LAMINATED MAGNETS —

IMPEDANCES AND BEAM DYNAMICS

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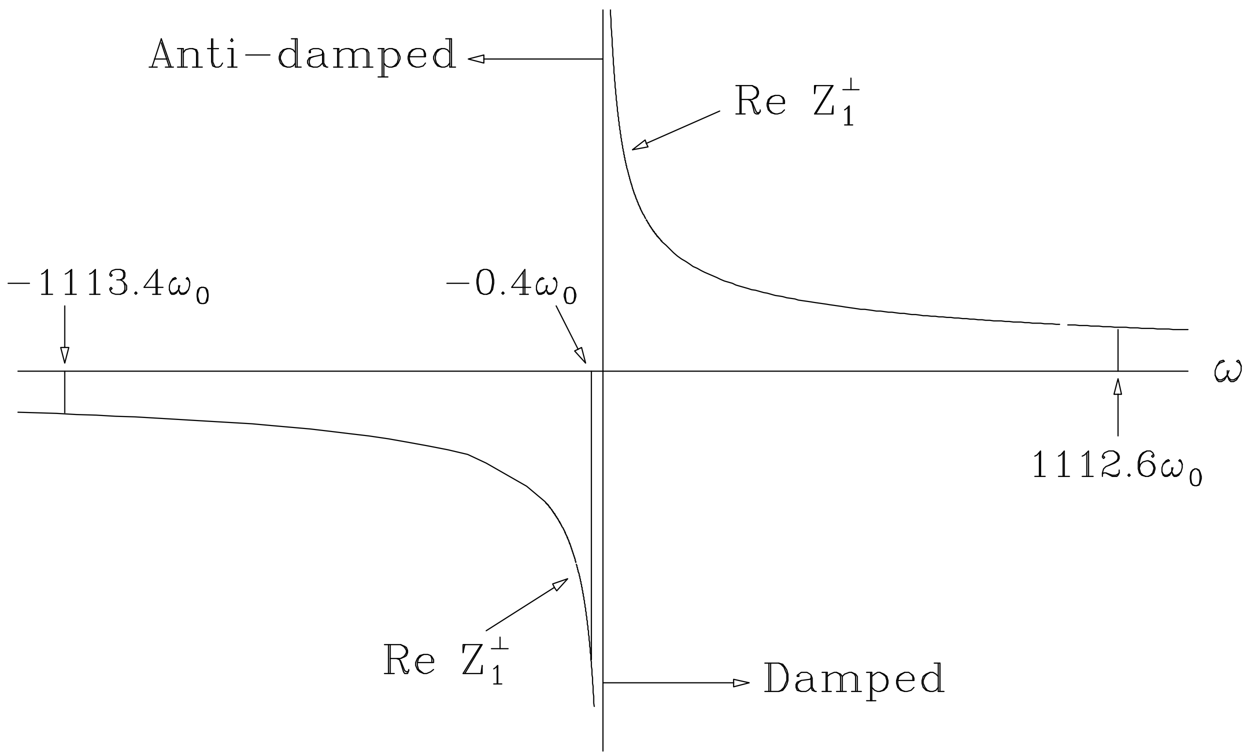
CONTENTS

INTRODUCTION

- Transverse coupled-bunch instabilities are observed in all accelerators with a full ring of bunches.
  For example, Main Ring, Tevatron in fixed target, etc.

- Driving force is $\mathcal{R}e\, Z_1^{\perp}$ of resistive wall.



- Our Booster consists of 96 laminated magnets without a beam pipe.

  $\mathcal{R}e\, Z_1^{\perp}$ must be very large.

  There is no transverse damper.

  However, transverse coupled-bunch instabilities have never been reported.

  Why?

- This paradox motivates me to study laminated magnets in detail.

BEHAVIOR OF $Z_1^{\perp}$

- For round pipe of radius b,

$$\frac{Z_0^{\parallel}}{L} = \left[1 + j\,\mathrm{sgn}(\omega)\right] \frac{1}{2\pi b} \frac{1}{\delta_c \sigma_c} \ .$$

- We rewrite it as

$$\frac{Z_0^{\parallel}}{L} = \frac{\mathcal{R}}{2\pi b} \ \text{with} \ \mathcal{R} = \frac{1 + j\,\mathrm{sgn}(\omega)}{\delta_c \sigma_c} \ .$$

  Surface impedance $\mathcal{R} = \dfrac{E_z}{H_x}$ at wall

- $\boxed{Z_1^{\perp} = \dfrac{2c}{b^2} \dfrac{Z_0^{\parallel}}{\omega}}$ $\longleftarrow$ Panofsky-Wenzel-like.

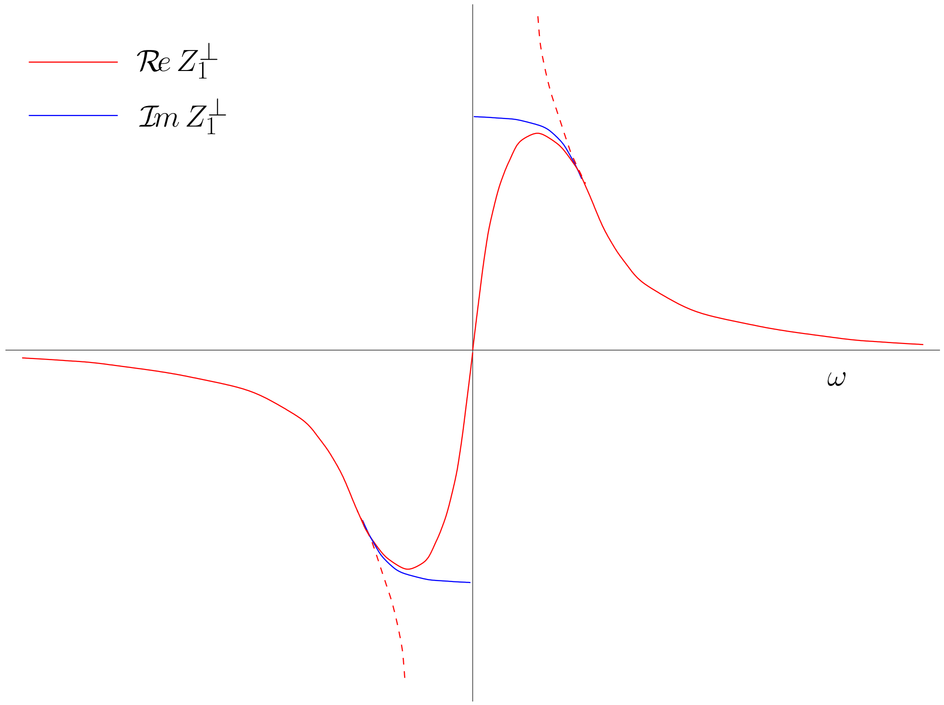
- $Z_1^{\perp}$ is an analytic function of frequency.

  It cannot diverge to $\pm\infty$ when $\omega = 0$.

- Correct behavior:

- $\boxed{\lim_{\omega\to 0} \frac{\mathcal{I}m\, Z_1^{\perp}}{L} = \frac{Z_0}{2\pi b^2}}$

  independent of wall conductivity σ_c.



- Bend-around takes place when $\left|(1+j)\frac{b}{\delta_c}\right| \sim 1$.

  SS: $\sigma_c = 0.5\times 10^7$ $(\Omega\text{-m})^{-1}$, $\mu_r \sim 1$, $b = 5$ cm, $f_{\text{bend}} \sim 10$ Hz.

  Very much less than f_0 of most rings $\longrightarrow$ no influence on collective instabilities.

- For laminated wall surface, $\mathcal{R}$ is very much larger.

  If $\mathcal{I}m\, Z_1^{\perp}$ approaches same limit, the bend-around must appear at a higher frequency.

- Thus laminated surface may not have $\omega^{-1/2}$ behavior in $\mathcal{R}e\, Z_1^{\perp}$ at low frequencies. Will not contribute to trans. coupled-bunch instabilities.

