MOMENTUM STACKING IN THE MAIN INJECTOR USING LONGITUDINAL BARRIERS

Longitudinal Barriers.

Potential function for rectangular voltage pulses expressed as a function of DT, time variable on the orbit synchronous rotation period $T_s$.

$$V(\Delta T)$$

The gradient of the potential is the effective force $eE$ volts per turn exerted on that part of the beam that is within a pulse period at any time.

$$H(\Delta E, \Delta T) = \left[ \frac{\eta}{2\beta^2 E_s} \right] \Delta E^2 - \left[ \frac{eV}{T_s} \right] \Delta T,$$

$$\frac{d(\Delta E)}{dt} = -\frac{\partial H}{\partial (\Delta T)} = \frac{eV}{T_s}, \quad \frac{d(\Delta T)}{dt} = \frac{\partial H}{\partial (\Delta E)} = \frac{\eta}{\beta^2 E_s} \Delta E.$$
disabled or de-Qed). Each batch is injected sufficiently far from a 'synchronous momentum' so that while the bunches de-bunch and the batch shears due to momentum spread, the entire batch moves sufficiently far in 66.67 ms so that another batch can be injected at or near the same azimuthal position on the next Booster cycle.

Longitudinal barriers are used to contain the injected batches in such a way that more than a full ring complement of batches can be injected while preserving clear space for continued injection. The barriers can also provide an arbitrary abort gap in the final distribution.

After injection of as many batches as possible (or desired), with the barriers containing the distribution, the injected beam is adiabatically captured into $h = 588$ (52.8 MHz), rf buckets.

If $\gamma$ in the MI is 21.8 at injection then $\eta = 0.00897$. For the batch to migrate 1.59 ms in 66.67 ms (simplest case), the minimum energy offset must be

$$\Delta E = \frac{B^2E_s \Delta T}{\eta T} = 23.85 \text{ MeV},$$

The energy (pc) acceptance of the Main Injector is assumed to exceed ±35 MeV, (" 0.39 %).

For this study we assume Booster longitudinal emittance is 0.07 eV-s per bunch; prior to extraction the Booster bucket height is reduced adiabatically to ±3 MeV, (i.e. bucket is nearly full). After extraction the bunches will shear and fill the 0.115 eV-s rectangle so there is an immediate minimum longitudinal dilution by a factor 1.64.

Plan 1. The simplest (and worst) method of barrier stacking.
Inject each Booster batch with minimum energy offset 24 MeV. The batches will shear and move such that successive batches can be injected at the same azimuth (with respect to some 'synchronous' point).

Place longitudinal barrier pulses such that the drifting batches are reflected as shown here.

After injection of ten booster batches the entire ensemble is adiabatically captured into $h = 588$, (52.8 MHz) rf buckets.

The energy spread of the captured protons is 60 Mev so the minimum longitudinal emittance of each recaptured bunch will be $S = (6 \times 10^7)(18.9 \times 10^9) = 1.13$ eV-s.

This is too large to represent an acceptable scenario.

The voltage required to generate the 32 MeV barriers, 7150 volts, is impractically large.

(The recycler program is proposing to use barrier pulses at 2000 volts.)

The not quite so bad method.
Reduce the barrier height to 23.7 MeV and add two additional barriers spaced by one Booster batch extent, 1.59 ms.

Inject just above the gap between the inside barriers. During one Booster cycle the injected batch will migrate along the flow lines and out of the injection area.

The energy spread of the injected batches is reduced to 30.7 MeV so the minimum longitudinal emittance of adiabatically captured bunches is reduced to 0.58 eV-s.

At reduced energy the slewing rate is reduced. More batches can be injected.
After injection of twelve batches, two of the barrier pulses can be switched off and an arbitrary abort gap preserved by the remaining two.

If the start of adiabatic capture is delayed for several booster cycle times the distribution will become more uniform but bunches will still be too large and 'hollow'.

The voltages indicated for barrier generation are still a bit too large. These could be reduced by half by making the pulses twice as long without adversely affecting the entire scenario.

