Chapter 3. Beam Optics

Alexandr Drozhdin, Carol J. Johnstone, Shoroku Ohnuma, and David Ritson

3.1. Lattice Design Considerations

Requirements for high beam intensity and beam power dictate the design of the Proton Driver lattice. This is particularly true in the arc part of the lattice where optical parameters determine the beam size.

3.1.1. Requirements and desired features

1. Avoidance of transition crossing. Although there are many ways to reduce beam loss during transition crossing [1], e.g., a γ_t-jump with a special quadrupole system, it is not possible to eliminate the loss completely. Furthermore, any source of longitudinal emittance dilution of the beam should be avoided in order to achieve a short bunch length in the extracted beam. The design presented here avoids transition crossing by choosing γ_t to be imaginary or the transition energy to be much higher than the extraction energy of the beam [2,3].

2. Large momentum acceptance. In order to make the bunch length short, it is necessary to rotate the bunch in longitudinal phase space. To meet the needs of a neutrino factory and a muon collider, the lattice should have momentum acceptance of $\pm 2.5\%$.

3. Large dynamic aperture. Tune spread within the beam due to space charge may be as large as 0.3 or more if the charge distribution of the injected beam has a peak at the center. To reduce this effect, the transverse charge distribution of the injected beam will be made as uniform as possible by the "painting" technique. As a result of painting, the beam emittance will be enlarged to 60π mm-mr (normalized, 100%). The dynamic aperture of the lattice is required to be twice this value for the entire momentum spread range, $\pm 2.5\%$.

4. Long straight sections. Three long straight sections P20, P40, and P60 will be used for injection/collimation, rf cavities, and extraction. Because of the high beam power and the rapid cycling operation, rf cavities must occupy at least 60 m of free space. At the same time, it is not desirable to place rf cavities near the injection/collimation devices or downstream of extraction devices. In the present design, P20 is used exclusively for injection/collimation and P40 for rf cavities. The extraction system occupies the downstream end of P60 but the upstream end of that long straight is available for rf cavities. Specific considerations given to the design of three long straight sections are:

   a) Superperiodicity. It is always desirable to avoid structural resonances in any machine and this is especially true when large-aperture magnets may contain a significant amount of systematic nonlinear field components. Although it is sufficient to have the same transfer matrix in three long straights for this purpose, *i.e.*, Twiss parameters matched to the arcs and the same phase advance, the design presented here utilizes the identical quadrupole arrangement in all three long straights. It may be possible to optimize each long straight for its intended use while preserving the superperiodicity. This will be studied in the future.

b) Choice of tunes. Since the phase advance in each arc is chosen to be 6π, the operating point in the tune diagram is determined by the phase advance in the long straights. Based on results from numerical studies, it has been decided to split horizontal and vertical tunes by one unit, which will avoid resonances of the type $2\nu_x - 2\nu_y = 0$. In addition, the fractional part of ν_x is larger than that of ν_y by 0.05 so that the operating point stays below the coupling resonance line. The integer part of the tunes should be chosen such that as many structural resonances as possible will be sufficiently away from the working point. The choice of (12.43, 11.38) in this design is nominal, and the performance of the real machine will eventually decide the working point.

c) Injection straight. The beam size at the stripping foil should not be too small in order to prevent an excessive temperature rise. At the same time, the large β function at the foil will contribute to emittance growth due to multiple Coulomb scattering. The compromise choice of β = 10 m in both directions satisfies these requirements. Drift spaces between the stripping foil and adjacent quadrupoles must be large enough to accommodate H^- injection and dumping of the H^0 component. The phase advance between the first and last kicker magnets for the painting, located on each side of the foil, should be close to π so that the required kicker strength is not excessive.

d) Collimation. Large dispersion and β function are necessary at the primary collimators. The phase advance over the collimation system should not be less than 180 degrees in both directions.

e) Extraction straight. The phase advance between kickers and septa as well as β functions at various extraction devices should be chosen carefully in order not to make excessive demands on magnets. The system should be able to accommodate at least twice the expected beam emittance, 120π mm-mr (normalized), so that there is no halo scraping in any of the extraction magnets.

f) RF straight. Although it has not been observed in proton machines, it is prudent to avoid any possibility of synchro-betatron coupling resonances [4], especially in view of a relatively large value of ν_s ($\approx$ 0.1) at low energies. The choice of a multiple of 2π as the arc phase advance assures zero dispersion in all long straights.

3.1.2. Constraints

Because of the large aperture in dipoles and quadrupoles, saturation at high field is a serious issue when they are in the same resonant circuit. Degradation of field quality is another concern. Based on experience gained in building and operating the Antiproton Accumulator, it has been decided to limit the peak dipole field to 1.5 T and the peak gradient of the quadrupoles to 8.9 T/m. The actual peak gradient is slightly less than this value for the chosen winding ratio of dipoles and quadrupoles. Expected performance of the magnets is discussed in Chapter 6 and the system requirements for the quadrupole tracking are presented in Chapter 7.

3.1.3. Other considerations

In an early stage of the design study, a lattice using combined function magnets was considered as a possible choice. It has been found that, with the requirements listed in Section 3.1.1, a lattice with combined function magnets cannot provide the needed flexibility. For one thing, the dispersion was too large to be usable for this machine. If one is to avoid transition crossing from 400 MeV (injection kinetic energy) up to 16 GeV (extraction kinetic energy), a lattice based on simple FODO cells in the arcs will not work. Instead, it is necessary to have cells with missing dipoles, or cells with doublets or triplets. Such lattice structures are often called "FMC" (flexible momentum compaction) [2].

3.2. Lattice Design

3.2.1. Lattice layout and nomenclature

The lattice layout is shown in Figure 2.2. It is composed of three arcs (P10, P30, P50) and three long straights (P20, P40, P60) with superperiod three. Each arc has four modules while each module is made of three different cells:

Cell_a: FODO (with different QF and QD) with two regular dipoles,
Cell_b: FODO (with different QF and QD), no dipoles,
Cell_c: mirror image of Cell_a but with one regular and one short dipole.

module = (a)(b)(c), half_arc = (module, –module), arc = (half_arc, –half_arc)

Note that arc can also be regarded as (half_arc, half_arc) since –half_arc (mirror image of half_arc) is identical to half_arc.

Three long straights, each approximately 64 m long, are identical and their structure is essentially of the FODO type except that distances between quadrupoles are varied according to the requirements of injection/collimation and extraction.

Nomenclature

The beam direction in the ring is clockwise. Starting from the arc (P10) upstream of the injection/collimation long straight (P20), arcs and long straights are arranged in a triangular shape: P10, P20, P30, P40, P50, and P60. Each item in the ring is referred to the quadrupole that is located immediately upstream of that item. Quadrupole names are "Q" followed by three numbers to identify their locations:

First number: section number, 1 to 6 (1 for P10, etc.).
Second number: module number, 1 to 4 in arcs, and 0 in long straights.
Third number: quadrupole ID, odd for QF and even for QD.