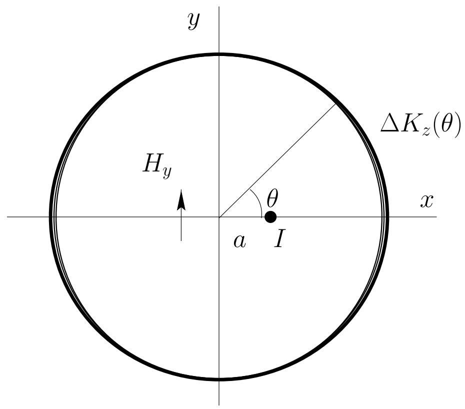
- Tasks:

  1. Need to understand the limit
  $$\lim_{\omega\to 0} \frac{\mathcal{I}m\, Z_1^{\perp}}{L} = \frac{Z_0}{2\pi b^2}\ .$$
  2. Need to compute bend-around freq. for laminated walls.

BY-PASS INDUCTANCE (L. Vos)

- A current I centered induces image current $I_b = -I$ or image surface current $I_b/(2\pi b)$.
- For perfectly conducting walls, current I offset by a induces surface current

$$K_z(\theta) = \frac{b^2 - a^2}{b^2 + a^2 - 2ba\cos\theta}\,\frac{I_b}{2\pi b}\ ,$$



- Want dipole contribution only,

$$\boxed{\Delta K_z(\theta) = \frac{I_b a}{\pi b^2}\cos\theta\ ,}$$

- Total image current on one side is

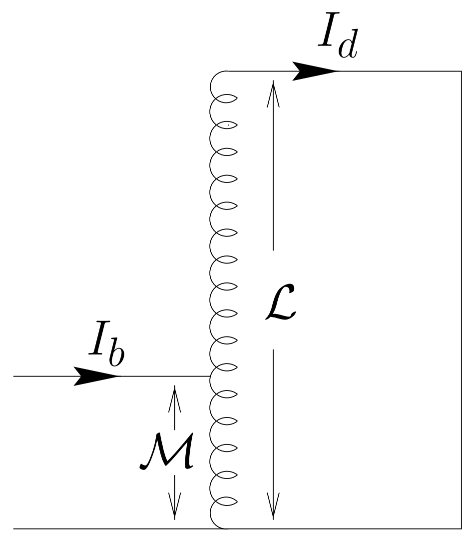
$$\boxed{I_d = \int_{-\pi/2}^{\pi/2} \Delta K_z(\theta) b d\theta = \frac{2I_b a}{\pi b}}$$

and $-I_d$ on the other side.

They are called *differential currents* and form a dipole loop.

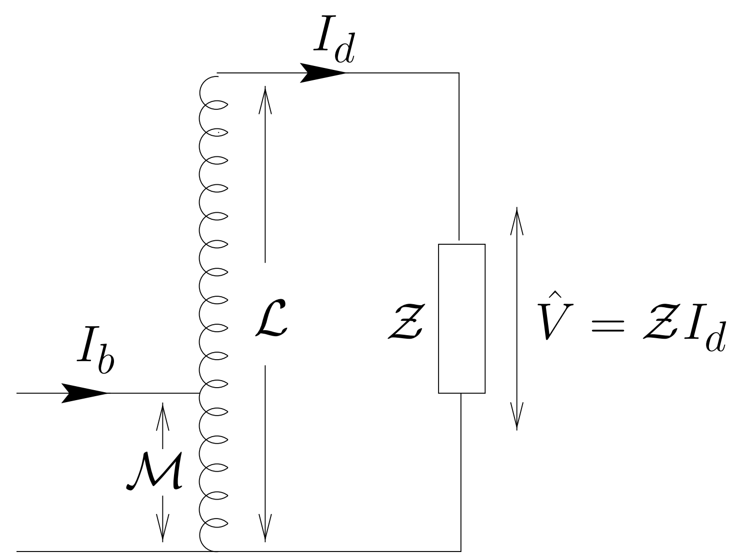
- There is magnetic field linking the loop $H_y(x) = -\int_{-\pi}^{\pi} \frac{\Delta K_z(\theta) b d\theta}{2\pi \alpha}\cos\phi = -\frac{I_b a}{2\pi b^2}$

- Flux link dipole image loop $\Phi_y = \int_{-b}^{b} B_y dx = 2bB_y = -\frac{\mu_0}{2} I_d$.



- I_d sees an inductance per length $\mathcal{L} = \frac{1}{2}\mu_0$.

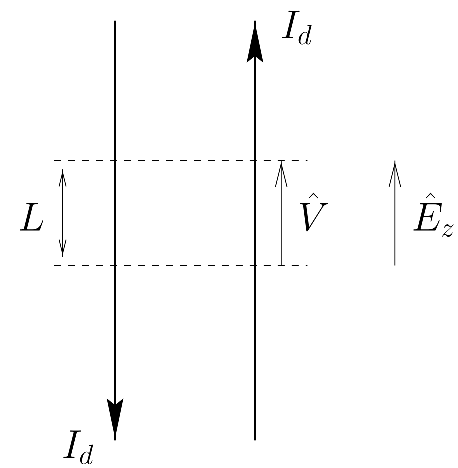
  Equivalent circuit: $\dfrac{\mathcal{M}}{\mathcal{L}} = \dfrac{I_d}{I_b} = \dfrac{2a}{\pi b}$



- With wall impedance, I_d sees $\mathcal{Z}$ in additional to $\mathcal{L}$.

  $$\frac{\mathcal{M}}{\mathcal{L}} = \frac{2a}{\pi b} \neq \frac{I_d}{I_b} = \frac{2a}{\pi b} \frac{j\omega\mathcal{L}}{j\omega\mathcal{L} + \mathcal{Z}}$$

  Need to express $\mathcal{Z}$ in terms of $Z_0^{\parallel}$.



- Voltage difference created by I_d loop for length L is

$$V(\theta) = 2\frac{\mathcal{R}L}{w} w \Delta K_z(\theta) = \frac{\mathcal{R}LI_d}{b}\cos\theta \ ,$$

Peak value is

$$\frac{\widehat{V}}{L} = \frac{\mathcal{R}I_d}{b} = 2\pi\frac{Z_0^{\parallel}}{L}I_d \qquad \longleftarrow \ \widehat{E}_z$$

But this is equal to $\mathcal{Z}I_d$; thus $\boxed{\mathcal{Z} = 2\pi\frac{Z_0^{\parallel}}{L} \ .}$

- Now the transverse impedance.

Horizontal force on source current is

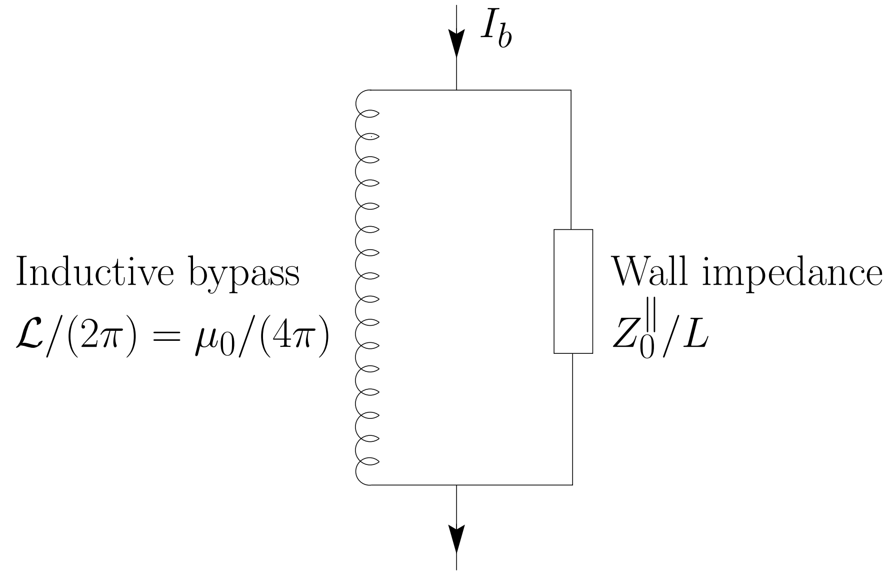
$$\frac{F_x}{e} = E_x - vB_y = -\frac{v}{j\omega}\frac{\partial E_z}{\partial x} = -\frac{v\mathcal{R}I_d}{2j\omega b^2} \ .$$

Transverse impedance is

$$\frac{Z_1^{H}}{L} = -\frac{F_x/e}{jIa\beta} = -\frac{c\mathcal{R}I_d}{2\omega b^2 Ia} = -\frac{c\pi}{\omega b}\frac{I_d}{Ia}\frac{Z_0^{\parallel}}{L} \ .$$

Recall that $\dfrac{I_d}{I_b} = \dfrac{2a}{\pi b}\dfrac{j\omega\mathcal{L}}{j\omega\mathcal{L}+\mathcal{Z}} \ .$

$$\frac{Z_1^H}{L} = \frac{2c}{\omega b^2}\frac{\frac{j\omega\mathcal{L}}{2\pi}\frac{Z_0^{\parallel}}{L}}{\frac{j\omega\mathcal{L}}{2\pi}+\frac{Z_0^{\parallel}}{L}} = \frac{2c}{\omega b^2}\frac{\frac{j\omega\mu_0}{4\pi}\frac{Z_0^{\parallel}}{L}}{\frac{j\omega\mu_0}{4\pi}+\frac{Z_0^{\parallel}}{L}}\ ,$$



- High frequencies: $\dfrac{Z_1^H}{L} = \dfrac{2c}{\omega b^2}\dfrac{Z_0^{\parallel}}{L}\ ,$ same as Panofsky-Wenzel-like relation.

