Thoughts on RF Voltage

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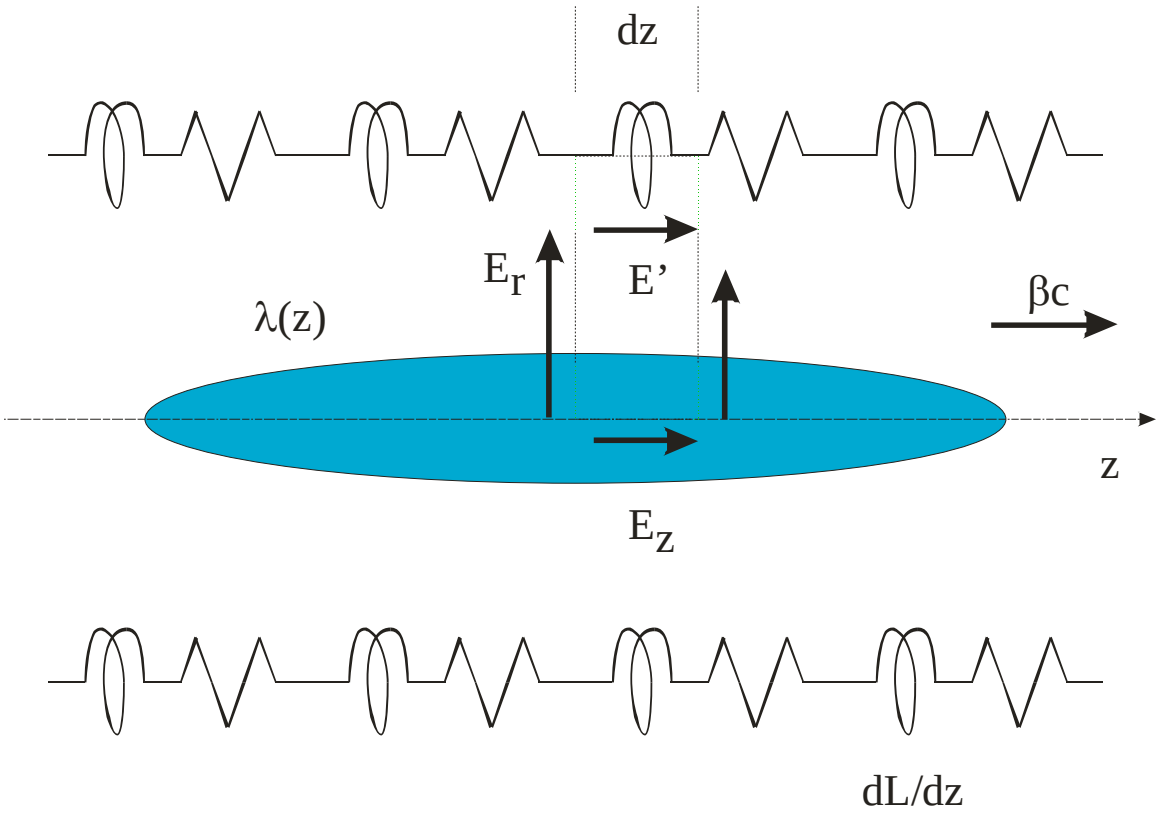
When we talk about voltage in the ring, we are really speaking about the electric field:

$$V_{ba} = -\int_a^b \overrightarrow{E_s} \cdot \overrightarrow{ds}$$

Beam potential

Taken from S. Hansen, et al., IEEE Trans. Nuc. Sci., **22**(3), 1975(1381)



Calculating the line integral around the closed path (and ignoring the resistance),

$$E_z = -e\frac{d\lambda(z)}{dz}\left[\frac{g_o}{4\pi\varepsilon_o\gamma^2} - \frac{dL}{dz}\beta^2c^2\right]$$

Note: L can be from both μ' and μ'' effects in ferrite and magnets and hence also be dissipative.

For the moment, let's also ignore the inductive contribution.

