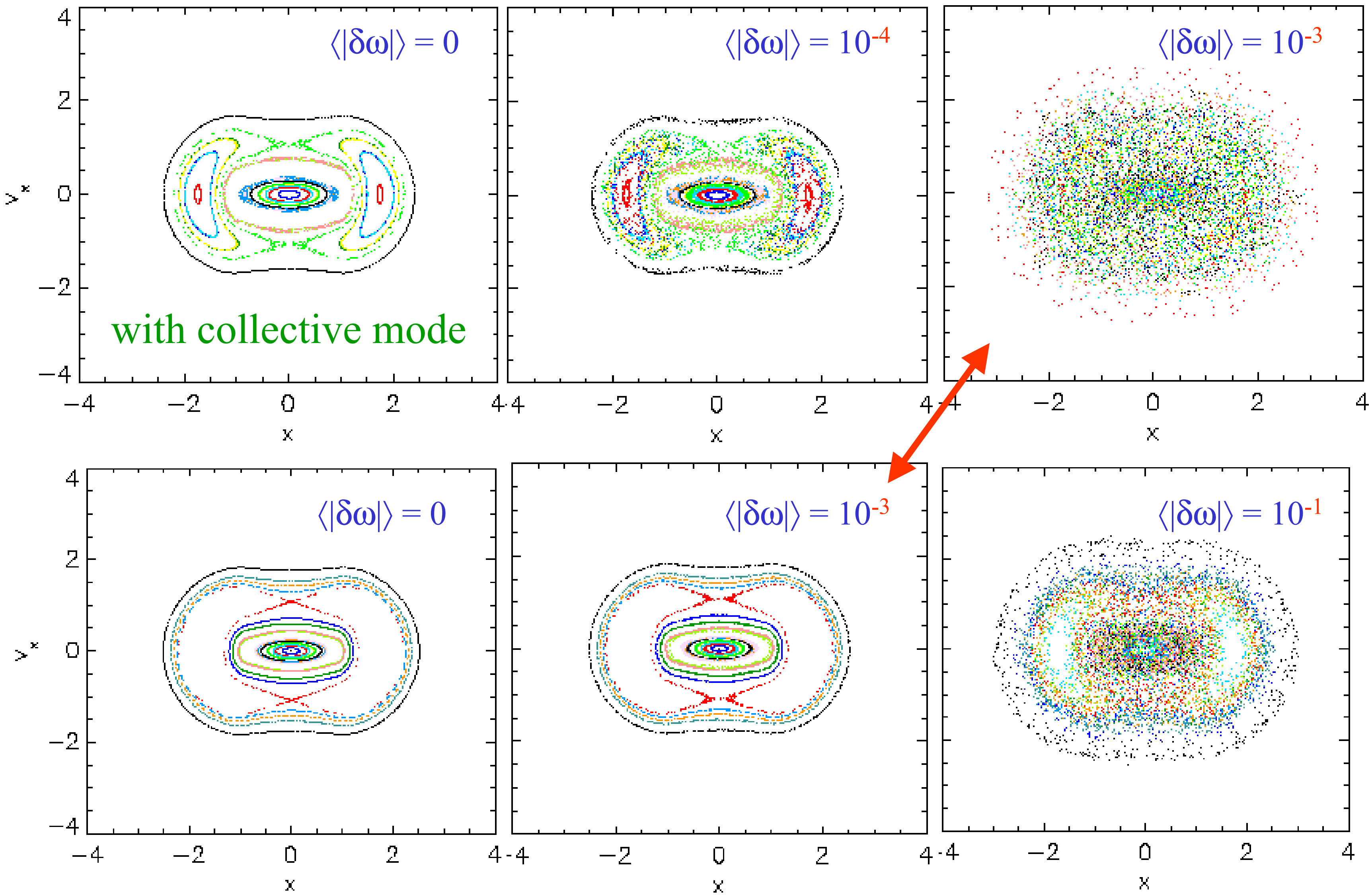
COLORED NOISE AND PHASE MIXING (Γ=0.05, t_c=80)



rms-mismatched: M = 1.1118, $\langle|\delta\omega|\rangle$ = .01

COLLECTIVE MODES ≠ RMS MISMATCH!

POINCARÉ SECTIONS (Γ=0.05, t_c=80, 18 orbits over ~250 t_D)



SUMMARY AND CONCLUSIONS

- Noise, an *unavoidable* phenomenon, can have ***major*** effects
  - expands the phase space
  - redistributes particles through phase space
  - affects Coulomb systems in general (e.g. galaxies, too!)

- Details ***do*** matter (halo being just one example)
  - control of rms properties is necessary but *not* sufficient
  - simulation codes must accommodate 'modes' at *all* scales
  - initial conditions are *critical* and must be specified accurately

- Collective modes affect dynamics ***differently*** from rms mismatch
  - phase-space tori are *much* more fragile
  - phase mixing is *much* faster and more voluminous