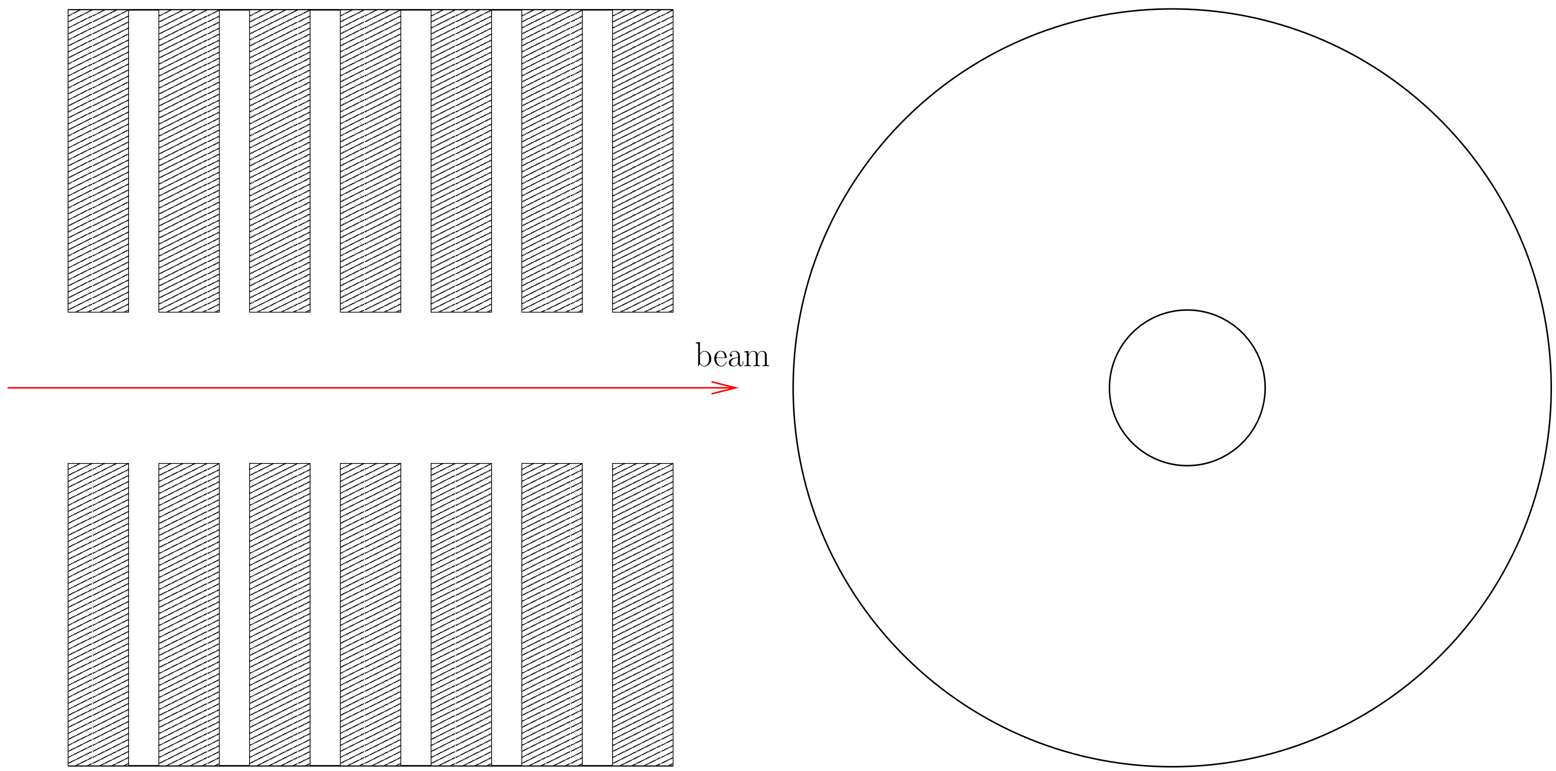
  Low frequencies: $\dfrac{Z_1^H}{L} = \dfrac{2c}{\omega b^2}\dfrac{j\omega\mu_0}{4\pi} = j\dfrac{Z_0}{2\pi b^2}$

  $\dfrac{\mathcal{L}}{2\pi} = \dfrac{\mu_0}{4\pi}$ is called *bypass inductance*.

- Bend-around frequency is given by $\dfrac{\omega\mu_0}{4\pi} \approx \left|\dfrac{\mathcal{R}}{2\pi b}\right|\ .$

  This gives correct bend-around freq for smooth resistive walls.

  For laminated surface, we need to compute $\mathcal{R}$.

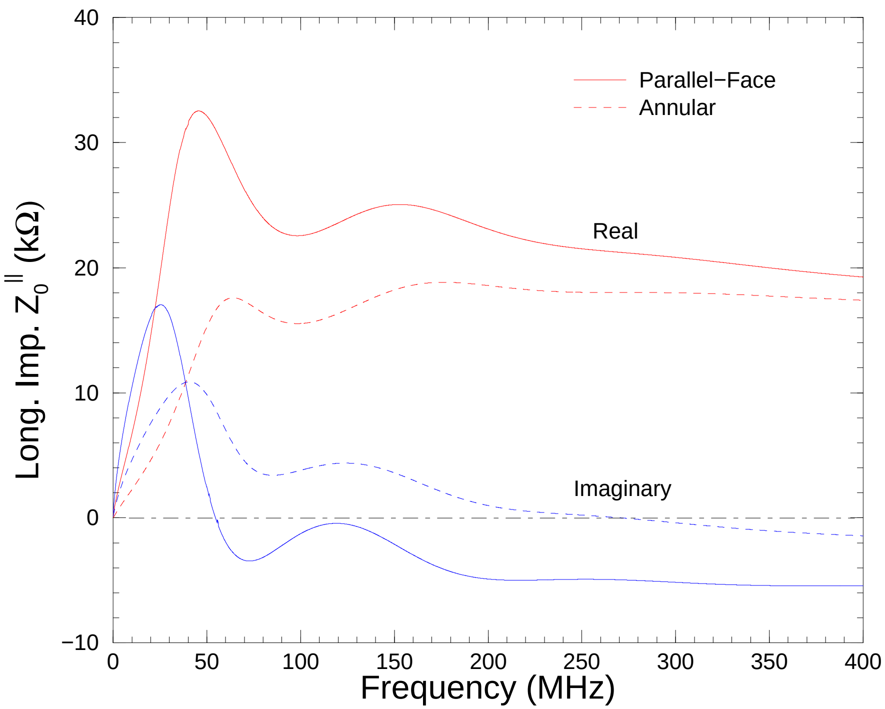

beam

IMPEDANCE OF LAMINATED MAGNET

Properties of $Z_0^{\|}$:

1. Annular approx. gives smaller $Z_0^{\|}$ because image current fans out evenly.
2. First resonance peak and a damped 2nd one.
3. $\mathcal{R}e\, Z_0^{\|}$ is not $\omega^{1/2}$ at low frequencies because of the bend-around influence. Resembles Crisp's measurement.
4. $\mathcal{R}e\, Z_0^{\|}$ reaches $\sim 30\ \Omega$, resembling Crisp's measurement.
5. $\mathcal{I}m\, Z_0^{\|}$ capacitive at high frequencies because image current flows across cracks instead of along laminations.
   Or cracks become capacitors.
   Crisp's measurement shows no capacitive behavior.