For simplicity, assume a parabolic line density of charge,

$$\lambda(z) = \frac{6N}{l_b^3}\left[\frac{l_b^2}{4} - z^2\right]$$

where N is the charge per bunch and l_b is the full bunch width, centered at z=0. At injection, the single turn space-charge voltage can be expressed as,

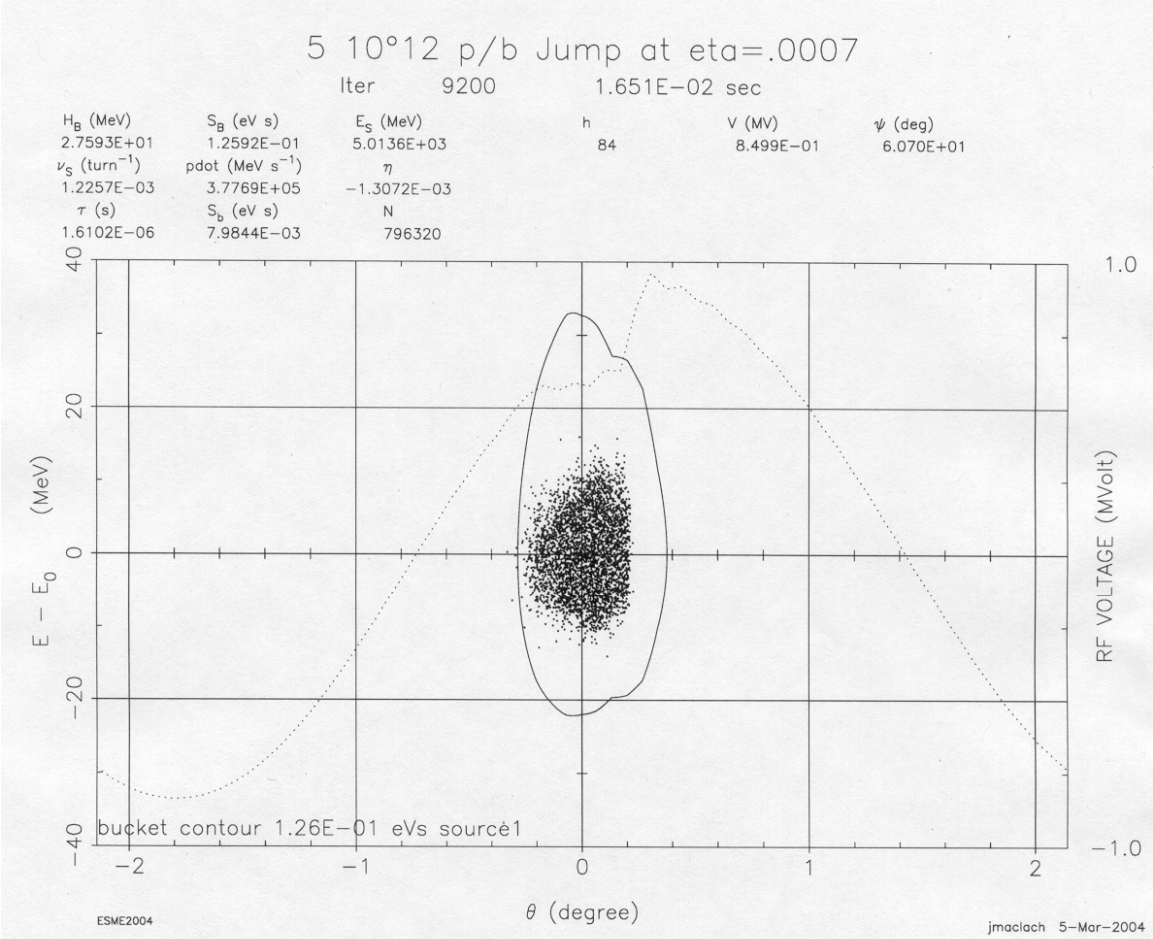
$$U_z = e\frac{g_o}{\varepsilon_o \gamma^2}\frac{6N}{R}$$
$$= 9.7 \text{ kV}$$

for N=5x10^{12} protons.