Examples: Arc P10: module 1: Q111, Q112, …, Q116
module 2: Q121, Q122, …, Q126, etc.

Long straight P20: Q201, Q202, …, Q208, etc.

Sextupoles are all in Cell_b so that in module 1 of Arc P10, they will be designated as (HS113, VS113, VS114, HS114). Note that VS113 is closer to Q114 and HS114 closer to Q115. As explained above, however, the convention is to refer to the upstream quadrupole.

3.2.2. Space allocations

The circumference of the ring is exactly 3/2 of the present Booster, namely, (3/2) × 474.203 m = 711.304 m. Three arcs occupy 3 × 173.18 m = 519.54 m and three long straights 3 × 63.92 m = 191.76 m. Spaces allocated to dipoles and quadrupoles are:

dipoles, regular (36):	186.0 m
dipoles, short (12):	50.3 m
quadrupoles in arcs (72):	102.7 m
quadrupoles in long straights (24):	29.7 m
free space in long straights:	147.6 m (total for 24 locations)
short straights in arcs (24):	77.8 m
mini straights in arcs (24):	11.5 m

Note that the lengths of dipoles given here are the path length of the beam within the dipoles and not necessarily their physical length. There are altogether four types of quadrupoles in the arcs and five types in the long straights. These nine types are different only in their lengths. It is hoped that this number will be reduced in future improvement of the lattice design. It should be noted also that a substantial amount of space is allocated to the magnet ends. For example, between a quadrupole and a dipole, 0.8 m is necessary to accommodate magnet ends and vacuum ports. Consequently, there is no space for sextupoles or trim dipoles in Cell_a. Sextupoles will be installed in the short straights of

Cell_b, altogether 48 of them occupying 14.4 m. Mini straights in Cell_c are for vertical steering magnets.

3.2.3. Various types of arc lattices and their comparisons

Several different candidates for the arc lattice, all satisfying the requirements of no transition crossing and zero dispersion in long straights, have been studied to see their relative merits and possible defects.

a) Doublet [5]. The merit of this choice is its simplicity. Only two types of quadrupoles are needed in the arcs. It has been found that, for reasonable values of dispersion and β function, the required quadrupole strength, 20 T/m, is far above a realizable value. Perhaps the most serious defect of this lattice is the lack of suitable locations for sextupoles. For our case, because of the large momentum range, chromaticity sextupoles are essential components of the ring.

b) Racetrack. It is possible to design a racetrack ring with the same modules as in a triangular ring. The superperiod of two may be less advantageous but this should not be a major concern. A triangular lattice has been chosen for the present design over a racetrack lattice for two reasons. 1) In a racetrack lattice, it is necessary to put injection/collimation and extraction in the same long straight, thereby complicating the design of this section. 2) For reasons explained in the next section, 3.2.4, the phase advance in each module is 270˚ and the total phase advance in each arc is a multiple of 2π. For a triangular lattice, there will be altogether $3 \times 4 = 12$ modules but $2 \times 8 = 16$ modules for a racetrack lattice. The total length of long straights will then be reduced.

c) Various choices of phase advance in FODO or DOFO modules. There are two ways to suppress dispersion in long straights. One is to introduce a special dispersion suppressor section, which usually requires dipoles of different lengths in order to reduce the maximum dispersion in the arc, and the other is to rely on a $(2n\pi)$ horizontal phase advance in each arc, which automatically assures zero dispersion at both ends of the arc. Although a dispersion suppressor provides more flexibility in choosing the phase advance per module, it has been decided to achieve zero dispersion in long straights by choosing the horizontal phase advance to be a multiple of 2π. After several combinations of horizontal/vertical phase advances per module were studied, 270˚/270˚ has been selected as the best in overall performance.

3.2.4. Arc design: 270˚/270˚ FODO modules

In selecting the phase advance per module, the overriding concerns have been the momentum aperture and the dynamic aperture. They are both determined essentially by the effects of chromaticity sextupoles as long as nonlinear field components of magnets are not excessive. This is particularly true when dispersion in the arc is not allowed to go above 3 m or so in order to limit the increase in beam size, D_x $(\Delta p/p)$. The choice of 270˚ as the horizontal phase advance per module is natural since four modules provide 6π. It also cancels all the driving terms of the third-integer resonances. For the vertical phase advance per module, choices other than 270˚ are possible. For example, either 135˚ or 180˚ will also cancel all the resonance driving terms arising from sextupoles. It has been found out, however, that there are no suitable locations for sextupoles with the choice of 135˚. As a consequence, the sextupole strength becomes large and their higher order effects reduce the momentum aperture as well as the dynamic aperture. The choice of 180˚ has an advantage in reducing the higher order effects of sextupoles. However, it produces a large amount of β beat, which is obvious from the relation $2\psi_y = 360˚$. Another case that has been studied is 270˚/270˚ but in DOFO. This is as good as FODO of the same phase advance giving good momentum and dynamic aperture. It has, however, a large maximum dispersion, 4 to 5 m, compared with 2.5 to 3 m for FODO.

3.2.5. Straight section design

Three straight sections P20, P40, and P60 are identical in beam optics to insure periodicity three. The structure is FODO but the cell lengths are varied according to the requirements of injection/collimation as well as of extraction. At present, there are five different types of quadrupoles but it may be possible to reduce this to two or three. Injection and collimation systems are located in P20 and the detailed description is given in Chapters 9 and 11. In Stage 1 of Phase I (see Chapter 5), most rf cavities will be in P40 but some will share P60 with the extraction devices. The extraction system and its hardware components are described in detail in Chapter 12.

3.2.6. Improvements

Work to improve the present lattice is still in progress. The dependence of zero dispersion in long straights on ($2n\pi$) horizontal phase advance in each arc may not be as robust as a standard dispersion suppressor. For example, the horizontal closed orbit for $\Delta p/p = 2.5\%$ oscillates between –4.5 mm and +3.7 mm in long straights where rf cavities are placed. When the beam intensity is high, this may cause longitudinal emittance dilution or even some beam loss. The dependence of residual dispersion in long straights on the horizontal phase advance in the arc is another concern. If the horizontal phase advance in an arc is changed by $\pm 0.05 \times (2\pi)$, the maximum residual dispersion may become as large as 0.15 m.

Lack of space for vertical steering magnets in Cell_a is another concern. In the middle of each arc, two vertical steering magnets are located side-by-side covering a phase advance of more than 180˚. As more space is needed for rf cavities in Stage 2, it may be necessary to redesign the long straights for a more efficient use of available free space.

With the present arc design, transition gamma is always either imaginary or comfortably above the extraction energy for the full range of $\Delta p/p = \pm 2.5\%$. This does not guarantee that the bunch rotation in longitudinal phase space, which is required to get a short bunch, will be free of distortion as the dependence of $\alpha \equiv 1/\gamma_t^2$ on $\Delta p/p$ is not favorable for such an operation. (See Section 3.3.1.) This defect will be taken into account in redesigning the dispersion suppression.