96 magnets of the Booster

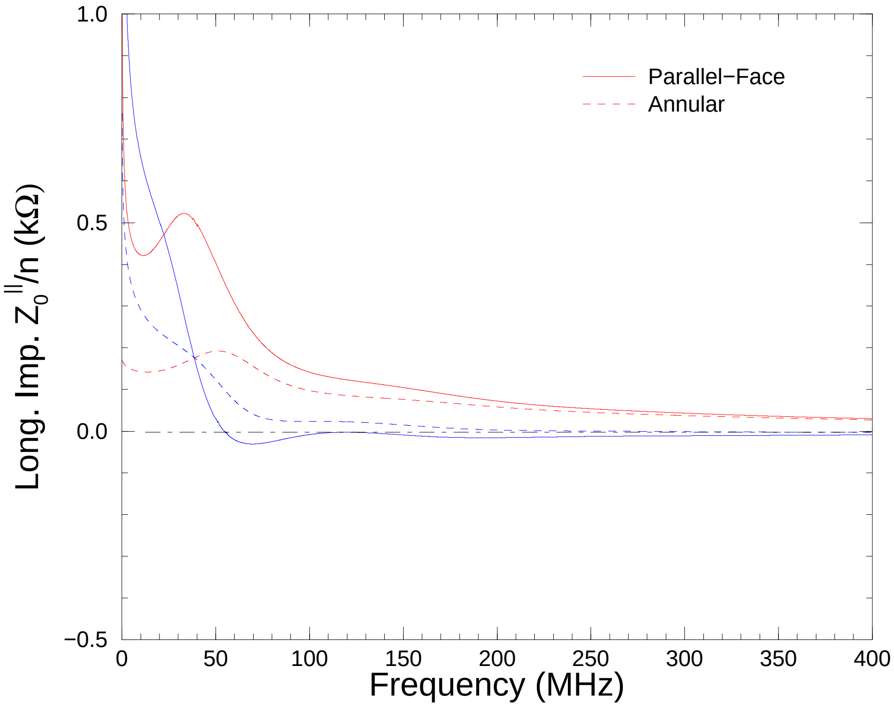


- To study instability, we need $Z_0^{\parallel}/n$.
- Typical half bunch length is $\hat{\tau} = 4$ ns.

  Need perturbing freq

  $f \gtrsim 1/(2\hat{\tau}) = 125$ MHz,

  $\mathcal{R}e\, Z_0^{\parallel}/n < 120\ \Omega$, $|\mathcal{I}m\, Z_0^{\parallel}/n| < 5\ \Omega$.



- **Microwave Instability**

$$\left|\frac{Z_0^{\parallel}}{n}\right| \lesssim \frac{|\eta| E}{eI_{\rm pk}\beta^2}\left(\frac{\Delta E}{E}\right)^2_{\rm FWHM} = \frac{8|\eta|(A/e)\widehat{\Delta E}}{3\pi e N_b \beta E}$$

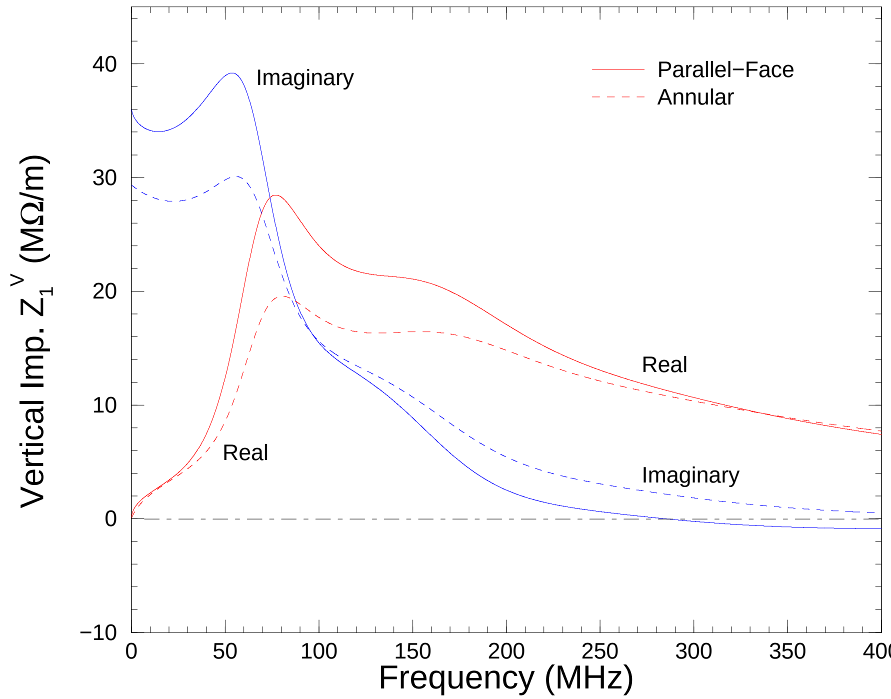
- At extraction, $A = 0.1$ eV-s, $N_b = 6\times 10^{10}$, $I_{\rm pk} = 3eN_b/(4\hat{\tau}) = 1.81$ A, ($\Delta E = 8$ MeV), $\eta = 0.0227$, get stability limit $|Z_0^{\parallel}/n| \lesssim 180\ \Omega$.

- At injection, $\eta = -0.4578$, same bunch area and energy spread $\longrightarrow |Z_0^{\parallel}/n| \lesssim 47500\ \Omega$.

- Beam will be unstable near transition unless passage is fast.

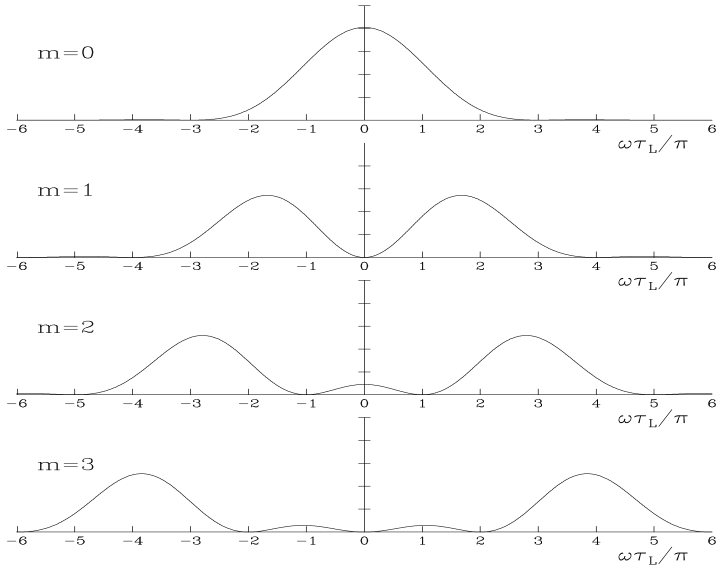
Transverse Impedance

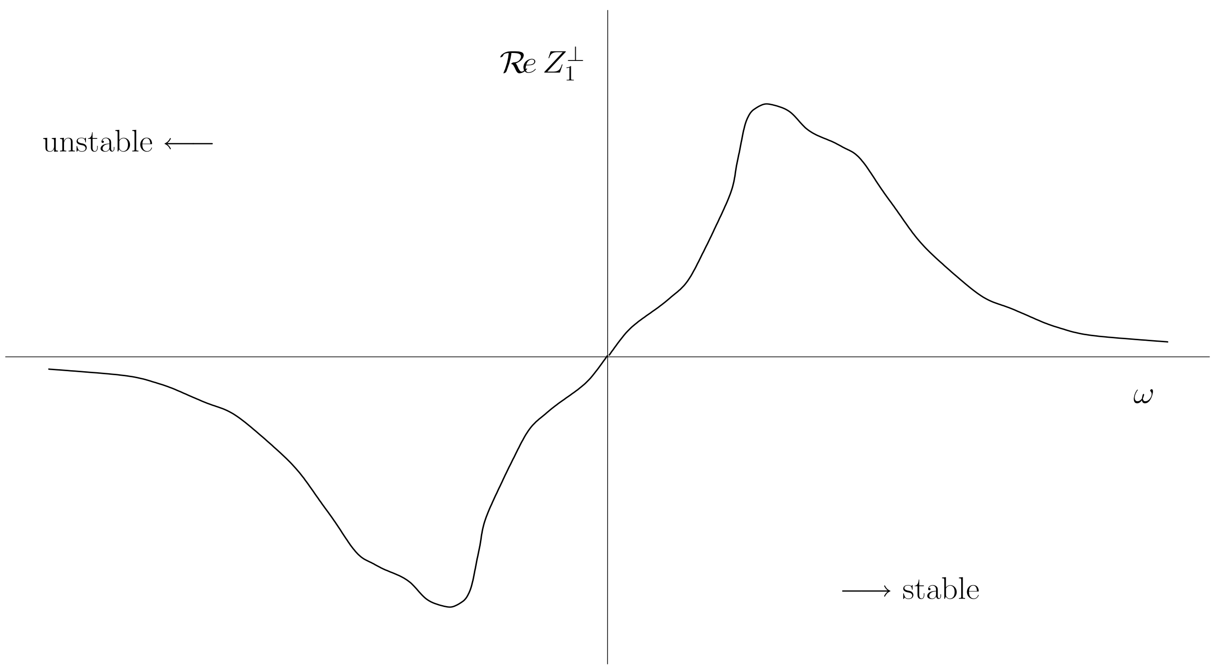
1. See inductive bypass at low frequencies.
   $\mathcal{R}e\, Z_1^{\perp}$ bends around to zero.
2. No $\omega^{-1/2}$ behavior at low frequency
   $\longrightarrow$ no trans. coupled-bunch instability.
3. Broadband ~ 20 MΩ/m
   from 50 to 200 MHz.
   Will drive head-tail instabilities.
4. Parallel-face approx. larger than annular approx.
5. Capacitive at high frequencies.



