Calculation of Blackbody Radiation Stripping using Hill-Bryant Method

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• The unit of the x-section is $a_0$

For Hydrogen,

$$a_0 = 0.529 \times 10^{-10} \text{ (m)}$$

$$\sigma_{peak} = \sigma_{max} = 4.2 \times 10^{-21} \text{ (m}^2\text{)}$$

• Threshold, $E_0 = 0.7542 \text{ (eV)}$
  (Electron Affinity of Hydrogen)

• As the $E_\gamma$ increases, the x-section rises to the power of 3/2 first, then, peaks at about 1.5 (eV)

• As the spectrum tails out, the structure persists from around 10.9 (eV) thru 14.35 (eV)
  (= 13.6 + 0.7542 eV).
• Photo-detachment of \( \text{H}^- \) just above threshold, 0.75 eV \((\text{dashed line})\), and in the presence of electric field \((\text{solid line})\).

• Photo-detachment sets off below detachment threshold, 0.75 eV, and the field-induced \textit{ripple-like structure} appears in the presence of electric field at higher \( E_\gamma \).
Collision Length Calculation (I)

Collision Length, \( L \sim \frac{1}{\langle \rho \sigma \rangle} \)

Differential Number Density of Photons by Bose-Einstein distribution can be approximated to Maxwell-Boltzmann distribution \( E_\gamma = | \vec{k} | \sim 1 \text{ (eV)} \), \( T_{\text{Room}} = \frac{1}{40} \text{ (eV)} \)

\[
\frac{d\rho}{d\gamma} = \frac{d^3k}{(2\pi)^3} \cdot \frac{2}{\exp(|k|/T) - 1} \approx 2 \cdot \frac{d^3k}{(2\pi)^3} \cdot \exp(-|\vec{k}|/T)
\]

\[
\frac{1}{L} = \frac{T^2}{2\pi^2\gamma} \cdot \int_0^\infty dE \cdot \sigma(E) \cdot (1 + \frac{E}{T_h}) \cdot \exp(-E/T_h)
\]

\[
T_h = T_R \cdot \sqrt{\frac{1 + \beta}{1 - \beta}} \approx 2\gamma T_R
\]

\[
\sigma(E) = \frac{8\sigma_{\text{max}}E_0^{3/2}(E - E_0)^{3/2}}{E^3}
\]

\[
\sigma_{\text{max}} = 4.2E - 21(m^2), \ E_0 = 0.7543\text{ (eV)}
\]

relativistic Doppler blue-shifted to \( T_h \), and Doppler-red-shifted effective temperature term is suppressed with given input parameters.
Collision Length Calculation (II)

- As the natural units \((\mathcal{C} = \hbar = \kappa_B = 1)\) is used in C. Hill’s equation, conversion factor, \((\hbar \mathcal{C})^3\), needs to be added to the pre-factor for numerical calculation.

\[
\frac{1}{L} = \frac{T^2}{2\pi^2 \gamma} \cdot \int_0^\infty dE \cdot \sigma(E) \cdot (1 + E/T_h) \cdot \exp\left(-\frac{E}{T_h}\right)
\]

\[
= \frac{8T^2 \sigma_{\text{max}} \cdot E_0}{2\pi^2 (\gamma\beta) (\hbar \mathcal{C})^3} \int_1^\infty d\varepsilon \frac{(\varepsilon - 1)^{3/2}}{\varepsilon^3} \left(1 + \left(\frac{E_0}{T_h}\right)\varepsilon\right) \cdot \exp\left(-\frac{E_0}{T_h}\varepsilon\right)
\]

\[
(\varepsilon = E/E_0)
\]

\[
\therefore L \approx 2000\,(\text{km})
\]

@ \(E = 8\,(\text{GeV}),\ T_{\text{room}} = 300\,(^\circ\text{K})\)
Loss Rate vs. H- Ion Kinetic Energy

calculated by Phil Yoon

Detachment Rate vs. H- Ion Kinetic Energy (I)

Detachment Rate (1/meter)

H- Ion Kinetic Energy (GeV)
Photodetachment of H- Ions from Blackbody Radiation

Loss Rate vs. H- Ion Kinetic Energy
Photo-Detachment Rate vs. Temperature

Detachment Rate vs. Temp.

Loss Rate (1/meter)

$E_{\text{kin}} = 8 \text{ (GeV)}$

$T_{\text{Room}} = 300 \text{ K}$

Li $N_2$ Boiling Point, 77 K

Temperature (K)
Concluding Remarks

• With the Hill-Bryant method, two different numerical calculations (Excel and Mathematica) agree well that the loss rate is about $5.0 \times 10^{-7} / \text{meter at } 8 \text{ GeV}$.

• The higher H$^-$ ion beam energy is, more likely to be stripped by thermal photons.

• When beam energy jumps to 8 GeV from 0.8 GeV, stripping by blackbody radiation and loss rate increase by 3 order of magnitude.

• As the temperature rises from Li Ni$_2$ boiling point to room temp. detachment rate increases by three order of magnitude.