3.3. Lattice Analysis

Beta functions and dispersion are shown in Figure 3.1. Maximum values are:

arc: $\beta_x = 35.4$ m, $\beta_y = 34.8$ m, $D_x = 3.0$ m
long straight: $\beta_x = 35.4$ m, $\beta_y = 39.0$ m, $D_x = 0.03$ m

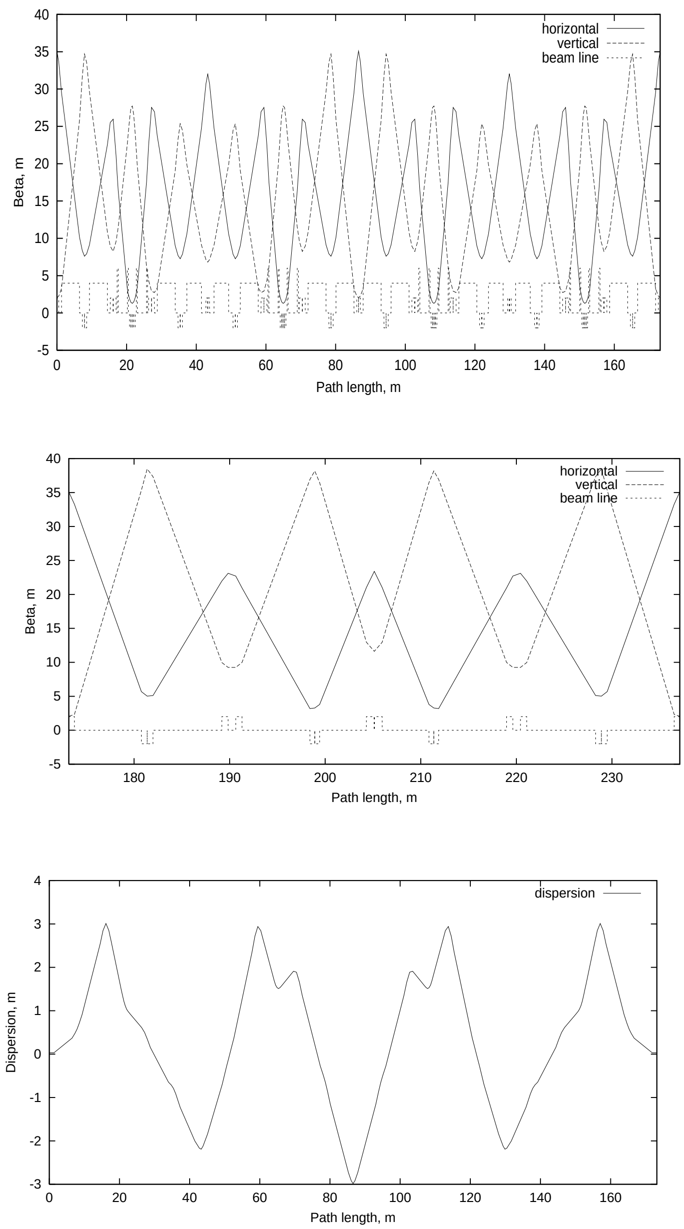


Figure 3.1. (top) β functions in the arc ; (center) β functions in the long straight; (bottom) dispersion in the arc

3.3.1. Chromatic properties

One important requirement for this lattice is the large momentum acceptance, Δp/p from –2.5% to 2.5%. Chromaticity correcting sextupoles are therefore essential in preventing the dependence of linear optical parameters (tunes, Twiss parameters, and dispersion) on the momentum deviation. Since β functions and dispersion are not allowed to be large in the arcs, the required sextupole strength tends to be large and the higher order effects may become a source of dynamic aperture limitation. The choice of phase advance, 270˚/270˚ per module, minimizes such effects.