- There will be transverse head-tail instabilities if $\xi \neq 0$.
- Let us look at the power spectra of the azimuthal modes. The Sacherer's approximate sinusoidal modes will be used.

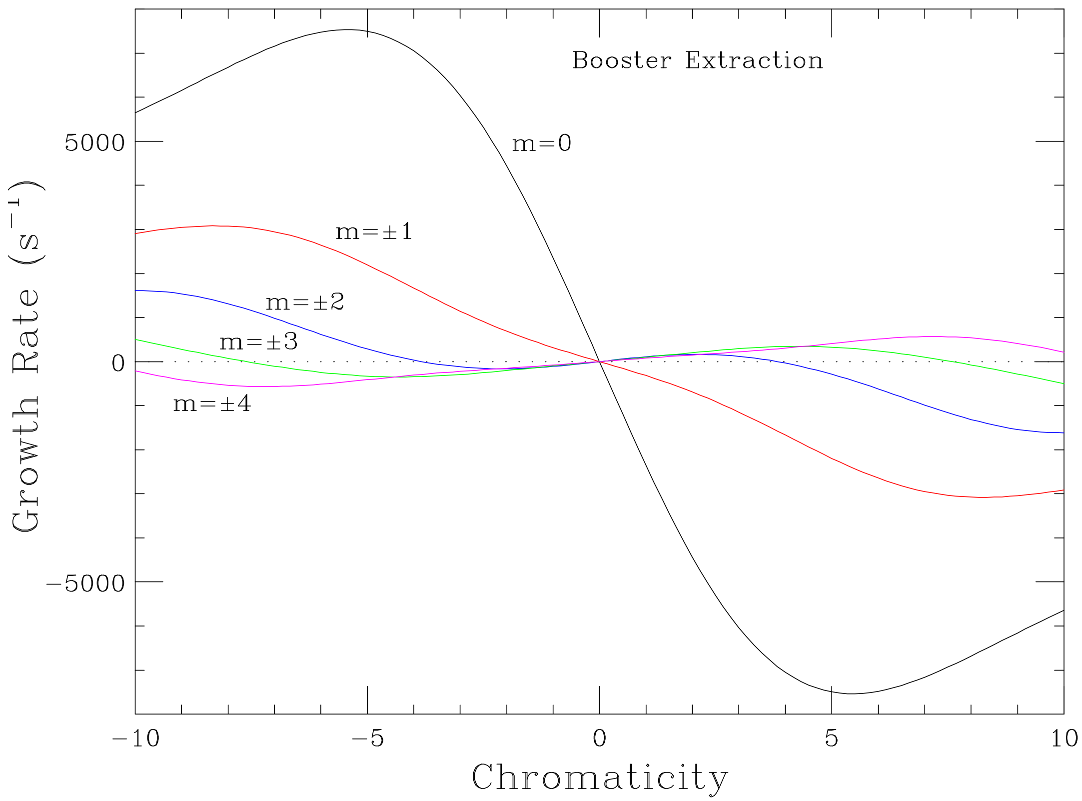
$$\boxed{\frac{\xi\omega_0}{\eta} \longrightarrow}$$





- At extraction, typical total bunch length $\tau_L = 8.0$ ns.
  Assume annular-ring approximation.

- At $\xi > 0$, $m = 0, \ \pm 1$ stable
  $m = \pm 2, \ \pm 3, \ \cdots$ unstable.
- At 6×10^{10} per bunch,
  minimum growth times:
  2.8 ms for $m = \pm 2$
  6.2 ms for $m = \pm 3$.



- Coherent tune shifts are downwards and small.

  $\Delta\nu_{\text{coh}} = -0.0033$ for $m = 0$

  $\Delta\nu_{\text{coh}} = -0.0010$ for $m = \pm 1$

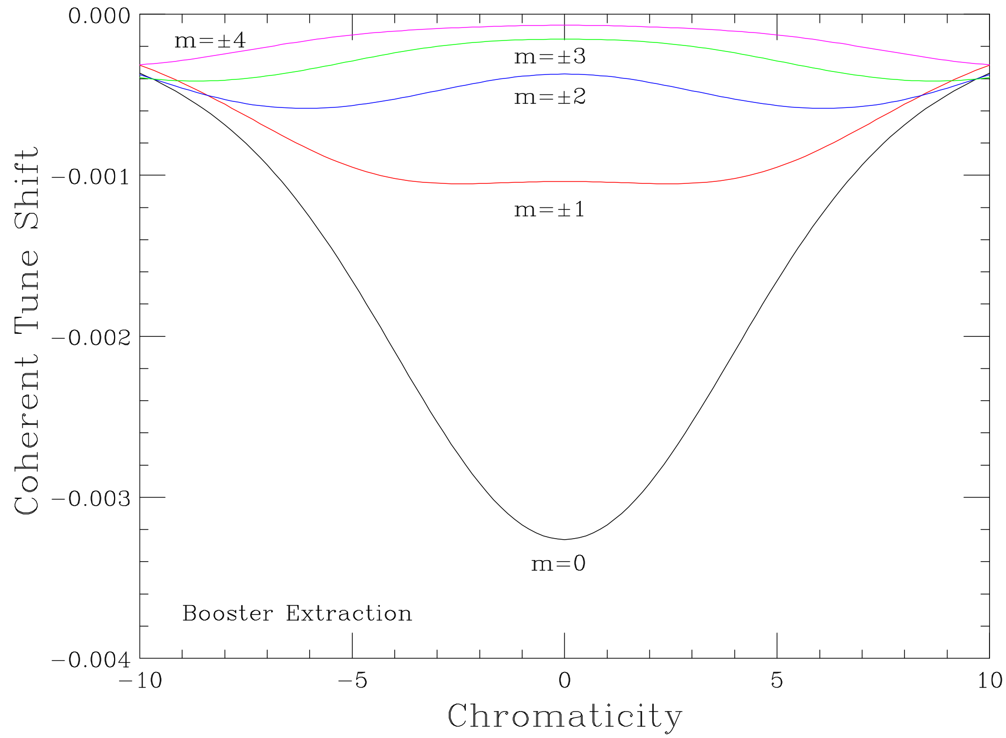
  $\Delta\nu_{\text{coh}} = -0.0006$ for $m = \pm 2$