When voltage is determined from integrating the longitudinal electric field around the ring, the total voltage is the same, irrespective of the number of bunches (or instantaneous current), since it is simply the amount of charge that matters.

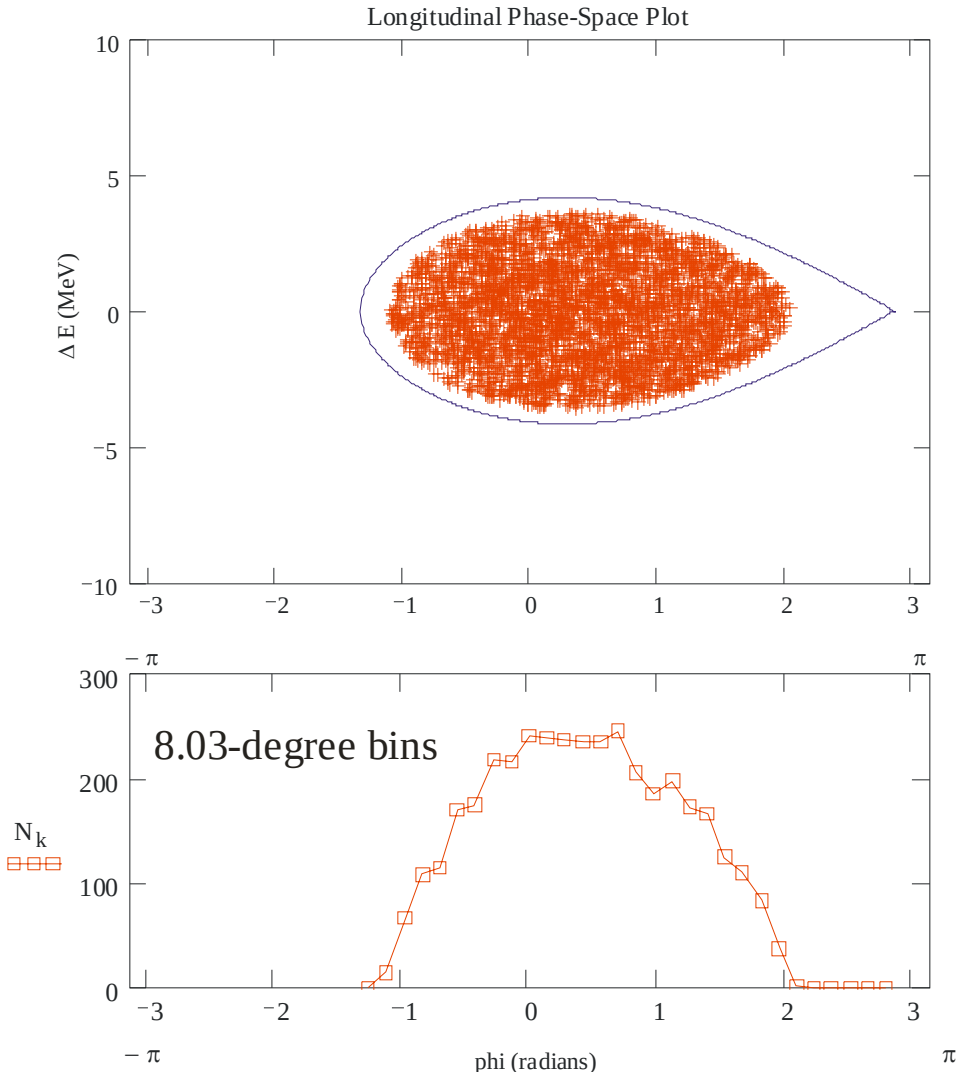
However, when the inductive component is included, then bunch structure or instantaneous current matters.

Question: What is ESME showing us?