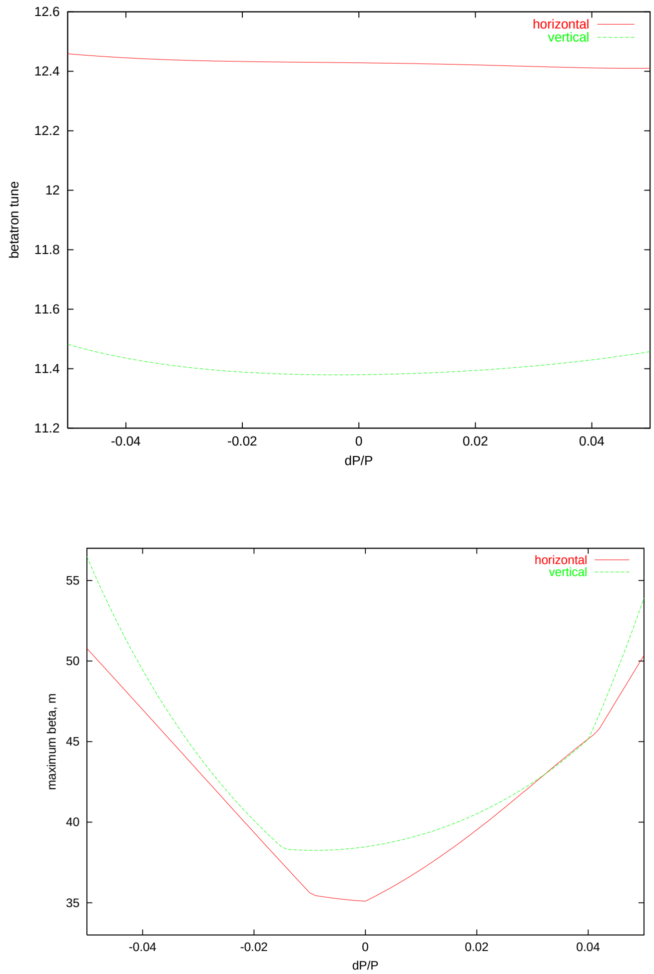


Figure 3.2. Tunes and maximum β as a function of Δp/p

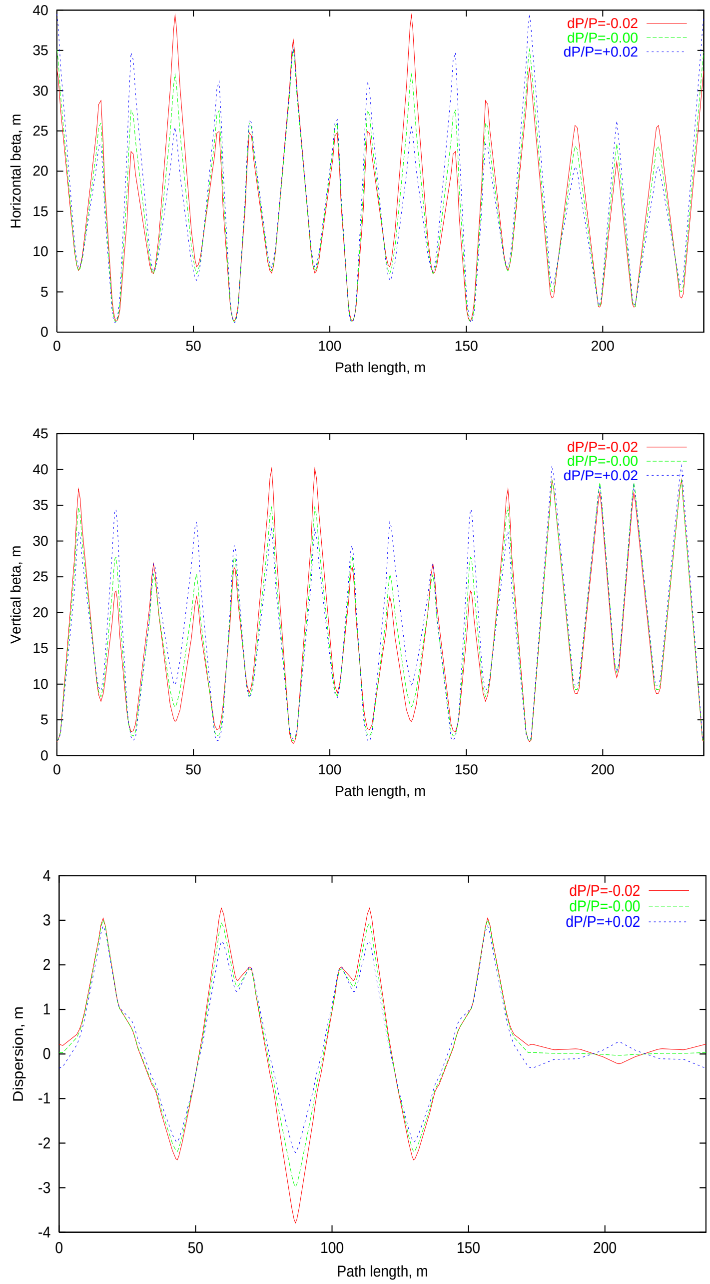
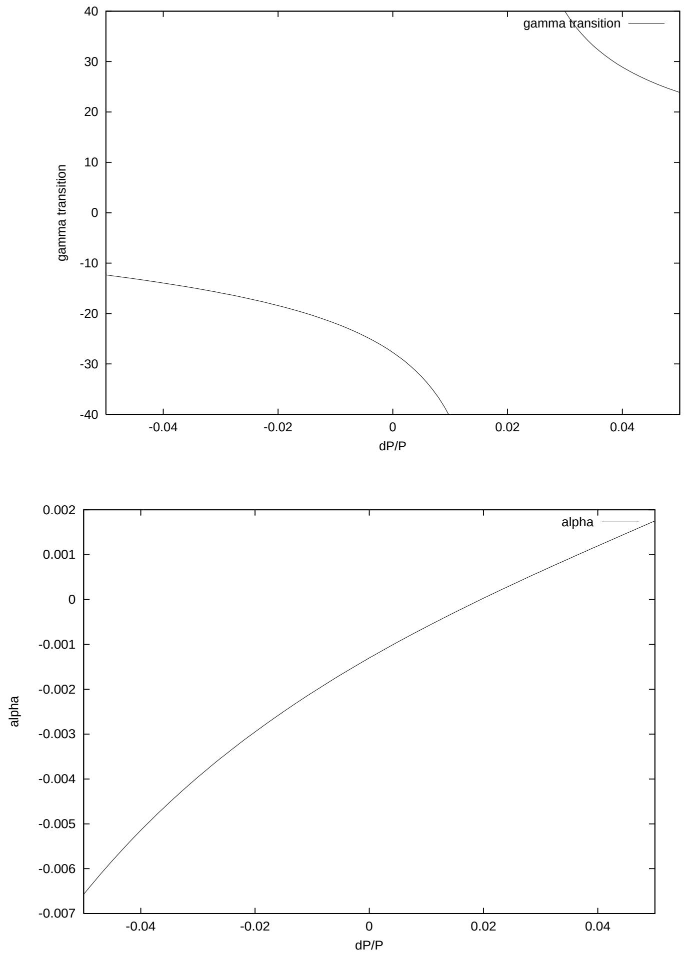


Figure 3.3. (top and center) β functions in one superperiod;
(bottom) dispersion in one superperiod

There are altogether 48 sextupoles, each 0.3 m long, located in the short straights of Cell_b where dipoles are missing. The required strengths are $B'' = 68.8\ T/m^2$ for HS and $-95.4\ T/m^2$ for VS. Tunes and the maximum β functions are shown in Figure 3.2 as a function of $\Delta p/p$ while in Figure 3.3, Twiss parameters in one superperiod are displayed. Transition gamma, γ_t, and the parameter $\alpha \equiv 1/\gamma_t^2$ are shown in Figure 3.4.



γ_t

Figure 3.4. (top) transition-gamma, γ_t ; (bottom) $\alpha \equiv 1/\gamma_t^2$

In Figure 3.4, for the sake of convenience, imaginary γ_t is plotted as a negative quantity. For all values of $\Delta p/p$ under consideration, γ_t is safely away from the energy of the extracted beam, $\gamma = 18.05$ for 16 GeV (KE). For bunch rotation, the important quantity is the momentum dependence of the pathlength C,

$$(\Delta C/C) = \alpha_0 (\Delta p/p) + \alpha_1 (\Delta p/p)^2 + \alpha_3 (\Delta p/p)^3 \tag{3.1}$$

where $\alpha_0 = -0.001302$, $\alpha_1 = 0.07302$, and $\alpha_3 = -0.3952$.

With these values, the momentum dependence of the revolution time T can be written as

$$(\Delta T/T) = -\,0.004370\,(\Delta p/p) + 0.07762\,(\Delta p/p)^2 - 0.4016\,(\Delta p/p)^3 \tag{3.2}$$

shown in Figure 3.5. This relation should be used to estimate the degree of deformation when the bunch is rotated before extraction at 16 GeV. Such a study is now in progress.

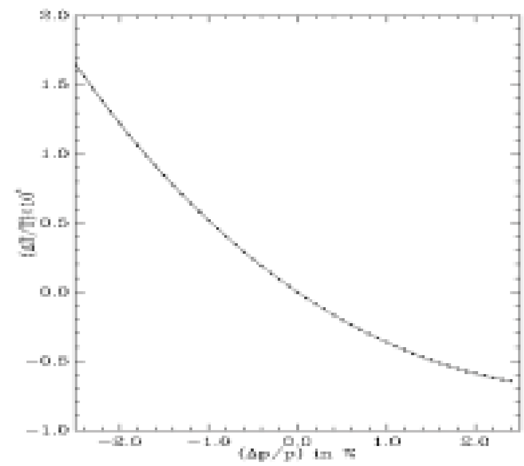


Figure 3.5 Dependence of revolution time as a function of (Δp/p)

3.3.2. Dynamic aperture

In the absence of any closed orbit distortion or magnet errors, the dynamic aperture has been found by tracking particles over 1000 revolutions and the results are shown in Figure 3.6.