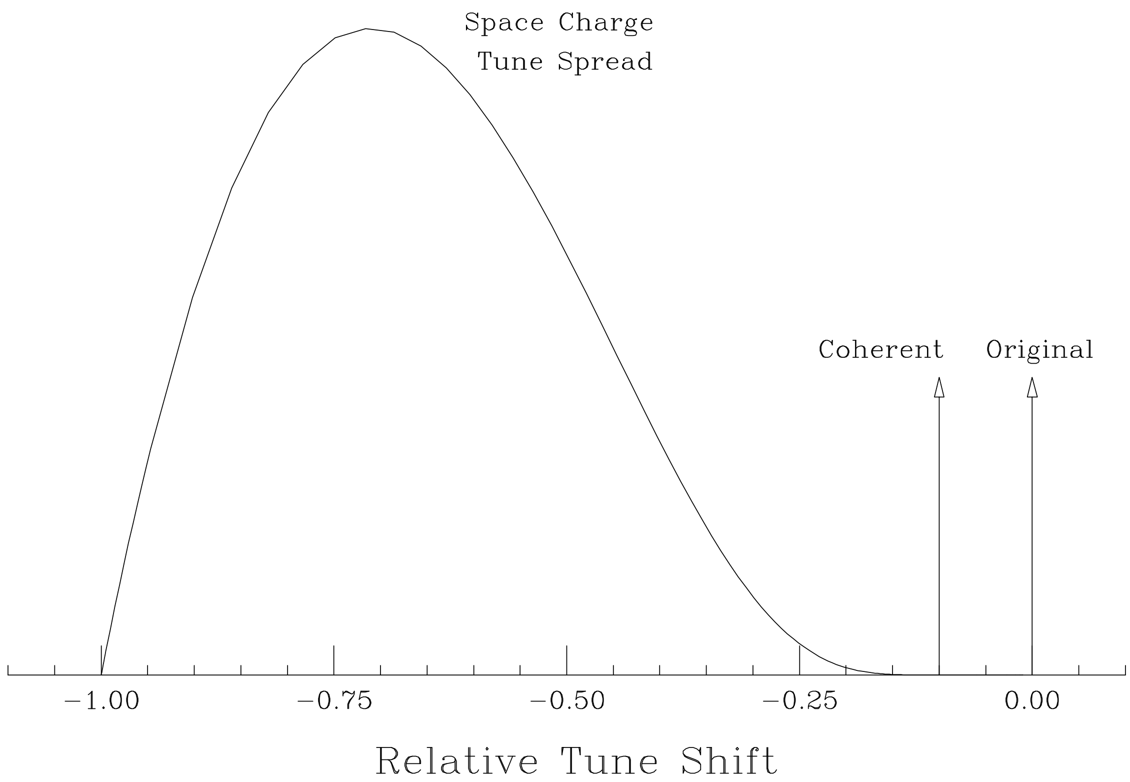
  $\Delta\nu_{\text{coh}} = -0.0004$ for $m = \pm 3$.

- Incoherent space-charge tune shift is large

  $$\Delta\nu_{\text{spch}}^{\text{max}} = \frac{3N_t r_p}{2\gamma^2\beta\epsilon_{_N} B} = -0.014$$

  assuming $\epsilon_{_N} = 2.7 \times 10^{-12} N_t$ πmm-mr, $N_t = 84 \times 6 \times 10^{10}$ is total number of p in beam.



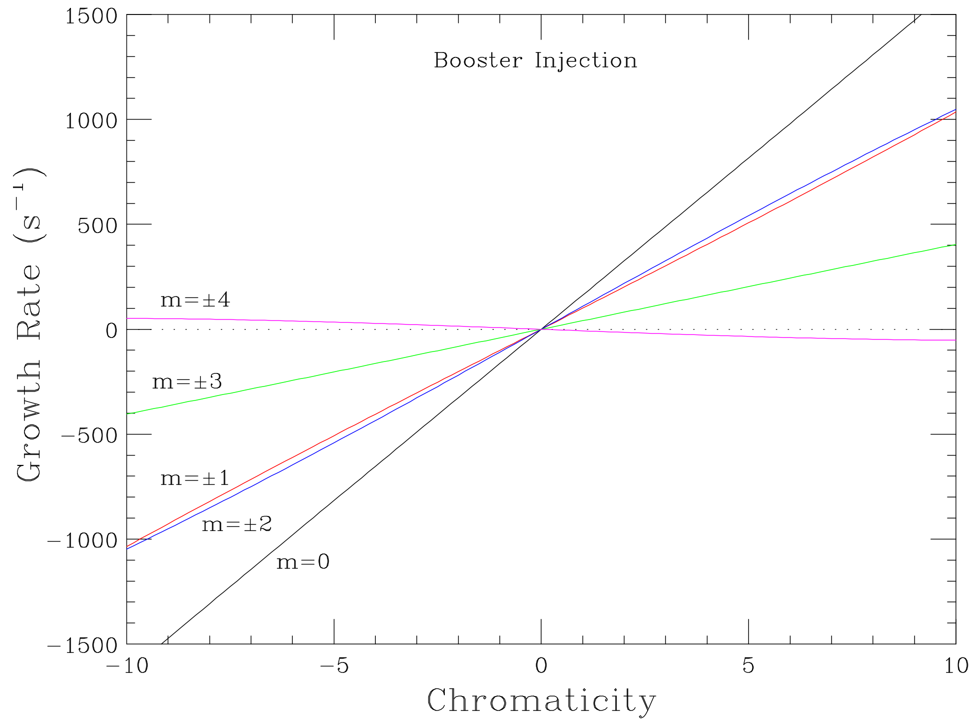


- Space-charge tune spread does not cover the coherent mode lines and cannot damp the growths.

- To Landau damp the growths, need tune spread of the order 0.01 from sextupoles or octupoles.

- Growth rates inversely proportional to E should be larger at injection.
- But $\eta = -0.392$ (vs. 0.023 at extraction). To shift spectra to broad peak of $\mathcal{R}e\, Z_1^{\perp}$:

  $\frac{\xi f_0}{\eta} \sim -100$ MHz requires $\xi = 132$.
- For $|\xi| < 10$, shift is small and so are growth rates.



- Bunch length is much longer, bunching factor 0.49 (0.22 at extraction).

- Power spectra at lower freq than $\mathcal{R}e\, Z_1^{\perp}$ peak for $m = 0,\ \pm 1,\ \pm 2,\ \pm 3$. They are stable for $\xi < 0$.

- Growth rate of $m = \pm 4$ is 75 s^{-1} (13 ms) at $\xi = -10$.

- Coherent tune shifts are small

  $\Delta\nu_{\text{coh}} = -0.014$ for $m = 0$

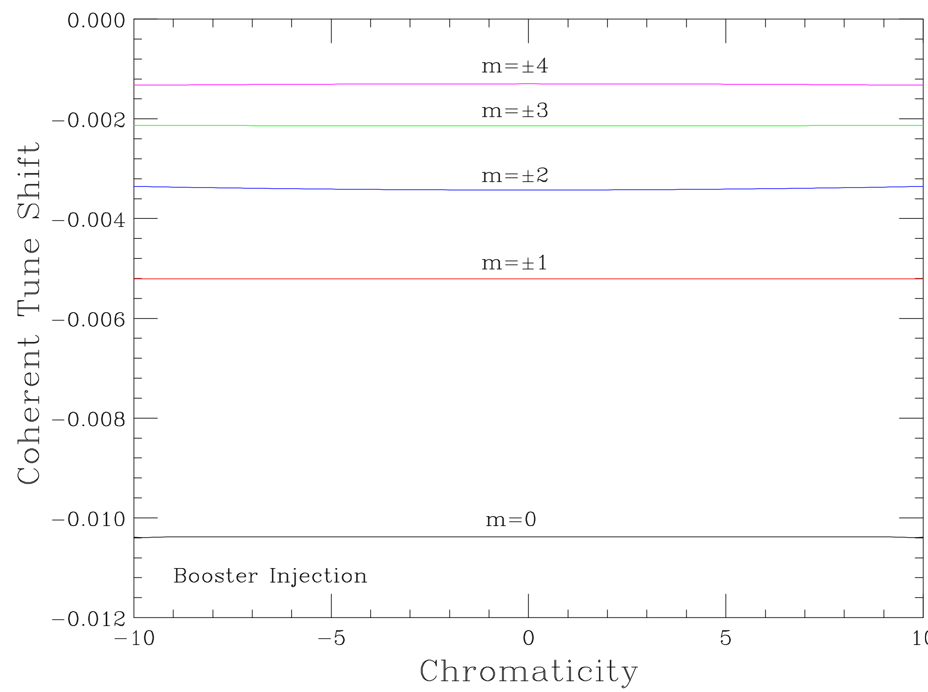
  $\Delta\nu_{\text{coh}} = -0.005$ for $m = \pm 1$

  $\Delta\nu_{\text{coh}} = -0.003$ for $m = \pm 2$

  $\Delta\nu_{\text{coh}} = -0.002$ for $m = \pm 3$

- Incoherent space-charge tune shift is large

  $\Delta\nu_{\text{spch}}^{\text{max}} \sim -0.4$.



