From what we know now, we may have a coherent picture to explain the beam loss observed at the early stage of the cycle (0-3 ms, about 30% loss) when the Booster operates at high intensities.

- **Longitudinal loss:**
  - The Booster momentum acceptance is small (~ ±0.15-0.2%). It is about the same as the linac beam momentum spread (±0.13%).
  - When the RF is up and the beam is being bunched, the momentum spread will increase and exceeds the acceptance, resulting in loss.

- **Transverse loss:**
  - The good field region of the magnets is small (~ ±1 inch).
  - Due to edge focusing of the orbit bumps and doglegs, the beta-functions and dispersion are larger than the design value. This reduces the maximum allowable beam emittance to about only ~8 π, while the linac beam emittance is about 7 π.
  - The situation is worsened by the space charge. It blows up the emittance during multi-turn injection. The beam is scraped transversely, resulting in loss.
• **The situation improves rapidly when energy** $E \uparrow$:
  o **Longitudinal:**
    - $\Delta E/E \downarrow$
    - $1/\beta^2 \downarrow$
    - $\Delta p/p = (1/\beta^2) \times \Delta E/E \downarrow \downarrow$
  o **Transverse:**
    - Dogleg focusing strength: $1/f = \theta^2 / 2L \propto 1/p^2 \downarrow \downarrow$
    - Beam size due to adiabatic damping: $\varepsilon = \varepsilon_N/\beta\gamma \downarrow$
    - Space charge effect $\propto 1/\beta\gamma^2 \downarrow \downarrow$
  o Therefore, the acceptance limit will be lifted when energy goes up.
  o The loss in the middle and late stage of the cycle is due to some other mechanisms (transition crossing, head-tail instability, coupled bunch instability, etc.).

• **Fast and slow loss:**
  o When the acceptance limit comes from physical aperture, the loss is instantaneous.
  o When it comes from dynamic aperture (poor field quality), the loss is slow.

• **Additional info needed in this study:**
  o Measurement - *Transverse acceptance* at 24 long and 24 short straights (first need to remove the r.m.s. current limit of the steering magnets)
  o Simulation - *Dynamic aperture* tracking with body and chromaticity sextupoles
  o Simulation - *ESME* for dynamic process with RF on (moving bucket)
• **Possible measures to reduce losses:**
  o To enlarge aperture:
    ▪ RF: from 2-1/4 inch to 5 inch *(an action item)*
    ▪ Kicker: (?)
  o To compensate dogleg effects:
    ▪ Increase distances between doglegs *(Sasha)*
    ▪ Relocate DOG13 *(global correction)* *(Weiren)*
    ▪ Local correction using wedge magnets *(Weiren)*
    ▪ Local correction using quads *(Chuck, Weiren)*
    ▪ Correction using solenoid? *(Francois)*
  o To reduce space charge effects:
    ▪ Painting
      ➢ Using falling side of the orbit bump pulse
      ➢ Using falling or rising side of $B(t)$
    ▪ Trade-off between linac current and injection turns
    ▪ Install inductive inserts *(an action item)*
    ▪ Install a 2nd harmonic RF system *(expensive!)*
  o To chop the beam at 38 MHz:
    ▪ Laser chopping at 750 keV *(Ray, Xi)*
Longitudinal Calculation before and after Bunching

• Before bunching:
  Beam momentum spread:
  \[ \Delta p/p = \pm 0.13\% \text{ (measured)} \]
  Beam energy spread:
  \[ \frac{\Delta E}{E} = \beta^2 \times \frac{\Delta p}{p} = \pm 0.066\% \quad (\beta = 0.713) \]
  \[ E = 1.338 \text{ GeV} \]
  \[ \Delta E = 1.77 \text{ MeV} = \pm 0.88 \text{ MeV} \]
  Beam length (1/84 of the circumference):
  \[ L_b = 26.4 \text{ ns} \]
  Beam emittance (1/84 of the total injected beam):
  \[ \varepsilon_L = \Delta E \times L_b = 0.047 \text{ eV-s} \]