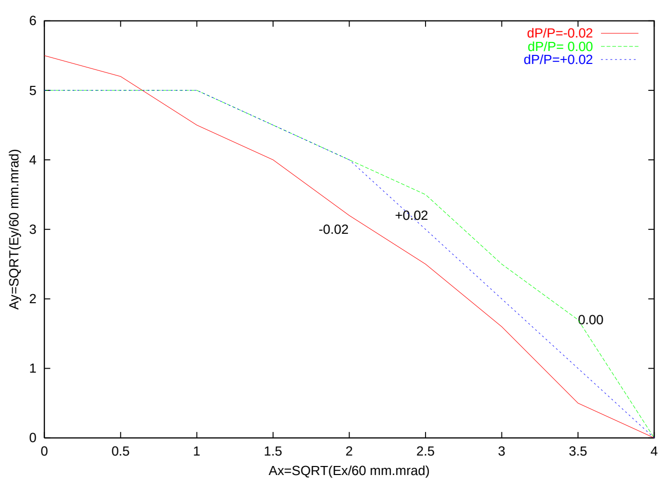


Figure 3.6. Dynamic aperture at injection

Scales used for amplitudes A_x and A_y are such that A_x or $A_y = 1$ corresponds to normalized emittance of 60π mm-mr, the design value. It is clear from Figure 3.6 that the dynamic aperture is more than five times this value in both directions, $A > \sqrt{5}$, for $\Delta p/p$ from –2% to +2%. During injection and acceleration, the momentum spread of the beam will be much less than $\pm 2\%$ until the bunch begins to be rotated immediately before extraction.

3.4. Magnet Errors and Corrections

A reliable estimate is possible for magnet field quality and alignment precision from experience gained in construction of the Accumulator. For those figures that were found to be difficult to achieve in the Accumulator, specifications will be relaxed so that the goals presented below are sound and realistic.

3.4.1. Field errors and their effects

3.4.1.1. Orbit distortions

For the Accumulator, it was possible to achieve the following alignment accuracies and dipole field uniformity [6]:

1) quadrupole misalignments: $\Delta_{rms} = 0.2$ mm in both directions.
2) dipole tilt: $\Delta'_{rms} = 0.2$ mrad; this will be relaxed to 0.5 mrad.
3) integrated dipole field uniformity: 2×10^{-4}; this will be relaxed to 5×10^{-4}.

With 90% probability and including the influence of higher harmonics [7], the expected closed orbit distortions are:

1) max $|x| = 74\ \Delta_{rms} = 14.8$ mm, max $|y| = 66\ \Delta_{rms} = 13.2$ mm.
2) max $|y| = 26\ \Delta'_{rms} = 13.0$ mm.
3) max $|x| = 29 \text{ m} \times (5 \times 10^{-4}) = 14.5$ mm.

Adding all in quadrature, one finds max $|x| = 21$ mm and max $|y| = 19$ mm.

3.4.1.2. Horizontal-vertical coupling

This will result from tilting of the quadrupoles. The figure achieved in the Accumulator is $\theta_{rms} = 0.2$ mrad. The coupling coefficient is defined by

$$k \equiv \frac{1}{4\pi}(2\theta_{rms})\sqrt{\sum \beta_x \beta_y (\frac{GL}{B\rho})^2} \tag{3.3}$$

where the summation is for all quadrupoles in the ring and (GL) is the integrated field gradient of each quadrupole. The amount of coupling in transverse amplitudes is

$$\frac{(2k)}{\sqrt{(2k)^2 + \Delta^2}} \tag{3.4}$$

where $\Delta \equiv$ fractional part of $(\nu_x - \nu_y)$. With $\theta_{rms} = 0.2$ mrad, $2k_{rms} = 0.0016$. If $\Delta = 0.05$, for example, the amount of amplitude coupling is 3.2% (rms). That is, if the horizontal excursion of the beam is 10 mm, the expected rms excursion of the beam in the vertical direction generated by the coupling is 0.3 mm. Correction of the closed orbit or other diagnostic procedures during commissioning will not be disturbed by this amount of coupling.

3.4.1.3. Systematic quadrupole effects

Natural chromaticity of the lattice can be found from the relations

$$\Delta\upsilon_x = 22.9\ \Delta_{QF} - 3.9\ \Delta_{QD}, \quad \Delta\upsilon_y = -5.3\ \Delta_{QF} + 23.7\ \Delta_{QD} \tag{3.5}$$

or, equivalently,

$$\Delta_{QF} = 0.0454\ \Delta\upsilon_x + 0.0074\ \Delta\upsilon_y, \quad \Delta_{QD} = 0.0101\ \Delta\upsilon_x + 0.0439\ \Delta\upsilon_y \tag{3.6}$$

where Δ_{QF} and Δ_{QD} are fractional changes in focusing and defocusing quadrupole gradients, respectively. Natural chromaticities are: $\xi_x = -(22.9 - 3.9) = -19$ and $\xi_y = -(-5.3 + 23.7) = -18.4$ for $\xi = \Delta\nu/(\Delta p/p)$. Focusing quadrupoles in arcs are important in keeping the dispersion in long straights small. If the third and the fifth quadrupoles (called QF3 in MAD input file) in all modules are changed by 0.5%, the dispersion in the long straights will be increased to 0.25 m while the same change in the first quadrupoles in all modules will cause an increase of 0.12 m. It may become necessary to depend entirely on quadrupoles in long straights to move the operating point to the optimum position. For quadrupoles in long straights alone, the natural chromaticity is given by

$$\Delta\upsilon_x = 5.0\ \Delta_{QF} - 0.68\ \Delta_{QD}, \quad \Delta\upsilon_y = -1.7\ \Delta_{QF} + 6.1\ \Delta_{QD} \tag{3.7}$$

or, equivalently,

$$\Delta_{QF} = 0.208\ \Delta\upsilon_x + 0.023\ \Delta\upsilon_y, \quad \Delta_{QD} = 0.057\Delta\upsilon_x + 0.170\Delta\upsilon_y \tag{3.8}$$

3.4.2. Specifications for magnet field quality

Specifications for magnet field quality will be similar to the ones adopted for the Accumulator magnets [8] but they will be relaxed somewhat since this machine is not a storage ring. It will not be necessary to have rotating-coil measurements of any magnet to find multipole contents. Rather, it is essential to make an accurate measurement of the integrated field covering both ends completely. For one thing, this will give a reliable value of the effective magnet length. Precise control of the integrated gradient is especially important for arc quadrupoles in order to achieve the desired phase advance per module.