Plan 3. The even better method.
The Booster can inject at selected times along the 'synchronous' orbit in the MI. In this plan we inject with minimum energy offset 12 MeV so that the minimum offset particles in each batch migrate by half a period, 0.795 ms, during each Booster cycle. The next Booster injection is placed just ahead of the migrated batch. In this way one should be able to inject twelve or maybe thirteen Booster batches.

Just adjacent to and following the injection point there is a 'moving barrier' that moves to earlier time (left), 0.795 ms during the following Booster acceleration cycle so that it ends up just adjacent to the next injected batch. This barrier is about 15 MeV so that it allows about half of the injected protons over the top (right), and reflects the other half backward (left). In a sense this amounts to 'phase displacement acceleration'. Because the barrier and the injected batch are moving through each other, slightly more voltage is required than indicated by the barrier height expression. (This is like a 'moving bucket factor'.)

There are also 8 MeV barriers (as shown), at the ends of the synchronous period.
The following two confusing figures are supposed to show the paths of particles starting at each end of the first injected batch, during the ensuing three Booster periods. No additional injected Booster batches are shown to avoid total confusion.
The moving barrier remains on during the next Booster cycle (~ 6000 MI turns) and it moves to earlier time exactly one-half of a Booster period during the next cycle. About half of the injected particles are inside of the moving barrier. They are accelerated to about 6.5 MeV positive energy. Note that at +6.5 MeV they do not move to earlier time as rapidly as the moving barrier, so the end of the barrier overtakes them. The remaining (outside) particles are also accelerated. The particles at -18 MeV move to about -6.7 MeV while under the influence of the moving barrier. Subsequent injected batches are deposited nearly symmetrically above and below the synchronous energy with full energy spread ~13.2 MeV. The moving barrier has outrun all of these particles so that their energy distribution will not change with time. The barrier at the left end of the MI rotation period can be adjusted in position to allow an abort gap.

At this point the 'leading' pulse cannot be moved further without moving into and decelerating the protons at the other end of the distribution. However, it can be 'stopped' and held in place. The 'following' pulse can be moved normally to accelerate what has just been injected. It will 'cancel' the leading pulse as it moves into it. The 'leading' pulse can be made narrower if the amplitude is adjusted such that the area remains constant. But the number of batches is limited by this pulse length problem. If this scenario is sufficiently attractive it can be improved with money by using narrower and correspondingly larger pulses for the moving barriers.
Just after injection of the twelfth batch. (The barrier pulses are now 6060 V and 0.5 ms.) Energy spread is still about 6.7 MeV.

The minimum longitudinal emittance per bunch that can be expected to result from adiabatic capture of this ensemble is the product of the energy spread and the rf period. Alternatively, just divide the energy spread by the rf frequency. If the rf frequency is to be 52.8130 MHz then for this case $S_{\text{min}} \sim 0.25 \text{ eV-s}$. Adiabatic capture into 53 MHz buckets will increase the energy spread by nearly a factor of two. Still well within the energy acceptance of the MI.

In this situation the rf frequency and/or harmonic number might be changed to suit some other program. If this were to be done, the azimuthal part of the ring occupied by beam could be reduced prior to adiabatic capture, by moving the end barriers together (slowly). This, of course, would result in an increase in momentum spread, but there is plenty of acceptance left.

Alternatively, the harmonic number of the MI could be unchanged but the Booster could have been operated at a different (probably lower) harmonic number.

Conclusion:

Using moving longitudinal barriers generated by microsecond pulses from 2.5 to 5 kV (admittedly a bit difficult) it appears feasible to inject up to 12 Booster batches into the MI with only moderate growth in longitudinal emittance.

If the booster can reach $4 \times 10^{12}$ protons per cycle with $\leq 0.09 \text{ eV-s}$ per bunch, this implies acceleration up to $6 \times 10^{13}$ protons per cycle in the MI with reasonable longitudinal emittance.