- It is not possible to have tune spread as large as 0.4 to Landau damp modes $|m| \geq 4$.

  But if ξ is small, e.g. $\xi = -2$, growth time becomes 67 ms, too long to worry.

- Near transition η is small and the shift of power spectra will be large.

  To have growth rates under control, $|\eta|$ must be small near transition and change sign across transition.

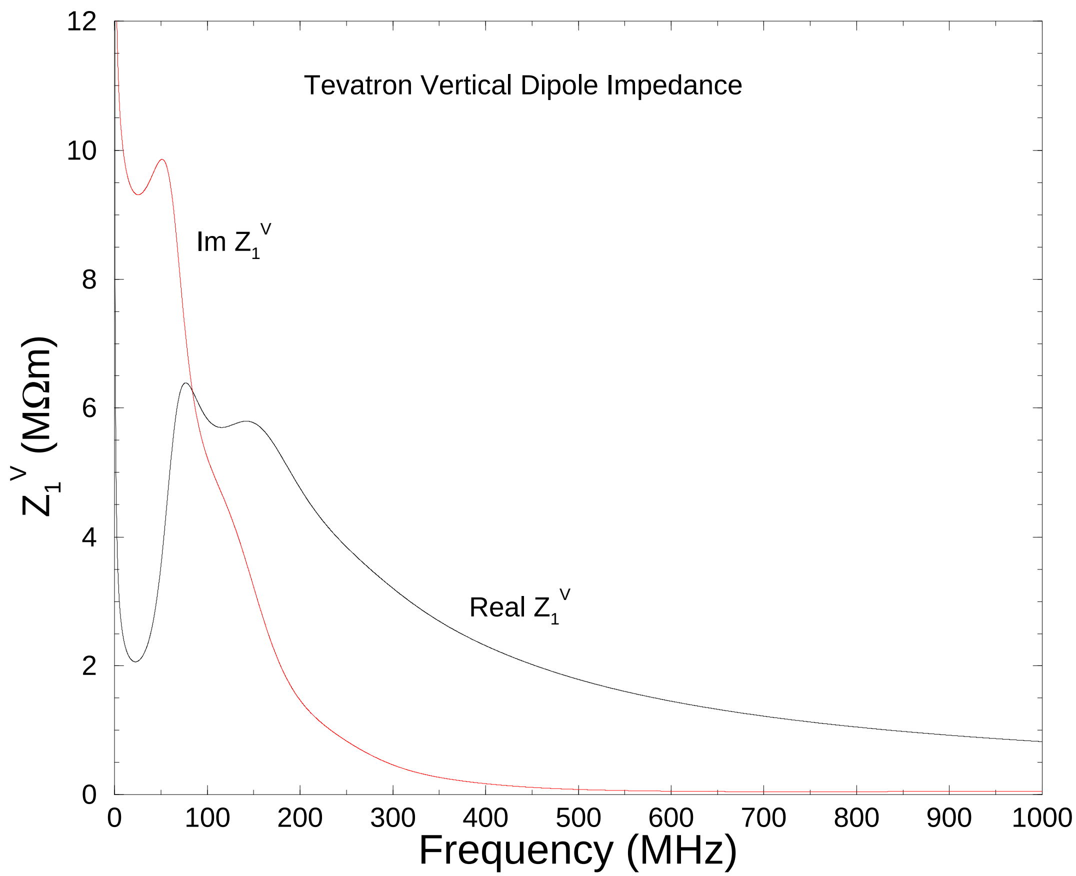
CONCLUSION

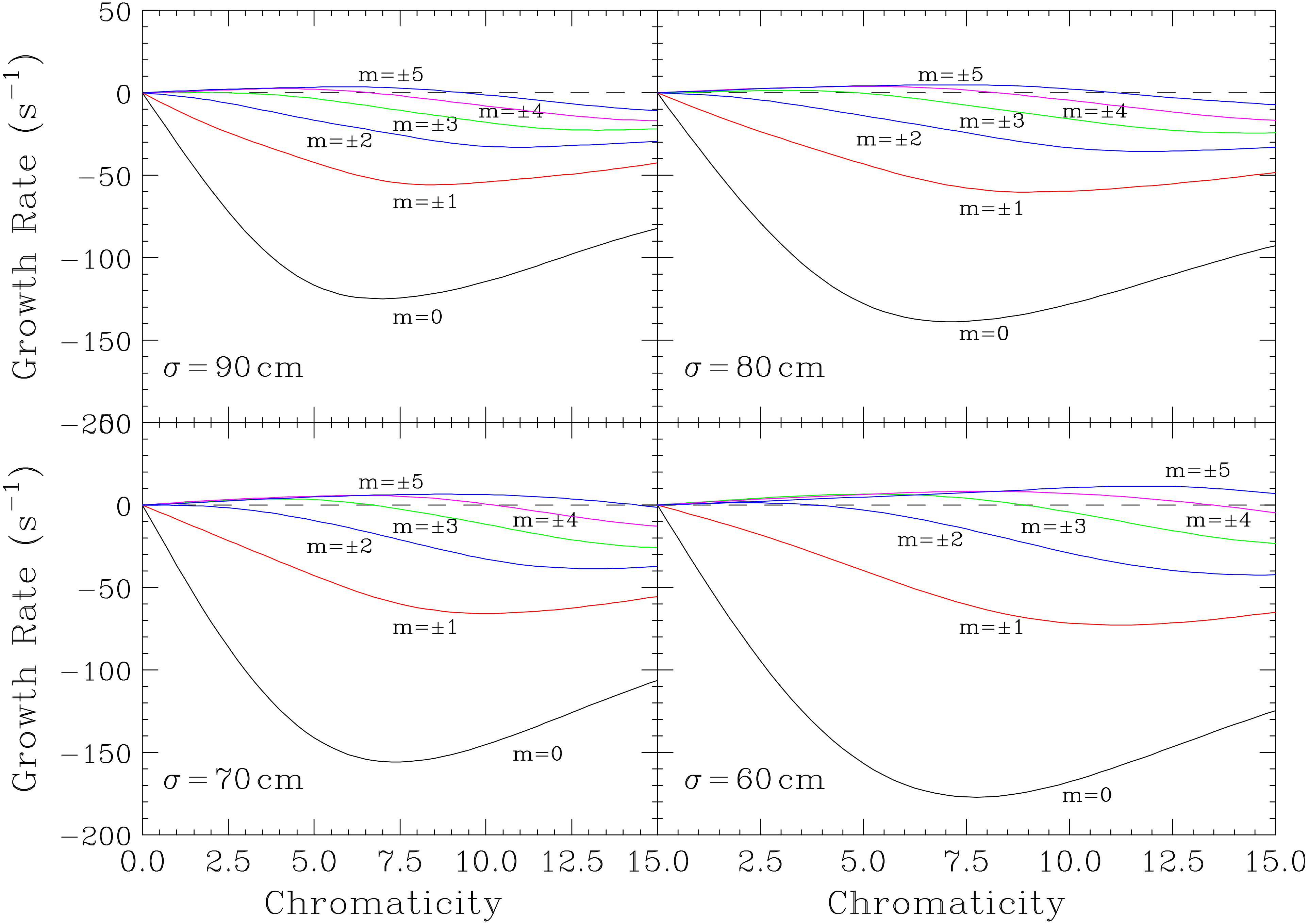
- The inductance bypass plays an important role in the transverse impedance of laminated magnets.

- Laminated magnets will not contribute to trans. coupled-bunch instabilities. But will drive head-tail instabilities.