The specification for magnet field quality will be in terms of "flatness", which is defined as

$$\text{flatness at } x = x_o \equiv \left| \frac{\int B_y(x_o, y = 0; z)\,dz}{\int B_y(x = y = 0; z)\,dz} - 1 \right| \tag{3.9}$$

for dipoles. Integrals are over the entire field of a magnet including the fringe field at both ends. For quadrupoles, the field gradient $G \equiv \partial B_y/\partial x$ is to be used in place of B_y. For large aperture magnets in the Accumulators, the specifications were, for dipoles: 2×10^{-4} at x = 3.75 inches; and for quadrupoles: 2×10^{-3} at x = 4 inches.

For the Proton Driver, the corresponding specifications are

$$\text{dipoles: } 5 \times 10^{-4} \text{ at } x = 4 \text{ in; quadrupoles: } 5 \times 10^{-3} \text{ at } x = 4 \text{ in.} \tag{3.10}$$

The consequence of these specification will be discussed in Section 3.4.3. It should be emphasized that these are NOT rms values but should be treated as the maximum allowed values for any magnet. If averaged over the magnet, they are equivalent to

$$\text{dipoles: } |B''/B_o| = 0.10 \text{ m}^{-2} \text{ if sextupoles alone,} \tag{3.11}$$
$$|B''''/B_o| = 110 \text{ m}^{-4} \text{ if decapoles alone;}$$
$$\text{quadrupoles: } |G''/G_o| = 1.0 \text{ m}^{-2} \text{ if octupoles alone,} \tag{3.12}$$
$$|G'''/G_o| = 29 \text{ m}^{-3} \text{ if dodecapoles alone.}$$

It is of course better to have a separate estimate of sextupole and decapole contents in dipoles, and of octupole and dodecapole contents in quadrupoles. It is doubtful, however that, unlike in storage rings, multipoles higher than octupole will play any significant role in this ring. In what follows, it will be assumed that the nonlinear field is all sextupole in dipoles and all octupole in quadrupoles. If needed, one can estimate these multipole contents separately by measuring the integrated bend field or the integrated gradient field at, say, x = ± 2 in, ± 3 in, and ± 4 in. and fitting data by least squares. Unlike rotating coil measurements, this procedure is not exact but it is more practical. If measurements show a significant amount of decapole content, it will be necessary to evaluate its impact on the momentum dependence and amplitude dependence of tunes. This is straightforward as the needed formulas already exist [9].

In bending magnets, sextupole field in the "body" can have both systematic and random components but it is mostly systematic at magnet ends. In quadrupole magnets, octupole field is expected to be predominantly random in the "body" and systematic at ends unless there is a breakdown of quadrupole symmetry common to all quadrupoles.

3.4.2.1. Magnet end field

It has been noticed in the design of muon storage rings at Fermilab as well as at CERN [10] that the dynamic aperture of the ring is reduced drastically when the quadrupole edge field is taken into account. On close inspection, it has been confirmed that this happens when the variation of the β function is abnormally steep. One obvious cure is to lengthen the magnet and reduce the β variation. For the Proton Driver lattice, the variation of β is everywhere normal and the edge field of quadrupoles or dipoles should not affect the dynamic aperture. Nevertheless, possible effects of field fall-off have been examined using the existing measurement data on the Accumulator large aperture quadrupoles and dipoles [11,12]. The study has confirmed that indeed the effects can be ignored as long as the flatness defined by Eq. (3.9) covers the entire magnet including both ends.

For accumulator quadrupoles and dipoles, the measured field fall-off can be expressed remarkably well by the function $f(z) = \dfrac{1}{1+\alpha(z+6)^p}$ where z (in inches) is the distance along the magnet axis measured from the endplate. This form is chosen such that f(z) = 1 at z = –6 in., that is, 6 in. into the magnet from the endplate. Two fitting parameters are

dipoles: $\alpha = 0.0134$, $p = 3.010$
quadrupoles: $\alpha = 1.955 \times 10^{-4}$, $p = 4.55$

It is interesting to note that the effective magnetic boundary is at z = –1″ for dipoles while it is at z = +1″ for quadrupoles. If one assumes that the body field is purely dipole or quadrupole, end field can be expressed as [11]

dipole $$B_y = B_o\{f_1(z) - \frac{1}{8} f_1''(z)(x^2+3y^2)\}, \qquad (3.13)$$

quadrupole $$\partial B_y/\partial x = G_o\{f_2(z) - \frac{1}{4} f_2''(z)(x^2+y^2)\}. \qquad (3.14)$$

Nonlinear components arising from the field fall-off are not pure sextupole, $B_y \propto (x^2-y^2)$, or pure octupole, $\partial B_y/\partial x \propto (x^2-y^2)$, but their effects on the particle motion are similar to those from pure multipoles.

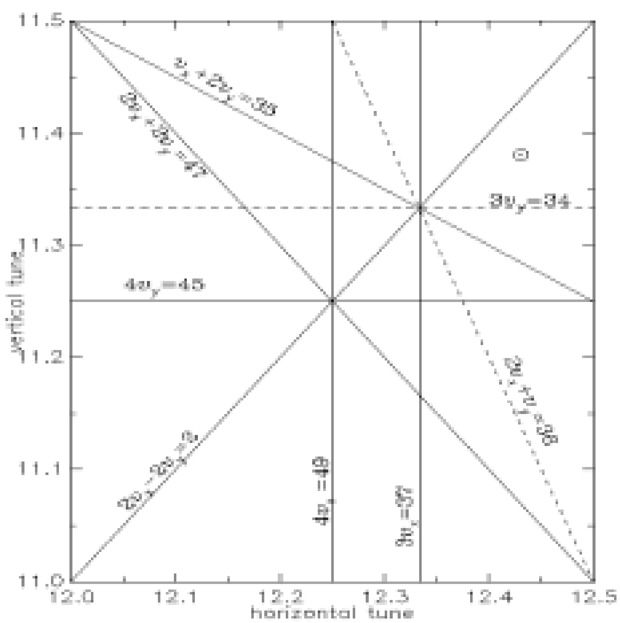
3.4.3. Resonances

In discussing possible resonances that may affect the beam, the major uncertainty is the amount of tune spread within the beam due to space charge. For a uniform charge distribution, the space charge tune shift varies little from particle to particle and the shift itself can easily be corrected by quadrupole trim windings. The tune spread is another matter. The core of the beam will experience the largest (negative) shift while the peripheral part will be affected mostly by the image force on the wall. If the normalized emittance stays constant, the real emittance goes down as $1/(\gamma\beta)$ and the tune shift caused by the self field will have a $1/(\gamma^2\beta)$ dependence on beam energy. For example, with $\Delta\upsilon = -0.30$ at injection, 400 MeV(KE), the tune shift due to self field will be down to less than –0.05 at 2 GeV(KE) and –0.01 at 5 GeV(KE). Regardless of what one does with the tune correction, tunes of the core part will always be the lowest. As the beam energy increases, tune of the entire beam will approach the value determined by quadrupole settings and, for $\Delta p/p \neq 0$, by the sextupole settings.