• After bunching:
  RF voltage: \[ V_{rf} = 800 \text{ kV} \]
  \[ \text{(console RF curve)} \]
  RF frequency: \[ f_{rf} = 37.87 \text{ MHz} \]
  Revolution frequency: \[ f_0 = 450.8 \text{ kHz} \]
  Harmonic number: \[ h = 84 \]
  Average machine radius: \[ R = 75.47 \text{ m} \]
  Slip factor: \[ \eta = 0.458 \]
  Bunch emittance: \[ \varepsilon_L = 0.047 \text{ eV-s} \]
  \[ (\text{assuming no blowup during rf capture!}) \]
Bunch length:

\[ L_b = \left\{ \beta \varepsilon_L \frac{c^2}{f_0} \right\}^{1/2} \times \left\{ \frac{\eta}{2\pi \hbar E V_{\text{rf}}} \right\}^{1/4} \]

= 11.5 ns

Bunch energy spread:

\[ \Delta E = 4.1 \text{ MeV} = \pm 2.05 \text{ MeV} \]

\[ \Delta E/E = \pm 0.15\% \]

Bunch momentum spread:

\[ \Delta p/p = \left(1/\beta^2\right) \times \Delta E/E = \pm 0.3\% \]

→ More than a factor of 2 increase in \( \Delta p/p \) due to bunching (even without assuming any emittance dilution). This value exceeds the momentum acceptance of the Booster (\( \sim \pm 0.15-0.2\% \)), resulting in beam loss.

- **Bonus results:**

Bucket area:

\[ A_{\text{rf}} = (8R/c) \left\{ 2E V_{\text{rf}} / \pi \hbar^3 \eta \right\}^{1/2} = 0.1 \text{ eV-s} \]

Bucket height:

\[ H_{\text{rf}} = \pm A_{\text{rf}} \times \frac{2\pi f_{\text{rf}}}{8} = \pm 3 \text{ MeV} \]

Synchrotron tune:

\[ \nu_s = \frac{f_s}{f_0} = \left\{ \frac{\hbar \eta V_{\text{rf}}}{2\pi \beta^2 E} \right\}^{1/2} = 0.085 \]

Synchrotron period:

\[ 1/\nu_s = 12 \text{ turns} \]

\[ i.e., \text{ 3 turns for 1/4 rotation.} \]
Transverse Calculation

- Acceptance is reduced due to orbit bumps and doglegs:
  \[ A = \{\beta_{\text{max}} \times \varepsilon_N/\beta\gamma\}^{-1/2} + D_{\text{max}} \times \Delta p/p + \text{c.o.d.} \]

At injection (400 MeV):
- Good field region (horizontal) = ± 1.2 inch (TM-405)
- \( \beta\gamma = 1.0 \)
- \( \Delta p/p = \pm 0.13\% \) (measured)
- c.o.d. = closed orbit distortion = 3 mm (optimal)

Without orbit bumps and doglegs:
- \( \beta(x)_{\text{max}} = 33.7 \text{ m}, \ D_{\text{max}} = 3.19 \text{ m} \)
- Max allowable beam emittance: \( \varepsilon_N(x) = 16 \pi \text{ mm-mrad} \)

With orbit bumps and doglegs:
- \( \beta(x)_{\text{max}} = 48.7 \text{ m}, \ D_{\text{max}} = 6.84 \text{ m} \)
- Max allowable beam emittance: \( \varepsilon_N(x) = 8 \pi \text{ mm-mrad} \)
- → a factor of 2 reduction in acceptance due to large \( \beta \) and \( D \)!

- Beam emittance is increased during multi-turn injection due to space charge:
  - Linac beam emittance is about 7 \( \pi \) (Elliott, PD2 report, Ch. 8)
  - At 43 mA \( \times \) 10 turns, the emittance is blown up by about 50\% (Ray's IPM data)

- Put these two effects together, the beam is scraped in transverse plane, resulting in beam loss.