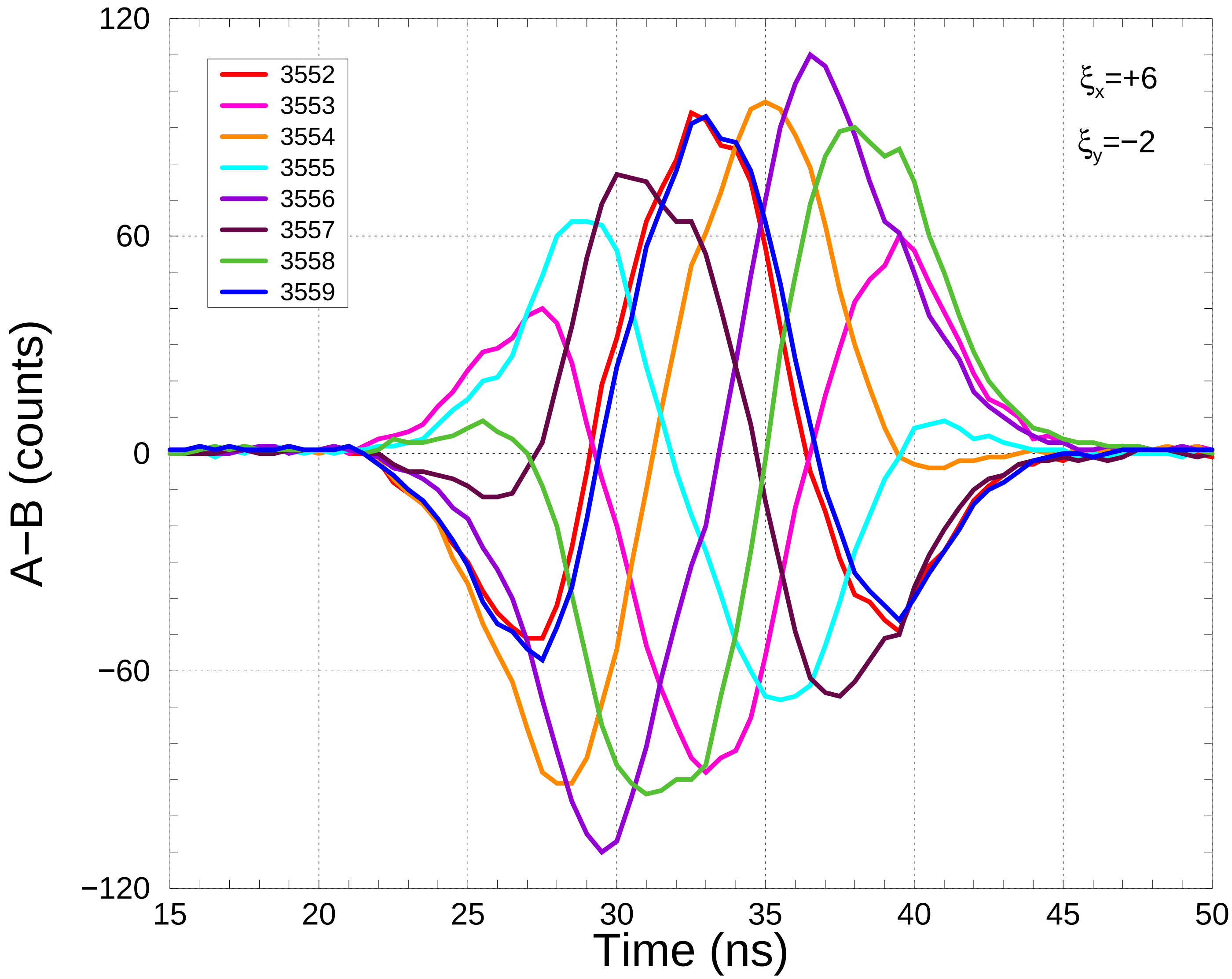
Tevatron at injection

At 2.6×10^{11} per bunch





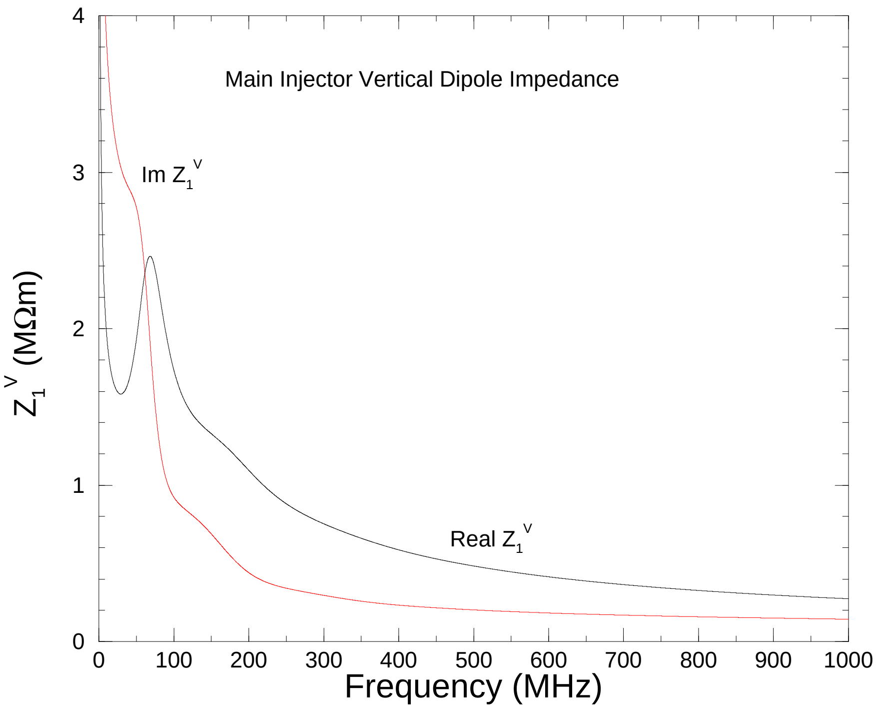
Growth Rate (s^{-1})
Chromaticity
$\sigma = 90$ cm
$\sigma = 80$ cm
$\sigma = 70$ cm
$\sigma = 60$ cm
m=±5
m=±4
m=±3
m=±2
m=±1
m=0

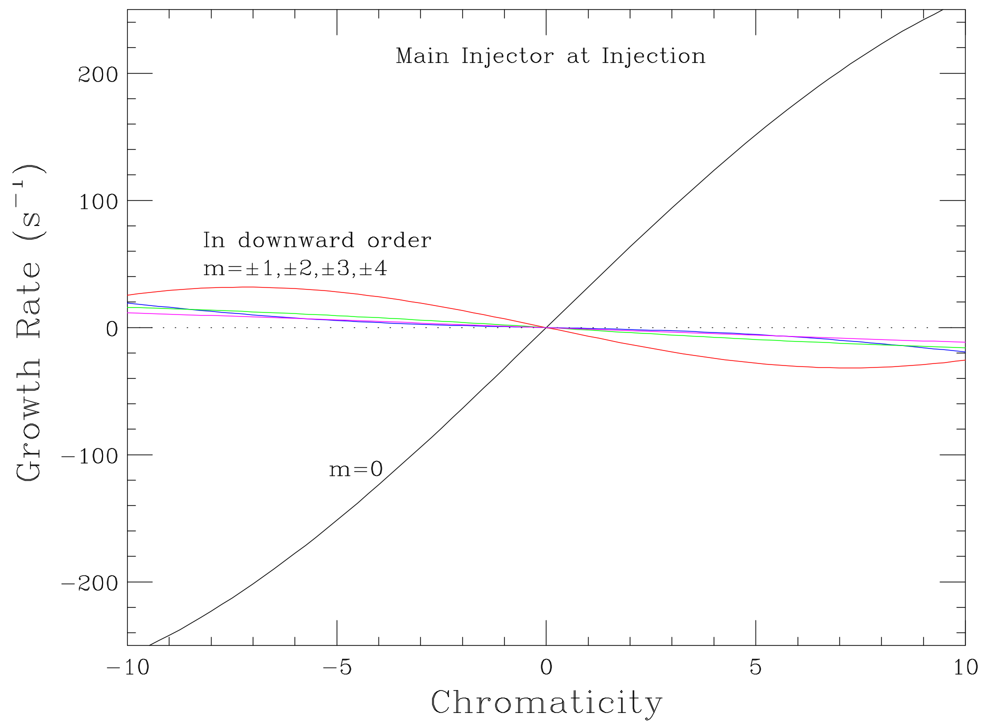


120
60
0
−60
−120
15
20
25
30
35
40
45
50
A−B (counts)
Time (ns)
3552
3553
3554
3555
3556
3557
3558
3559
ξx=+6
ξy=−2

Main Injector at injection

6×10^{10} per bunch, $V_{\rm rf} = 1$ MV





- Although there are many Lambertson magnets in the Main Injector, they will not add on to trans. coupled-bunch instabilities.

- Lambertson magnets will drive head-tail instabilities.
  However, growth rates are very slow.