3.4.3.1. Tune diagram

Low order resonances in the area $\nu_x = 12.0 - 12.5$ and $\nu_y = 11.0 - 11.5$ are displayed in Figure 3.7 with the nominal working point at (12.43, 11.38). If the maximum amount of tune shift is assumed to be $\Delta\nu_x = -0.2$ and $\Delta\nu_y = -0.25$ at injection, the range of ν_x is from 12.23 to 12.43 and ν_y from 11.13 to 11.38. Resonances up to the fourth order experienced by some part of the beam will be

normal sextupole: $3\nu_x = 37$, $\nu_x + 2\nu_y = 35$;
skew sextupole: $3\nu_y = 34$, $2\nu_x + \nu_y = 36$;



normal octupole: $4\nu_x = 49$, $4\nu_y = 45$, $2\nu_x + 2\nu_y = 47$, $2\nu_x - 2\nu_y = 2$.

Figure 3.7. Nominal working point and low-order resonances; solid lines: normal sextupole and octupole; dashed lines: skew sextupole

Difference resonances, except for $2\nu_x - 2\nu_y = 2$, are generally considered unimportant. It is also unlikely that magnets would have a significant amount of skew octupole contents unless quadrupole tilting is large. It is to be noticed that, of these resonances, only two, $2\nu_x + \nu_y = 36$ and $4\nu_y = 45$, are structural. Furthermore, there is no reason to suspect that the skew sextupole field will be systematic so that $4\nu_y = 45$ is the only structural resonance for the present lattice with three-fold symmetry. The importance of the resonance $2\nu_x - 2\nu_y = 2$ is not well understood. It is known from simulations that resonance $2\nu_x - 2\nu_y = 0$ may cause some emittance growth in the presence of a strong space charge force. Tunes are therefore split by one unit. How much split in the fractional parts is needed is not known but 0.05 for the design values should be sufficient. Because of the negative tune shifts from space charge, the working point is chosen below the diagonal line.

3.4.3.2. Low order resonances

Half-integer resonances

It is important to see how close one can place the working point to the half-integer lines. The stopband width is proportional to the deviation of the integrated gradient of each magnet from its design value:

$$|\Delta\nu|_{rms} = 6 \times (\Delta K/K)_{rms}\,; \quad K \equiv (1/B\rho)\int G\,dz.$$

With $(\Delta K/K)_{rms} = 3 \times 10^{-3}$, which is achievable, the rms stopband width will be less than 0.02. The working point, which is 0.05 away from the half-integer lines, will therefore be quite safe. The nominal design values of the tune are (12.43, 11.38).

Sextupole components in dipoles: $|(B''/B_o)|_{av} = 0.10\ m^{-2}$. See Eq. (3.11).

Contributions to the chromaticity are

$$\Delta\xi_x = +0.24 \text{ and } \Delta\xi_y = -0.17 \text{ from all regular dipoles,}$$
$$\Delta\xi_x = -0.17 \text{ and } \Delta\xi_y = +0.12 \text{ from all short dipoles.}$$

These values are negligible compared with the natural chromaticities, which are approximately –20. See Eq. (3.5). Because of the phase advance per module of 270˚/270˚, there will be no contribution to any third-integer resonance driving terms from the average sextupole field in dipoles. Driving terms are cancelled in each arc separately so that the periodicity is immaterial. This is true for contributions from the chromaticity correcting sextupole magnets as well, resulting in a comfortable dynamic aperture.

Octupole components in quadrupoles: $|(G''/G_o)| = 1.0\ m^{-2}$. See Eq. (3.12).

There are three issues to be examined in the presence of octupole contents in quadrupoles: 1) momentum dependence of the tunes, 2) amplitude dependence of the tunes, and 3) fourth order resonances.

1) Tune *vs.* momentum [9]. From each quadrupole, the contribution is

$$\Delta\nu = \left(\frac{1}{8\pi}\right)\left(\frac{GL}{B\rho}\right)\left(\frac{G''}{G_0}\right)\beta x_c^2 \tag{3.15}$$

where $x_c \approx D_x(\Delta p/p)$ is the closed orbit for $(\Delta p/p)$. From this relation, by simply adding the contribution from each quadrupole, one finds for all quadrupoles combined

$(\Delta p/p)$	$\Delta\nu_x$	$\Delta\nu_y$
– 2.5%	0.019	0.001
+2.5%	0.015	0.001

This includes contributions from body and ends. The contribution from the ends alone is $\Delta\nu_x = 0.007$ for $(\Delta p/p) = 3\%$. Since this is not a storage ring, these values should be acceptable. It is possible that the specification given in Eq. (3.10) for quadrupoles is needlessly too relaxed. If one goes back to the original specification used for the Accumulator quadrupoles, that is, the maximum flatness at 4 in. = 2×10^{-3} instead of 5×10^{-3}, the momentum dependence will be reduced proportionately and $\Delta\nu$ will be less than 0.008 for all values of $(\Delta p/p)$.

2) Tune *vs.* amplitudes. For amplitudes $A = \sqrt{\beta\varepsilon}$, the dependences of the tune on amplitudes are proportional to [9]

$$\Delta\nu_x \propto (3/8)\beta_x^2\varepsilon_x - (3/4)\beta_x\,\beta_y\,\varepsilon_y\,,\ \ \Delta\nu_y \propto (3/8)\beta_y^2\varepsilon_y - (3/4)\beta_x\,\beta_y\,\varepsilon_x \tag{3.16}$$

The maximum value of $\Delta\nu$ with ε_x and ε_y = 120 π mm-mr, twice the beam emittance at injection, is less than 0.006. The contribution from the ends alone is less than 0.0006.

3) Fourth order resonances. When the beam energy reaches 5 GeV or so, the tunes will be close to the nominal design values, 12.43 and 11.38, and the working point is safely away from all fourth order resonances. This is important. Fourth order resonances have been found to be responsible for the extraction beam loss in ISIS [13]. Below that, because of the tune spread within the beam due to space charge, a structural resonance $4\nu_y = 45$ may exist as the phase advance for the arc, 270˚/270˚ per module, is the most unfavorable choice. The contributions from each magnet to the resonance driving term simply add up. Fortunately, the magnitude of the driving term is not excessive and the required strength for harmonic correction magnets is manageable. See Section 3.4.4.3. It is hoped that the effect of this resonance during acceleration is a minor emittance dilution and not a beam loss.

3.4.3.3. Magnet sorting

In Section 3.4.3.2, it has been emphasized that the average sextupole content in dipoles does not excite any third integer resonance because of the favorable phase advance per module. This will not be the case if the random sextupole content is dominant. It is, however, still possible to reduce substantially the resonance driving terms of $3\nu_x$ and $\nu_x + 2\nu_y$ simultaneously by a suitable arrangement of dipoles.

In each arc, there are twelve regular dipoles and four short dipoles. For both resonances, the relevant phase distance between the first and the seventh regular dipoles, between the second and the eighth, and so on, is almost exactly 180˚. The same phase relation is true for the first and the third, and the second and the fourth short dipoles. If two magnets with similar sextupole strengths are placed as a pair at these locations, the magnitude of the resonance driving term will be reduced substantially. In general, it is not possible to achieve a perfect cancellation but the arrangement will certainly avoid the possibility of unlucky combinations. If it is necessary, mixing regular and short dipoles in this sorting scheme is worth considering.