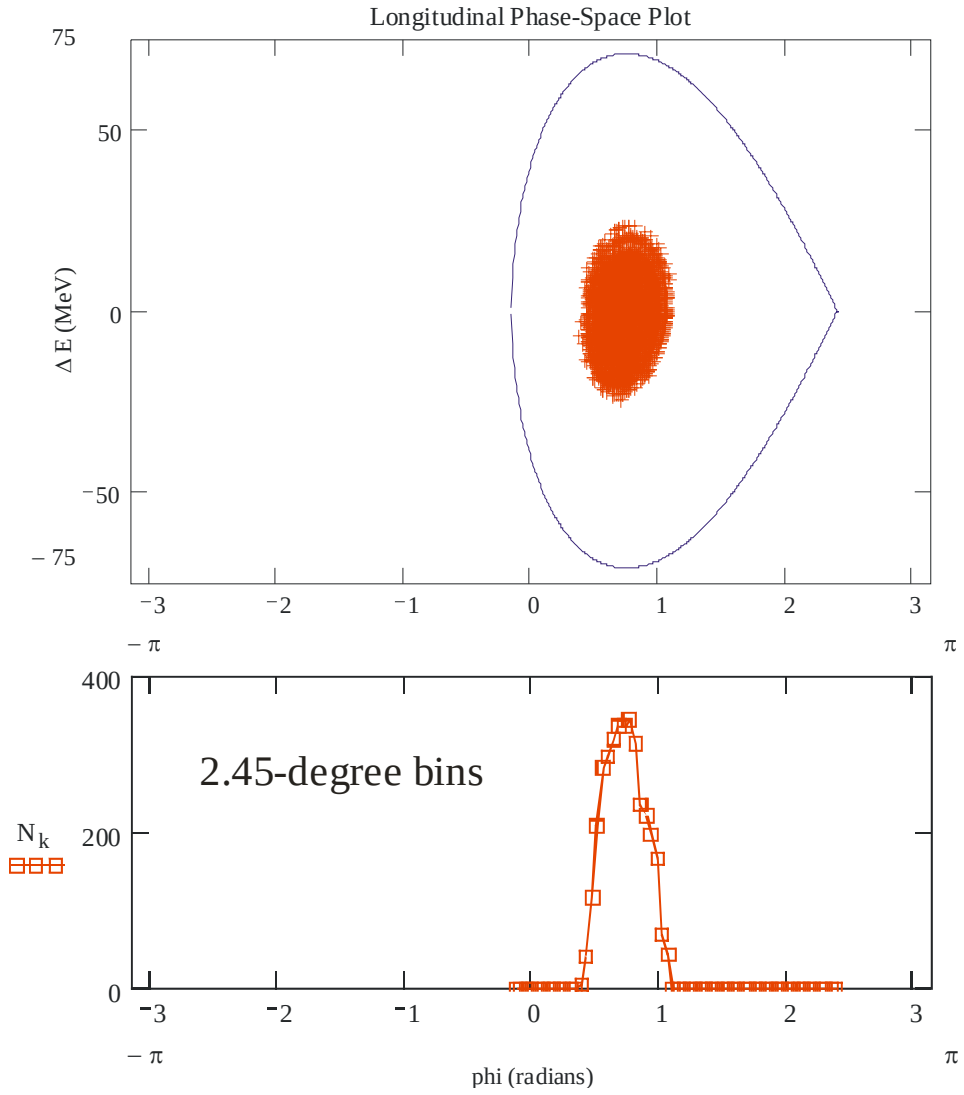


A bunch of $6 \cdot 10^{10}$ protons just before transition. The dotted curve is the sum of rf potential and the collective potential calculated from Crisp magnet impedance measurements and perfectly conducting wall space charge impedance.

CAPTURE simulation of booster at 5 ms



CAPTURE simulation of booster at 16.6 ms – just before transition



Comments:

The ESME abscissa displayed is ±180°/h = ±2.14° of the whole booster or ±180° for a single rf bucket.

CAPTURE only includes effects from bunch space-charge—its effect on the separatrix is imperceptible at 4 GeV.

The deformation from the bunch according to ESME is due almost entirely to the wall impedance term.