Sorting is not practical for suppressing the driving term of the $4\nu_y = 45$ resonance since all defocusing quadrupoles have the same phase.

3.4.4. Requirements for correction systems

Requirements for correction systems are based on the analyses of magnet errors presented in Section 3.4.1. Chromaticity correcting sextupoles are considered as essential items of a synchrotron and they are not included in the discussion of correction systems.

3.4.4.1. Steering magnets

In Section 3.4.1.1, the expected closed orbit distortion has been estimated to be approximately 20 mm in both directions. Most likely, this is an overestimate and there is a definite possibility that a closed orbit would be established without any use of steering magnets. Such an orbit with large excursions will, of course, be unsuitable for stable operation of the machine, and a correction system is essential for assuring the necessary dynamic aperture.

The specification common to horizontal and vertical directions is that the maximum kick angle of each steering element should be 5 mr. The maximum horizontal and vertical deflections generated by this kick will be 65 mm and 100 mm, respectively. For horizontal kick, there will be special windings in each dipole. The required kick angle is 3.6% of the bend angle of regular dipoles and 4.5% of short dipoles. Horizontal orbit correction is then possible up to the highest energy. Vertically, the kick will be provided by 25 cm-long steering magnets. With the maximum field of 0.26 T, the required kick angle of 5 mrad will be available up to 3 GeV (kinetic energy). If it is required to correct the orbit vertically beyond this energy, it will be necessary to realign some quadrupoles. Based on the BPM readings of the established closed orbit, it is possible to select the optimum combination of a specified number of quadrupoles for realignment.

One defect of the present lattice is that, unlike a lattice composed of regular FODO cells, the phase distance between two adjacent correctors can be uncomfortably large.

Horizontally, there is no dipole in Cell_b and the phase over this cell is 163˚. This can be reduced by installing in Cell_b a steering magnet similar to the vertical one but the reduction is not significant, 146˚ instead of 163˚. At other locations, the typical phase distance is 45˚. One alternative scheme is to depend on steering magnets alone located in the short straights of Cell_b and in the mini straights of Cell_c. There will be ten magnets in each arc, eight in Cell_b and two in Cell_c. Even with this arrangement, the largest phase distance between two adjacent kicks is 138˚.

There will be eight vertical correctors in an arc, four each in Cell_b and in Cell_c. The same defect exists in the vertical direction as well. In Cell_a, there is no space for installing correctors. In the middle of each arc, there are two consecutive Cell_a′s and the phase distance to cover them is 230˚. It is clear that the customary local three-bump algorithm will not work there and a more comprehensive means of straightening the orbit with more than three correctors may be required.

In long straights, three vertical correctors and four or five horizontal correctors are sufficient to cover the entire length.

3.4.4.2. Tune adjustment

Experience gained in Accumulator and Main Injector operation clearly indicates that if magnets are constructed carefully, it is not necessary to have the flexibility to explore a wide range of tune values. For the present machine, tune adjustment is needed primarily for three reasons: maintenance of the 270˚/270˚ phase advance per module in arcs, compensation of space charge detuning, which is expected to be less than 0.4 even for a peaked charge distribution of the bunch, and minimization of beam loss during extraction. It has been found in ISIS that precise control of the tune is essential to reduce the beam loss to less than one part in 10^4 during extraction when an orbit bump is introduced [13]. The bump moves the beam toward the stray field of the septum magnet and also far off center in some quadrupoles.

Tune adjustment will be done by special windings in each quadrupole. At least two power supplies in each arc, one for controlling all QFs and one for QDs, are desirable so that the phase advance in each arc can be adjusted independently. Similarly, each long straight should have two power supplies, although adjustment of the phase advance in P40, where rf cavities are located, is not important. Arc quadrupoles will be responsible for maintaining the phase advance per module in arcs while quadrupoles in long straights will be used to locate the working point at the optimum position in the tune diagram. It is not possible to predict where this optimum position will be a priori. It is, however, unlikely that tunes would have to be moved by more than 0.2 or 0.3 from the nominal design values.

3.4.4.3. Harmonic corrections: sextupoles and octupoles

As has been discussed in Section 3.4.3, all structural resonances are avoided except for one fourth order resonance, $4\nu_y = 45$. The possibility of being affected by nonstructural resonances depends primarily on the amount of tune spread within the beam. There are two third order and two fourth order resonances of this type: ($3\nu_x = 37$, $\nu_x+2\nu_y = 35$) and ($4\nu_x = 49$, $2\nu_x+2\nu_y = 47$). Nonstructural third order resonances are excited predominantly by random sextupole components in dipoles, and the sorting scheme to reduce their driving terms has been described in Section 3.4.3.3. If necessary, it is possible to install two families of sextupoles, which are identical to chromaticity correcting sextupoles but much weaker, for harmonic correction. This, however, should wait until there is a clear indication of such a resonance in the actual machine. In the Accumulator, one sextupole resonance of the type $\nu_x+2\nu_y$ has been found to affect the beam and a correction system has been installed to eliminate its harmful effect.

For fourth order resonances, magnet sorting is not available. One can estimate the required strength of correction octupoles from the expected octupole contents in quadrupoles. (See Section 3.4.3.2.) If six correctors are installed in two families at $\beta = 20$ m, for example, the required strength for each corrector is $(B'''\ell) \approx 85$ T/m^2. If $\ell = 0.3$ m, the poletip field at $r = 10$ cm is 500 G. It is unlikely that such a system will ever be needed. In any case, these correctors should be installed in long straights where dispersion is zero or very small so that the correction of the chromaticity will not be affected by them.

References

[1] J. Wei, Handbook of Accelerator Physics and Engineering, edited by A. Chao and M. Tigner, p. 281.

[2] S.Y. Lee, et al., Phys. Rev. **E48**(1993), 3040.

[3] U. Wienands, et al., Proc. 1992 HEACC (Hamburg), p. 1070.

[4] A. Piwinski, Handbook of Accelerator Physics and Engineering, p. 72.

[5] G. Rees, private communication.

[6] F. Mills, private communication.

[7] C. Bovet, et al., A SELECTION OF FORMULAE AND DATA USEFUL FOR THE DESIGN OF A.G. SYNCHROTRONS, CERN/MPS-SI/Int. DL/70/4. 23 April, 1970. p. 24.

[8] S. Ohnuma, $\bar{p}$ Note #346 (October 24, 1983).

[9] S. Ohnuma, $\bar{p}$ Note #324 (July 1983).

[10] E. Keil, private communication.

[11] S. Ohnuma, $\bar{p}$ Note #380 (April 18, 1984) and #380A (April 5, 1985).

[12] D. Harding, private communication.

[13] G. Rees, private communication.